AC Modeling of Power Stage in Flyback Converter

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ABSTRACT

This application report provides a step-by-step procedure for constructing the AC model for the power stage of a Flyback DC/DC Converter in Continuous Conduction Mode. The model is used in the TIna simulator to plot and verify the control-to-output transfer function.

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1 Introduction

The flyback DC-DC converter is derived from the basic buck-boost converter. The conversion ratio for this converter is given by,

$$\frac{Vout}{Vin} = n.\frac{D}{1-D}$$

As seen from this equation, the conversion ratio is similar to that of the buck-boost converter with an included factor of *n*, which is the turns ratio of the flyback transformer.

1.1 How it Works

In a flyback converter, when the current ramps up in the primary, energy is built up in the transformer core. When the FET switches off, this energy is dumped into the secondary, allowing current to flow in the secondary, so the primary current collapses immediately to zero. This causes an immediate rise in current in the secondary, after which, the current in the secondary ramps down linearly as it discharges, supplying energy to the load. The current in the secondary when the FET switches off, is proportional to the peak in the primary, and is determined by the turns ratio.

The circuit topology of the flyback converter is as shown in Figure 1.

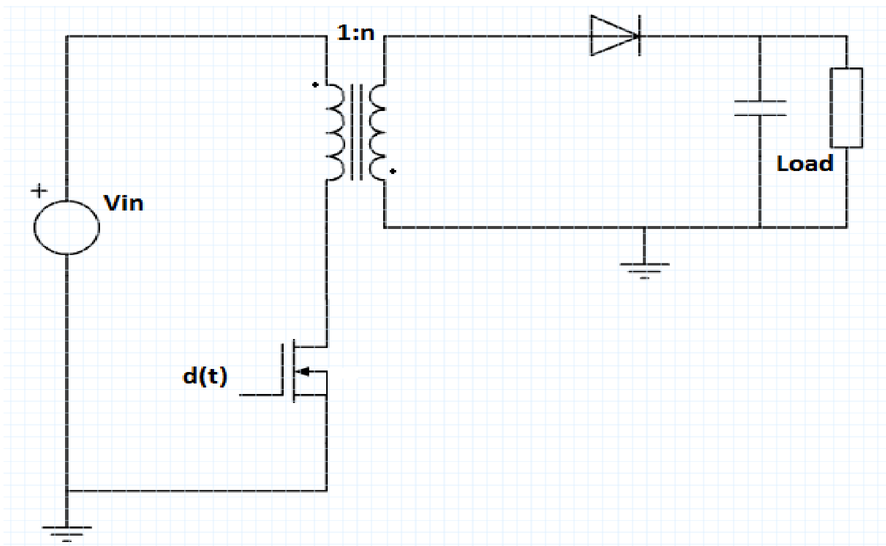


Figure 1. Basic Topology of a Flyback Converter

2 Circuit Manipulations

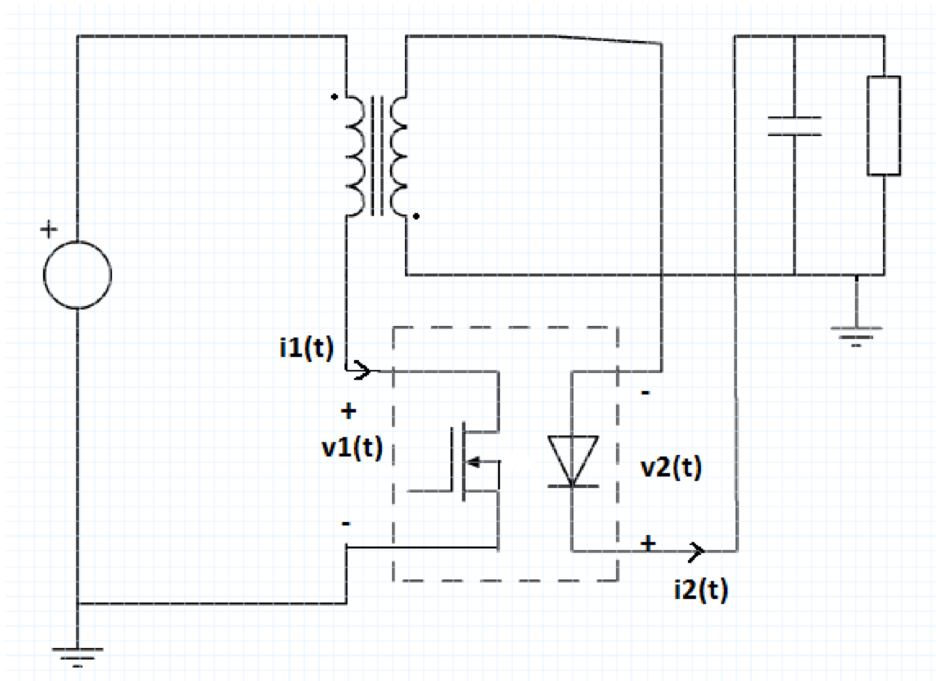
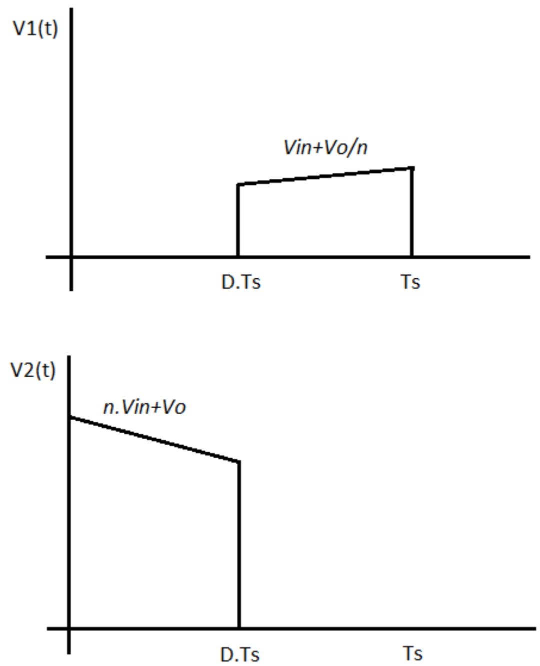


Figure 2. Separating Out the Non-Linear Switch Elements

The switching elements (MOSFET and Diode) are shown in the dashed rectangular box. These are non-linear switching elements. The box is treated as a 2-port network with input voltage $v_1(t)$, input current $i_1(t)$, output voltage $v_2(t)$, and output current $i_2(t)$.

The waveforms for $v_1(t)$ and $v_2(t)$ over one switching period, T_s, are shown in Figure 3. A simplifying assumption of zero MOSFET and diode conduction drop has been made for obtaining the waveform shown below.



Note: The slope of the voltage waveform in the figure is exaggerated for the sake of clarity.

Figure 3. Voltage Waveforms for the 2-Port Network

Similarly, the waveforms for $i_1(t)$ and $i_2(t)$ are shown in Figure 4.

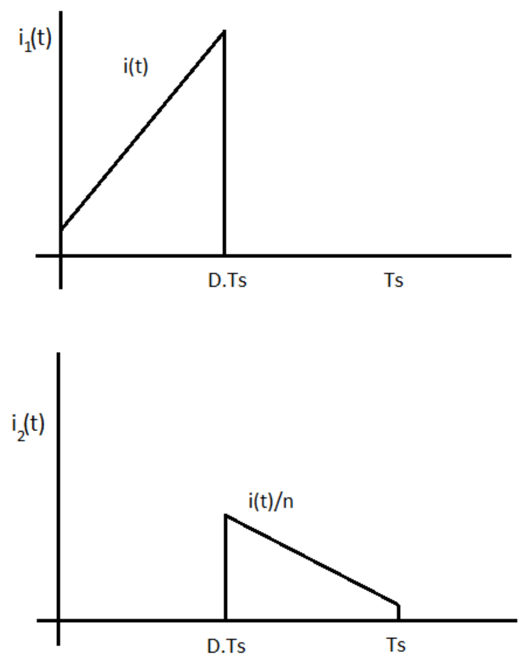


Figure 4. Current Waveforms for the 2-Port Network

3 Construction of the Mathematical Model

The waveforms in Figure 3 and Figure 4 are averaged over one switching period, which effectively removes the switching harmonics. The switching harmonics have no role of consequence in the development of the AC model. Observing the waveform, it can be seen that:

$$< \boldsymbol{V_1(t)} >_{Ts} = \boldsymbol{d'(t)} * \{\boldsymbol{V_{in}} + \boldsymbol{Vo}/\boldsymbol{n}\}$$

$$< \boldsymbol{V_2(t)} >_{Ts} = \boldsymbol{d(t)} * \{\boldsymbol{nV_{in}} + \boldsymbol{Vo}\}$$

Dividing the above equations, we get the following relation:

$$< \boldsymbol{V_1(t)} >_{Ts} \Big/ < \boldsymbol{V_2(t)} >_{Ts} = \frac{\boldsymbol{1}}{\boldsymbol{n}} * \frac{\boldsymbol{d'(t)}}{\boldsymbol{d(t)}}$$

Hence, we have:

$$< \boldsymbol{V_1(t)} >_{Ts} = \frac{\boldsymbol{1}}{\boldsymbol{n}} * \frac{\boldsymbol{d'(t)}}{\boldsymbol{d(t)}} *< \boldsymbol{V_2(t)} >_{Ts} \quad (1)$$

Equation 1 is a non-linear one, since it contains products of time-dependent variables. To obtain a linear relation, the variables are perturbed about the quiescent operating point.

$$\boldsymbol{d(t)} = D + \hat{d}$$

$$< \boldsymbol{V_1(t)} >_{Ts} = V_1 + \widehat{v_1}$$

$$< \boldsymbol{V_2(t)} >_{Ts} = V_2 + \widehat{v_2}$$

Using the preceding equations in (1):

$$\boldsymbol{V_1} + \widehat{\boldsymbol{v_1}} = \frac{\boldsymbol{1}}{\boldsymbol{n}} * \frac{D' - \hat{d}}{D + \hat{d}} * (\boldsymbol{V_2} + \widehat{\boldsymbol{v_2}})$$

$$\boldsymbol{V_1} * \boldsymbol{D} + \boldsymbol{V_1} * \widehat{\boldsymbol{d}} + \widehat{\boldsymbol{v_1}} * \boldsymbol{D} + \widehat{\boldsymbol{v_1}} * \widehat{\boldsymbol{d}} = \frac{\boldsymbol{1}}{\boldsymbol{n}} * \{\boldsymbol{V_2} * \boldsymbol{D'} - \boldsymbol{V_2} * \widehat{\boldsymbol{d}} + \widehat{\boldsymbol{v_2}} * \boldsymbol{D'} - \widehat{\boldsymbol{v_2}} * \widehat{\boldsymbol{d}}\}$$

Since the perturbations are small, the non-linear terms $\widehat{\boldsymbol{v_1}} * \widehat{\boldsymbol{d}}$ and $\widehat{\boldsymbol{v_2}} * \widehat{\boldsymbol{d}}$ are small enough to be ignored. Upon rearranging and grouping terms appropriately:

$$(\boldsymbol{V_1} + \widehat{\boldsymbol{v_1}})\boldsymbol{D} + \boldsymbol{V_1} * \widehat{\boldsymbol{d}} = \frac{\boldsymbol{1}}{\boldsymbol{n}}[(\boldsymbol{V_2} + \widehat{\boldsymbol{v_2}}) * \boldsymbol{D'} - \boldsymbol{V_2} * \widehat{\boldsymbol{d}}\,] \quad (2)$$

The equation is re-arranged into the below form,

$$\boldsymbol{V_1} + \widehat{\boldsymbol{v_1}} = \frac{\boldsymbol{1}}{\boldsymbol{nD}}[(\boldsymbol{V_2} + \widehat{\boldsymbol{v_2}})\boldsymbol{D'} - \widehat{\boldsymbol{d}}(\boldsymbol{V_2} + \boldsymbol{nV_1})] \quad (3)$$

Note: Equating the quiescent terms in Equation 2, we have $\boldsymbol{V_1} * D = \frac{\boldsymbol{1}}{\boldsymbol{n}}\boldsymbol{V_2} * \boldsymbol{D'}$.

Substituting this relation for $\boldsymbol{V_2}$ in Equation 3,

$$\boldsymbol{V_1} + \widehat{\boldsymbol{v_1}} = \frac{\boldsymbol{1}}{\boldsymbol{nD}}[(\boldsymbol{V_2} + \widehat{\boldsymbol{v_2}})\boldsymbol{D'} - \widehat{\boldsymbol{d}}(\frac{\boldsymbol{nD}}{\boldsymbol{D'}}\boldsymbol{V_1} + \boldsymbol{nV_1})]$$

$$V_1 + \widehat{v_1} = \frac{1}{n}[(V_2+\widehat{v_2})\frac{D\prime}{D} - \widehat{d}/_D\,(\,(\frac{D+D'}{D'})nV_1)]$$

Now, **D+D’=1**.

So the above equation finally reduces to:

$$V_1 + \widehat{v_1} = \frac{1}{n}[(V_2+\widehat{v_2})\frac{D\prime}{D} - \frac{\widehat{d}}{DD\prime} * nV_1)] \tag{A}$$

Using the same procedure for the 2-port network currents $i_1(t)$ and $i_2(t)$, Equation B is obtained:

$$I_2 + \widehat{i_2} = \frac{1}{n}[(I_1+\widehat{i_1})\frac{D\prime}{D} - \frac{\widehat{d}}{DD\prime} * nI_2)] \tag{B}$$

Equations (A) and (B) are the relations for the input voltage and output current of the 2-port network and are used to obtain the linear model for the averaged switch network.

These relations can be implemented by a network consisting of an ideal transformer and an independent voltage and current source, both driven by duty cycle variations, d(t), as shown in Figure 5.

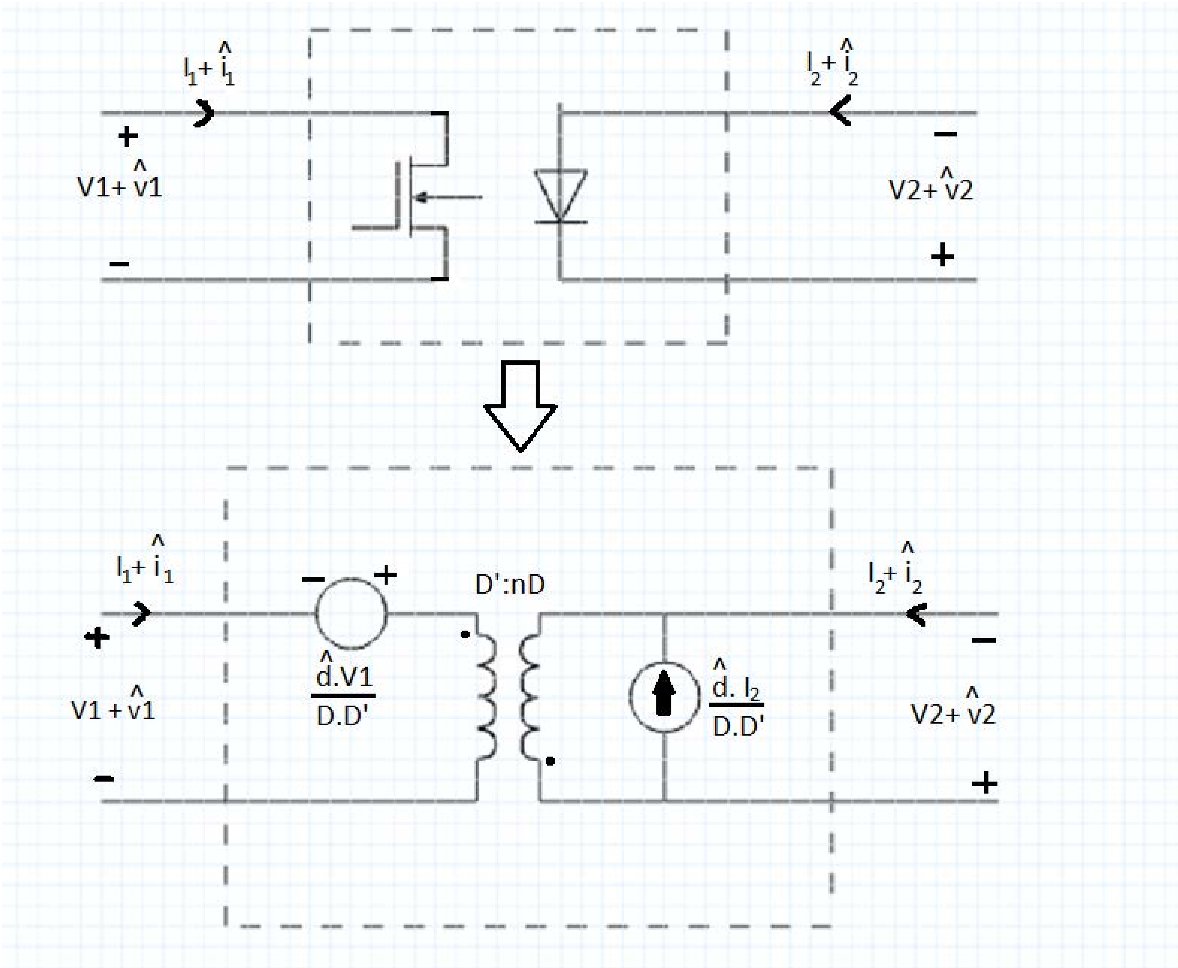


Figure 5. Replacing the Non-Linear Switch Network with an Equivalent Linear Network

The linear model for the switching element thus obtained is used in place of the MOSFET and diode in the rectangular dashed box of Figure 2 to obtain the circuit shown in Figure 6.

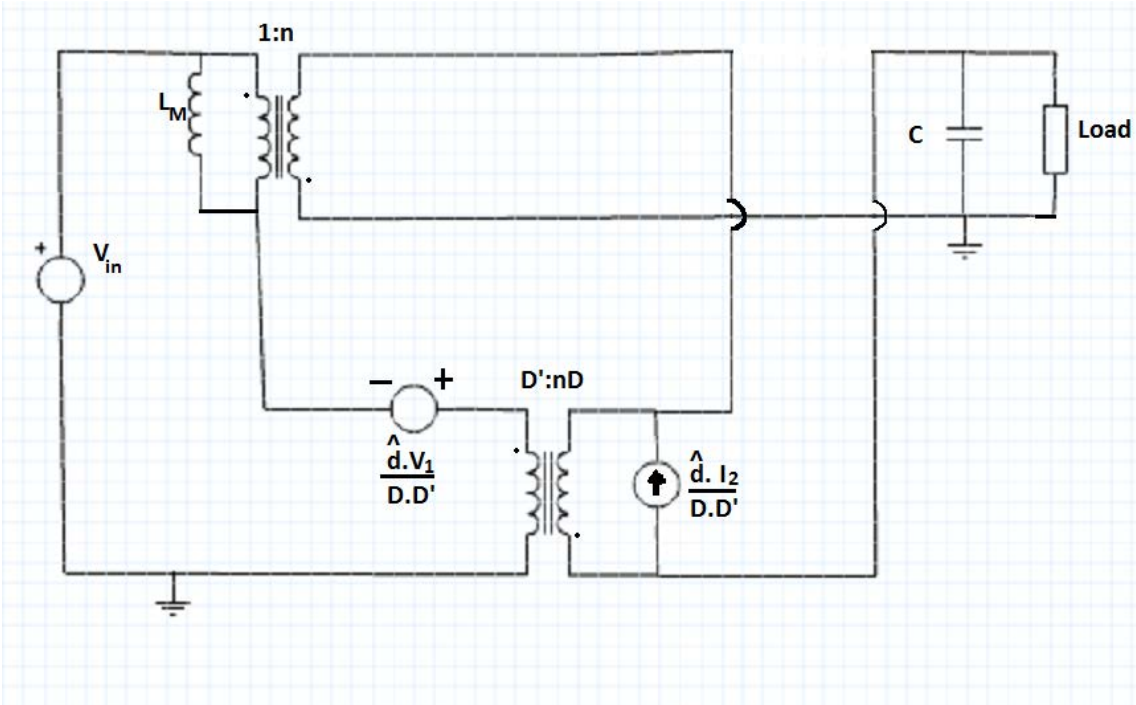


Figure 6. Complete Linear Circuit for Flyback Converter

The inductance L_M indicates the magnetizing inductance, and is used with an ideal transformer of turns ratio 1:n.

4 Simulation and Results

Example:
Consider a flyback converter designed for the following specifications: $V_{in} = 5V,\ V_{out} = 10V$. A load resistance of 10 Ω is connected at the output, so the load current drawn is 1 A. The magnetizing inductance of the flyback transformer is 6 µH, and load capacitance used is 500 µF.

The turns ratio n=4, hence, the duty cycle, D, required to produce a 10-V output is D = 0.33.

The quiescent value for v_1 (t), is $V_1 = D'.(V_{in} + V_o/n)$, as seen from waveform.

Similarly, $I_2 = I_{load}$

Hence, V_1= 4.95 V.

Also, I_2= 1 A.

Now, for modeling in TIna, the voltage and current sources controlled by d(t) are implemented using the VCVS and VCCS sources respectively, as shown in Figure 7.

The gain of the VCVS source is set to be $\frac{V_1}{D.D\prime}$ (since the voltage source obtained in the derived ac model is $\frac{V_1}{D.D\prime} * \hat{d}$)

so, the gain of VCVS source is 22.31.

The gain of VCCS source must be $\frac{I_2}{D.D\prime}$ (again, according to derived model).

The gain of VCCS source is calculated to be 4.5.

The turns ratio of the transformer (of the switch model) is nD/D' = 2.

Verification:

For a flyback converter, the location of RHPZ and LC resonant pole, is at[(1)]:

$$f_{RHPZ} = {(R_o * D'^2)}\Big/{(n^2 * 2\pi * L_M * D)}$$

$$f_{LC} = {D'}\Big/{(n * 2\pi * \sqrt{L_M C})}$$

Also, the gain at DC is, $G_{d0} = V_0/DD'$

Using the example values in the above equations, we should have the following:

f_{RHPZ} = 21.88 kHz

$$f_{LC} = 883.3Hz$$

G_{d0} = 45.09

$$G_{d0}|dB = 33.2dB$$

The circuit is constructed in TIna Simulator as shown in Figure 7.

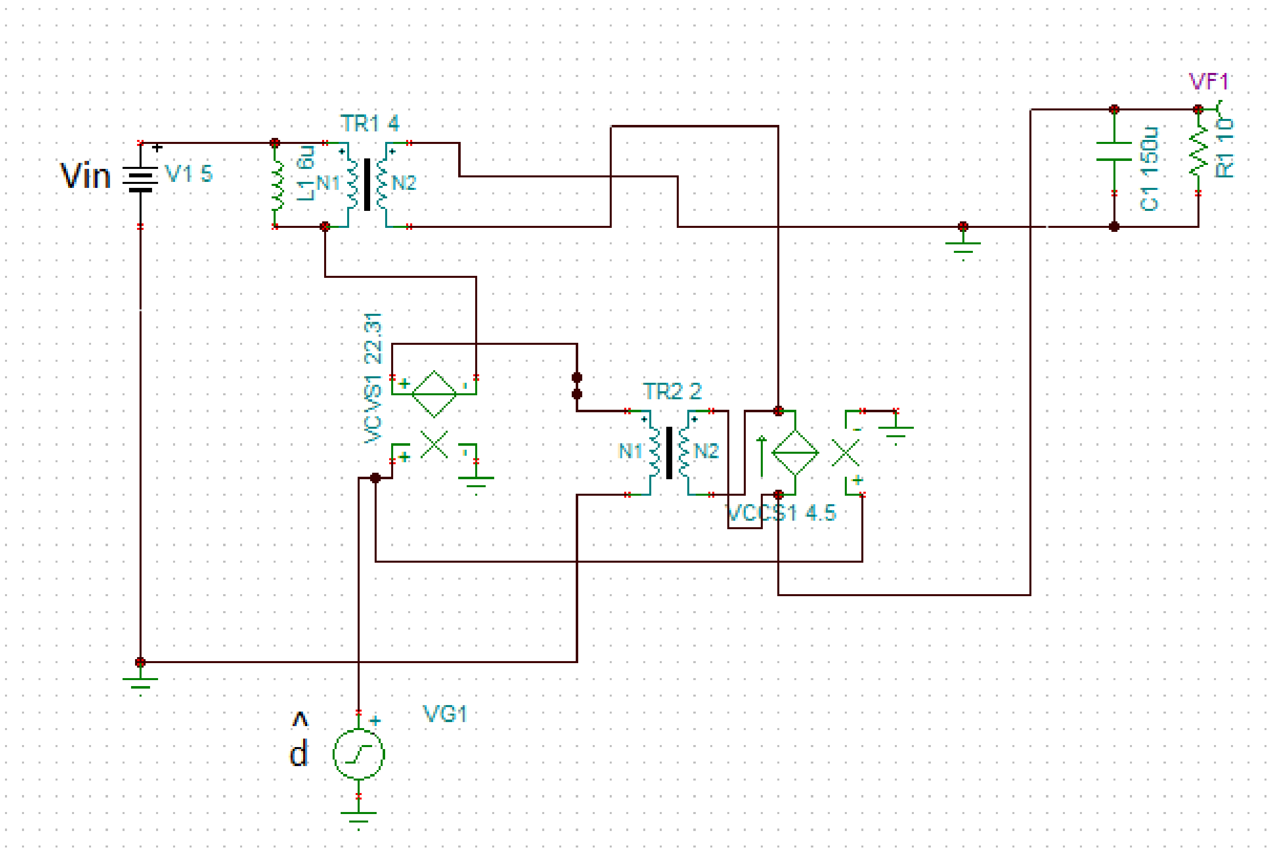


Figure 7. Implementation of the Example Case in TIna Simulator

Running the AC analysis, the bode plot for the control-to-output transfer function is obtained as shown in Figure 8.

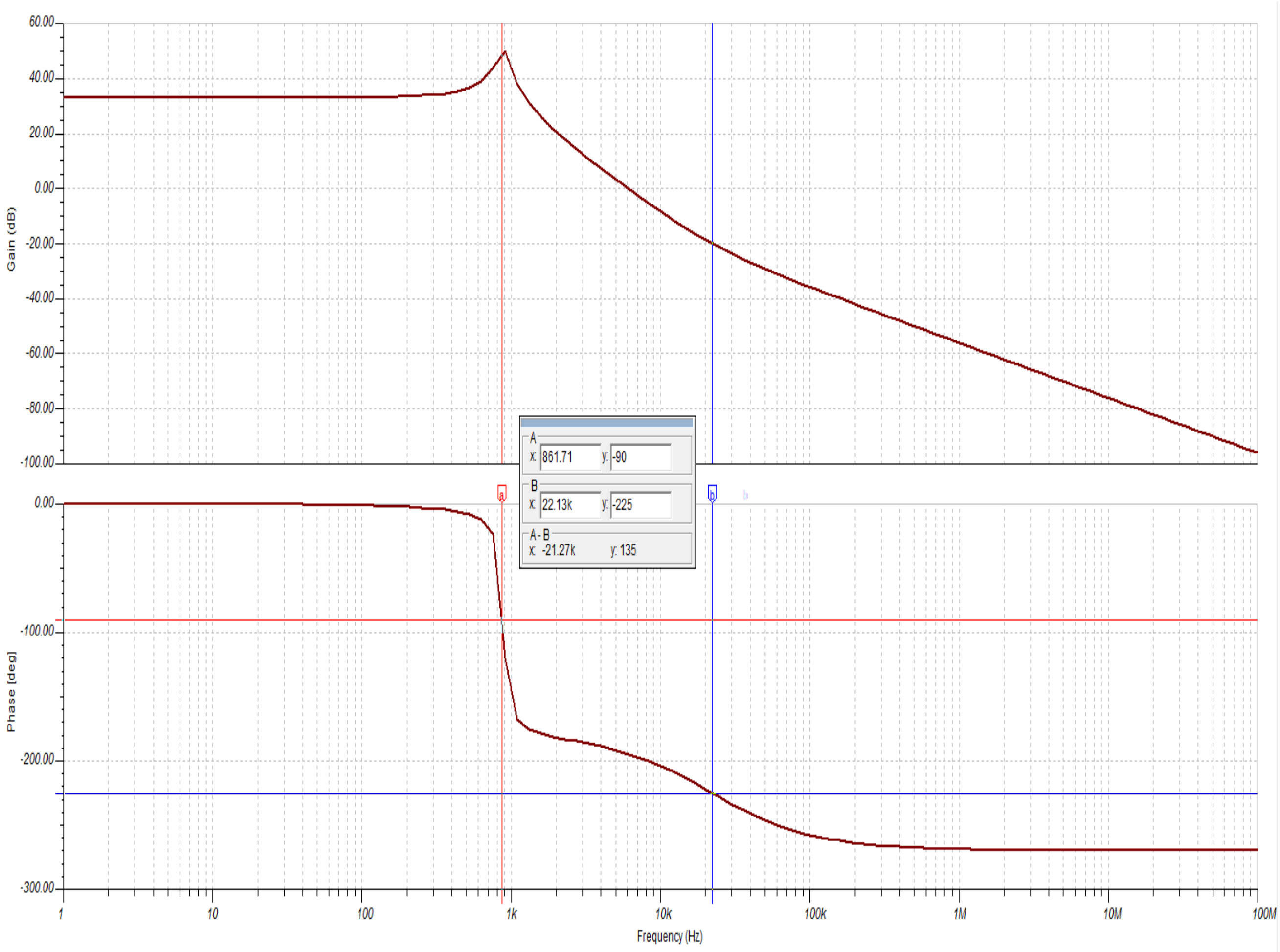


Figure 8. Control-to-Output Transfer Function Plotted using TIna

As seen from Figure 8, the LC resonant pole appears at around 862 Hz.

The RHPZ is located at around 22.13 kHz.

Gain at DC is 33 dB. Hence, it is seen that the response obtained from the model matches closely with the theoretical calculations.

5 Conclusion

The AC model of the power stage for a flyback converter can be constructed following the steps shown in this application report. The response of the model, implemented in a TIna Simulator, is found to be in alignment with what is theoretically expected.

6 References

1. Basso, C., "How to Keep a FLYBACK Switch Mode Supply Stable with a Critical-Mode Controller," ON Semiconductor™, *AN1681/D Application Note*, September 2000 – Rev. 0

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