

Operational amplifier gain stability, Part 1: General system analysis

By Miroslav Oljaca, *Senior Applications Engineer*,
and Henry Surtihadi, *Analog Design Engineer*

Introduction

The goal of this three-part series of articles is to provide a more in-depth understanding of gain error and how it can be influenced by the actual parameters of an operational amplifier (op amp) in a typical closed-loop configuration. This first article explores general feedback control system analysis and synthesis as they apply to first-order transfer functions. This analysis technique is then used to calculate the transfer functions of both noninverting and inverting op amp circuits. The second article will focus on DC gain error, which is primarily caused by the finite open-loop gain of the op amp as well as its temperature dependency. The third and final article will discuss one of the most common mistakes in calculating AC closed-loop gain errors. Often, during circuit analysis, system designers have the tendency to use DC-gain calculation methods for AC-domain analysis, which provides worse results than the real performance of the circuit. With these three articles, the system designer will have the simple tools required to determine the overall closed-loop gain error for any specific op amp by using its data-sheet parameters.

Steady-state sinusoidal analysis and Bode plots

Before the main topic of this article is discussed, it is appropriate to briefly review the concepts of sinusoidal-frequency analysis and Bode plots. These two concepts will be used repeatedly throughout this series of articles.

It is often useful to characterize a circuit by measuring its response to sinusoidal input signals. Fourier analysis can be used to reconstruct any periodic signal by summing sinusoidal signals with various frequencies. Thus, the circuit designers can gather useful information about a circuit's response to various input signals by characterizing its response to sinusoidal excitations over a wide frequency range.

When a linear circuit is driven by a sinusoidal input signal of a specific frequency, the output signal is also a sinusoidal signal of the same frequency. The complex representation of a sinusoidal waveform can be used to represent the input signal as

$$v_1(t) = V_1 \times e^{j(\omega t + \phi_1)},$$

and the output signal as

$$v_2(t) = V_2 \times e^{j(\omega t + \phi_2)}.$$

V_1 and V_2 are the amplitudes of the input and output signals, respectively; and ϕ_1 and ϕ_2 are the phase of the input and output signals, respectively. The ratio of the output signals to the input signals is the transfer function, $H(j\omega)$.

In a sinusoidal steady-state analysis, the transfer function can be represented as

$$H(j\omega) = |H(j\omega)| \times e^{j\phi(\omega)}, \quad (1)$$

where $|H(j\omega)|$ is the magnitude of the transfer function, and ϕ is the phase. Both are functions of frequency.

One way to describe how the magnitude and phase of a transfer function vary over frequency is to plot them graphically. Together, the magnitude and phase plots of the transfer function are known as a Bode plot. The magnitude part of a Bode diagram plots the expression given by Equation 2 on a linear scale:

$$|H(j\omega)|_{\text{dB}} = 20 \log_{10} |H(j\omega)| \quad (2)$$

The phase part of a Bode diagram plots the expression given by Equation 3, also on a linear scale:

$$\phi = \angle H(j\omega) \quad (3)$$

Both the magnitude and the phase plots are plotted against a logarithmic frequency axis.

The benefit of plotting the logarithmic value of the magnitude instead of the linear magnitude of the transfer function is the ability to use asymptotic lines to approximate the transfer function. These asymptotic lines can be drawn quickly without having to use Equation 2 to calculate the exact magnitude and can still represent the magnitude of the transfer function with reasonable accuracy.

As an example, consider a first-order (single-pole) transfer function,

$$H(j\omega) = \frac{1}{1 + j \frac{\omega}{\omega_0}}, \quad (4)$$

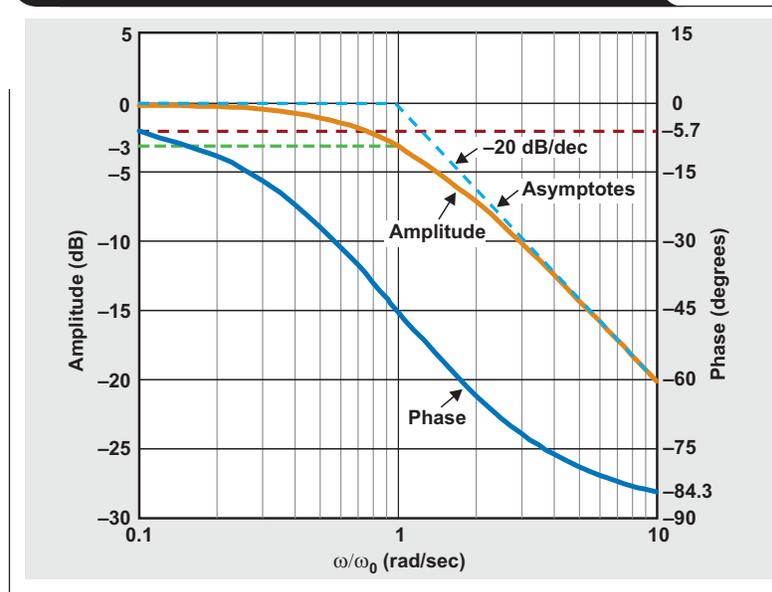
where ω_0 is the angular cutoff frequency of the system. The magnitude, in decibels, of the transfer function from Equation 4 can be described by Equation 5:

$$|H(j\omega)|_{\text{dB}} = 20 \log \left| \frac{1}{1 + j \frac{\omega}{\omega_0}} \right| \quad (5)$$

The transfer function, $H(j\omega)$, is a complex function of the angular frequency, ω . To calculate the magnitude, both real and imaginary portions of the function need to be used:

$$|H(j\omega)|_{\text{dB}} = 20 \log \frac{1}{\sqrt{1 + \frac{\omega^2}{\omega_0^2}}} \quad (6)$$

Equation 6 shows that at a frequency much lower than ω_0 , the magnitude is near 1 V/V or 0 dB. At frequency

Figure 1. Bode plot of single-pole transfer function

$\omega = \omega_0$, the magnitude drops to $1/\sqrt{2} = 0.707$, or roughly -3 dB. Above this frequency, the magnitude rolls off at a rate of -20 dB/decade.

Both the real and imaginary parts of the transfer function can be used to calculate the phase response as

$$\varphi(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right). \quad (7)$$

Similarly, when the frequency is much lower than ω_0 , the phase is 0° . At the frequency $\omega = \omega_0$, the phase is -45° . Finally, when the frequency is much higher than ω_0 , the phase levels off at -90° .

Figure 1 shows the Bode plot of the first-order transfer function just described. Notice the use of the two asymptotic lines to simplify the magnitude plot of the transfer function. At the intersection of the two asymptotic lines, the simplified magnitude curve is off from the actual magnitude by about 3 dB. At frequencies much lower or much higher than ω_0 , the error is negligible.

Deriving noninverting and inverting transfer functions

For simplicity, all the feedback networks in this article are shown as resistive networks. However, the analysis shown here will also be valid when these resistors are replaced with complex feedback networks.

Figure 2 depicts a typical noninverting op amp configuration. The closed-loop gain of the amplifier is set by two resistors: the feedback resistor, R_F , and the input resistor, R_I . The amount of output voltage, V_{OUT} , fed back to the feedback point is represented by the parameter β . The feedback point is the inverting input of the op amp. As stated, the β network is a simple resistive feedback network. From Figure 2, β is defined as

$$\beta = \frac{V_{FB}}{V_{OUT}} = \frac{R_I}{R_I + R_F}. \quad (8)$$

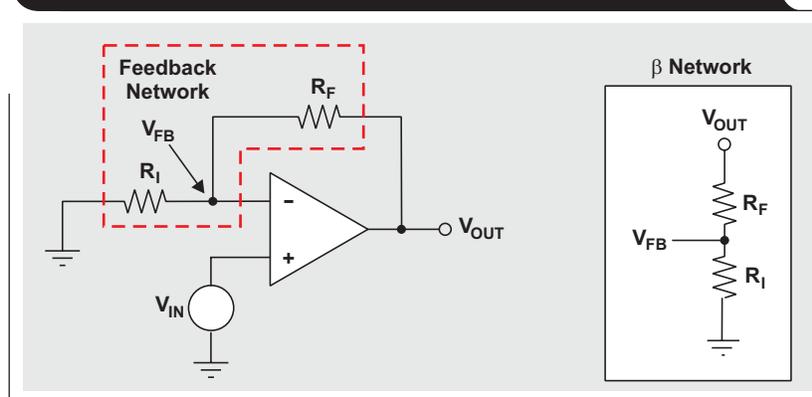
Figure 2. Typical noninverting op amp circuit with feedback

Figure 3 shows the control-loop model of the circuit in Figure 2. The parameter A_{OL} is the open-loop gain of the op amp and is always specified in any op amp data sheet. The control-loop model from Figure 3 can be used to express the closed-loop gain as

$$A_{CL} = \frac{V_{OUT}}{V_{IN}} = \frac{A_{OL}}{1 + \beta \times A_{OL}} \quad (9)$$

Assuming that this model is of a first-order system, the open-loop gain of an op amp as a function of angular frequency can be described as

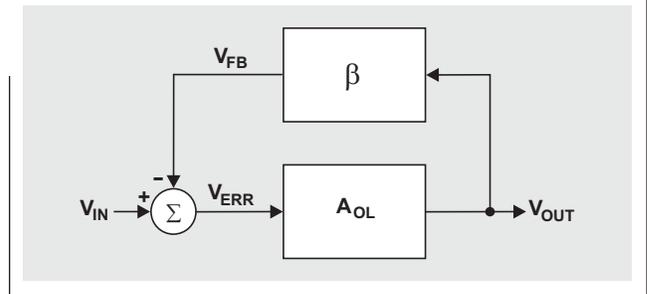
$$A_{OL}(j\omega) = \frac{A_{OL_DC}}{1 + j \frac{\omega}{\omega_0}} \quad (10)$$

The parameter A_{OL_DC} in Equation 10 is the open-loop gain of the op amp at a low frequency or at the DC level. The dominant pole of the op amp is given by the angular frequency, ω_0 , or equivalently by $f_0 = \omega_0/2\pi$.

The Bode plot of the open-loop gain expression from Equation 10 is presented in Figure 4. Asymptotic curves are used in this figure to create a simplified version of the actual open-loop response. Now it is possible to express the closed-loop gain from Equation 9 in the frequency domain by replacing the parameter A_{OL} with Equation 10. After a few algebraic steps, the closed-loop transfer function can be written as

$$A_{CL}(j\omega) = \frac{\frac{A_{OL_DC}}{1 + \beta \times A_{OL_DC}}}{1 + j \frac{\omega}{\omega_0} \times \frac{1}{1 + \beta \times A_{OL_DC}}} \quad (11)$$

Figure 3. Control-loop model of noninverting op amp circuit



$A_{CL}(j\omega)$ is a complex function of the angular frequency, ω . Also recall that to calculate the magnitude, both real and imaginary portions of the function need to be used in the same way that Equation 6 was obtained:

$$|A_{CL}(j\omega)|_{dB} = 20 \log \frac{\frac{A_{OL_DC}}{1 + \beta \times A_{OL_DC}}}{\sqrt{1 + \frac{\omega^2}{\omega_0^2} \times \frac{1}{(1 + \beta \times A_{OL_DC})^2}}} \quad (12)$$

If the angular frequency, ω , is replaced with $2\pi f$, the closed-loop transfer function from Equation 12 can be rewritten as

$$|A_{CL}(jf)|_{dB} = 20 \log \frac{\frac{A_{OL_DC}}{1 + \beta \times A_{OL_DC}}}{\sqrt{1 + \frac{f^2}{f_0^2} \times \frac{1}{(1 + \beta \times A_{OL_DC})^2}}} \quad (13)$$

Figure 4. Open-loop gain vs. frequency of typical op amp

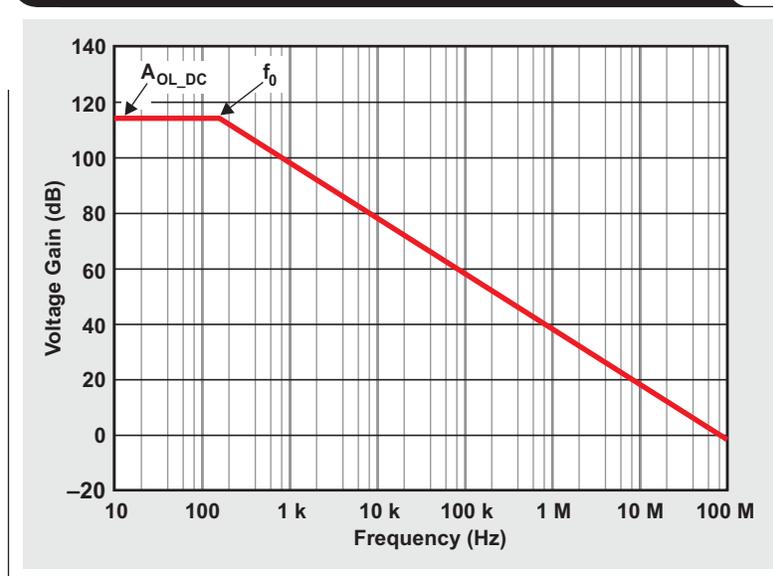


Figure 5 depicts a typical inverting op amp configuration. As with the analysis of the noninverting configuration, simple resistor networks are used that can be replaced with more complex functions. The closed-loop gain of the amplifier is again set by two resistors: the feedback resistor, R_F , and the input resistor, R_I . The amount of output voltage, V_{OUT} , fed back to the inverting input is again represented by β . In the inverting configuration, there is an additional signal arriving at the inverting node as a result of the input signal. The amount of this signal is represented by α .

For the inverting op amp configuration, α is defined as

$$\alpha = \frac{V_{FB}}{V_{IN}} = \frac{R_F}{R_I + R_F}, \quad (14)$$

while β is defined by Equation 8.

Figure 6 shows the control-loop model of the circuit in Figure 5. This model can be used to express the closed-loop gain of the circuit as

$$A_{CL} = \frac{V_{OUT}}{V_{IN}} = \frac{-\alpha \times A_{OL}}{1 + \beta \times A_{OL}}. \quad (15)$$

Substituting the A_{OL} term from Equation 10 into Equation 15 yields the closed-loop gain expression with its dependency on angular frequency:

$$A_{CL}(j\omega) = \frac{-\alpha \frac{A_{OL_DC}}{1 + \beta \times A_{OL_DC}}}{1 + j \frac{\omega}{\omega_0} \times \frac{1}{1 + \beta \times A_{OL_DC}}} \quad (16)$$

As before, the transfer function, $A_{CL}(j\omega)$, is a complex function of the angular frequency, ω . To calculate the magnitude, both real and imaginary portions of the function must again be used:

$$|A_{CL}(j\omega)|_{dB} = 20 \log \frac{\alpha \frac{A_{OL_DC}}{1 + \beta \times A_{OL_DC}}}{\sqrt{1 + \frac{\omega^2}{\omega_0^2} \times \frac{1}{(1 + \beta \times A_{OL_DC})^2}}} \quad (17)$$

If the angular frequency, ω , is replaced with $2\pi f$, the closed-loop transfer function from Equation 17 can be rewritten as

$$|A_{CL}(jf)|_{dB} = 20 \log \frac{\alpha \frac{A_{OL_DC}}{1 + \beta \times A_{OL_DC}}}{\sqrt{1 + \frac{f^2}{f_0^2} \times \frac{1}{(1 + \beta \times A_{OL_DC})^2}}} \quad (18)$$

Figure 5. Typical inverting op amp circuit with feedback

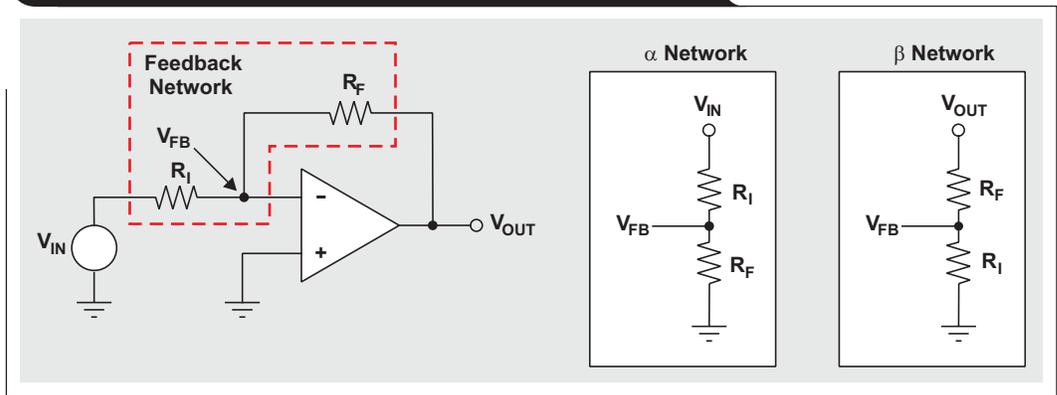
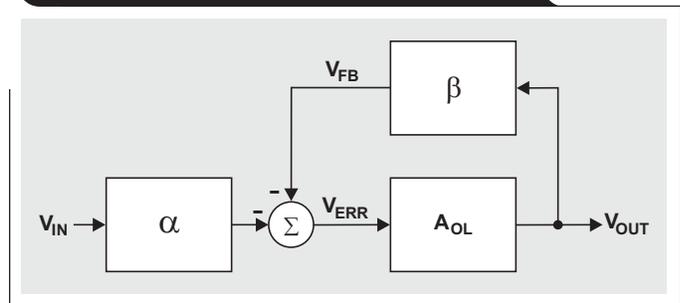


Figure 6. Control-loop model of inverting op amp circuit



Assuming that there is a first-order response from the op amp, the complete closed-loop equations for the non-inverting and inverting gain amplifiers are respectively represented by Equation 13 and Equation 18.

Conclusion

This article has explored feedback control system analysis and synthesis as they apply to first-order transfer functions. The analysis technique was applied to both noninverting and inverting op amp circuits, resulting in a frequency-domain transfer function for each configuration. In Parts 2 and 3 of this series, these two transfer functions will be used to analyze the DC and AC gain error of closed-loop op amp circuits. Part 3 will help circuit designers avoid the mistake of using DC-gain calculations for AC-domain analysis.

References

1. John J. D'Azzo and Constantine H. Houpis, *Feedback Control System Analysis and Synthesis* (New York: McGraw-Hill, 1966).
2. James W. Nilsson and Susan Riedel, *Electric Circuits* (Upper Saddle River, NJ: Prentice Hall, 2007).

Related Web site
amplifier.ti.com

TI Worldwide Technical Support

Internet

TI Semiconductor Product Information Center Home Page

support.ti.com

TI E2E Community Home Page

e2e.ti.com

Product Information Centers

Americas	Phone	+1(972) 644-5580
Brazil	Phone	0800-891-2616
Mexico	Phone	0800-670-7544
	Fax	+1(972) 927-6377
	Internet/Email	support.ti.com/sc/pic/americas.htm

Europe, Middle East, and Africa

Phone	
European Free Call	00800-ASK-TEXAS (00800 275 83927)
International	+49 (0) 8161 80 2121
Russian Support	+7 (4) 95 98 10 701

Note: The European Free Call (Toll Free) number is not active in all countries. If you have technical difficulty calling the free call number, please use the international number above.

Fax	+ (49) (0) 8161 80 2045
Internet	support.ti.com/sc/pic/euro.htm

Japan

Phone	Domestic	0120-92-3326
Fax	International	+81-3-3344-5317
	Domestic	0120-81-0036
Internet/Email	International	support.ti.com/sc/pic/japan.htm
	Domestic	www.tij.co.jp/pic

Asia

Phone	
International	+91-80-41381665
Domestic	<u>Toll-Free Number</u>
Australia	1-800-999-084
China	800-820-8682
Hong Kong	800-96-5941
India	1-800-425-7888
Indonesia	001-803-8861-1006
Korea	080-551-2804
Malaysia	1-800-80-3973
New Zealand	0800-446-934
Philippines	1-800-765-7404
Singapore	800-886-1028
Taiwan	0800-006800
Thailand	001-800-886-0010
Fax	+886-2-2378-6808
Email	tiasia@ti.com or ti-china@ti.com
Internet	support.ti.com/sc/pic/asia.htm

Important Notice: The products and services of Texas Instruments Incorporated and its subsidiaries described herein are sold subject to TI's standard terms and conditions of sale. Customers are advised to obtain the most current and complete information about TI products and services before placing orders. TI assumes no liability for applications assistance, customer's applications or product designs, software performance, or infringement of patents. The publication of information regarding any other company's products or services does not constitute TI's approval, warranty or endorsement thereof.

E121709

All trademarks are the property of their respective owners.

IMPORTANT NOTICE

Texas Instruments Incorporated and its subsidiaries (TI) reserve the right to make corrections, modifications, enhancements, improvements, and other changes to its products and services at any time and to discontinue any product or service without notice. Customers should obtain the latest relevant information before placing orders and should verify that such information is current and complete. All products are sold subject to TI's terms and conditions of sale supplied at the time of order acknowledgment.

TI warrants performance of its hardware products to the specifications applicable at the time of sale in accordance with TI's standard warranty. Testing and other quality control techniques are used to the extent TI deems necessary to support this warranty. Except where mandated by government requirements, testing of all parameters of each product is not necessarily performed.

TI assumes no liability for applications assistance or customer product design. Customers are responsible for their products and applications using TI components. To minimize the risks associated with customer products and applications, customers should provide adequate design and operating safeguards.

TI does not warrant or represent that any license, either express or implied, is granted under any TI patent right, copyright, mask work right, or other TI intellectual property right relating to any combination, machine, or process in which TI products or services are used. Information published by TI regarding third-party products or services does not constitute a license from TI to use such products or services or a warranty or endorsement thereof. Use of such information may require a license from a third party under the patents or other intellectual property of the third party, or a license from TI under the patents or other intellectual property of TI.

Reproduction of TI information in TI data books or data sheets is permissible only if reproduction is without alteration and is accompanied by all associated warranties, conditions, limitations, and notices. Reproduction of this information with alteration is an unfair and deceptive business practice. TI is not responsible or liable for such altered documentation. Information of third parties may be subject to additional restrictions.

Resale of TI products or services with statements different from or beyond the parameters stated by TI for that product or service voids all express and any implied warranties for the associated TI product or service and is an unfair and deceptive business practice. TI is not responsible or liable for any such statements.

TI products are not authorized for use in safety-critical applications (such as life support) where a failure of the TI product would reasonably be expected to cause severe personal injury or death, unless officers of the parties have executed an agreement specifically governing such use. Buyers represent that they have all necessary expertise in the safety and regulatory ramifications of their applications, and acknowledge and agree that they are solely responsible for all legal, regulatory and safety-related requirements concerning their products and any use of TI products in such safety-critical applications, notwithstanding any applications-related information or support that may be provided by TI. Further, Buyers must fully indemnify TI and its representatives against any damages arising out of the use of TI products in such safety-critical applications.

TI products are neither designed nor intended for use in military/aerospace applications or environments unless the TI products are specifically designated by TI as military-grade or "enhanced plastic." Only products designated by TI as military-grade meet military specifications. Buyers acknowledge and agree that any such use of TI products which TI has not designated as military-grade is solely at the Buyer's risk, and that they are solely responsible for compliance with all legal and regulatory requirements in connection with such use.

TI products are neither designed nor intended for use in automotive applications or environments unless the specific TI products are designated by TI as compliant with ISO/TS 16949 requirements. Buyers acknowledge and agree that, if they use any non-designated products in automotive applications, TI will not be responsible for any failure to meet such requirements.

Following are URLs where you can obtain information on other Texas Instruments products and application solutions:

Products		Applications	
Amplifiers	amplifier.ti.com	Audio	www.ti.com/audio
Data Converters	dataconverter.ti.com	Automotive	www.ti.com/automotive
DLP® Products	www.dlp.com	Communications and Telecom	www.ti.com/communications
DSP	dsp.ti.com	Computers and Peripherals	www.ti.com/computers
Clocks and Timers	www.ti.com/clocks	Consumer Electronics	www.ti.com/consumer-apps
Interface	interface.ti.com	Energy	www.ti.com/energy
Logic	logic.ti.com	Industrial	www.ti.com/industrial
Power Mgmt	power.ti.com	Medical	www.ti.com/medical
Microcontrollers	microcontroller.ti.com	Security	www.ti.com/security
RFID	www.ti-rfid.com	Space, Avionics & Defense	www.ti.com/space-avionics-defense
RF/IF and ZigBee® Solutions	www.ti.com/lprf	Video and Imaging	www.ti.com/video
		Wireless	www.ti.com/wireless-apps

Mailing Address: Texas Instruments, Post Office Box 655303, Dallas, Texas 75265
Copyright © 2010, Texas Instruments Incorporated