Using Windowing With SNRBoost\textsuperscript{3G} Technology

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ABSTRACT

Coherency is a well-known requirement when using FFT techniques to examine the spectrum of the output of an analog-to-digital converter (ADC). SNRBoost\textsuperscript{3G} technology results in loss in coherency, and this can be seen as an unstable noise floor in the spectrum. Windowing of the ADC output is a well-known solution to restore coherency and stable spectrum.

However, windowing also modifies the amplitude of the fundamental signal and the SNR values. This masks the real improvement in SNR due to the SNRBoost\textsuperscript{3G} technology.

This application report explains the scaling caused by different window functions that can be used to recover the actual SNR of the original signal (prior to windowing).

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1 Introduction

FFT techniques are commonly used to examine the spectrum of the output of an analog-to-digital converter (ADC). One of the requirements when using FFT is to ensure coherency between the analog input signal and the sampling clock of the ADC.

When using an ADC with SNRBoost\textsuperscript{3G} technology, loss in coherency is observed. This can be seen as an unstable noise floor in the spectrum. Windowing of the ADC output is a well-known solution to restore coherency and stable spectrum. However, windowing also modifies the amplitude of the fundamental signal and the SNR values. This masks the real improvement in SNR due to SNRBoost\textsuperscript{3G} technology.

This application note explains the scaling caused by different window functions that can be used to recover the actual SNR of the original signal (prior to windowing).
2 **SNRBoost\textsuperscript{3G} Technology Causes Loss in Coherency**

SNRBoost\textsuperscript{3G} technology works by improving the noise within a select band of frequencies. At the same time, the noise degrades outside the selected frequency band (Figure 1).

![Figure 1. Spectrum of Signal With SNRBoost\textsuperscript{3G} On](image)

Note that although the noise in the selected band improves with SNRBoost\textsuperscript{3G} enabled, the noise over the entire band degrades (compared to SNRBoost\textsuperscript{3G} disabled). For example, the full-band SNR is 66.91 dBFS with SNRBoost\textsuperscript{3G} disabled and 32.73 dBFS with SNRBoost\textsuperscript{3G} enabled (Figure 2).

![Figure 2. Full-Band SNR Comparison With SNRBoost\textsuperscript{3G} Disabled and Enabled](image)

In the time domain, this SNR degradation shows up as an increase in the range of code variations. For example, the output code range is 1 LSB with SNRBoost\textsuperscript{3G} disabled and 113 LSBs with SNRBoost\textsuperscript{3G} enabled (Figure 3).
The increased noise results in loss of coherency. The FFT spectrum of this data has a noise floor that is not uniformly flat and changes from one capture to the next.

Windowing can be used to overcome the loss in coherency. Any of the common window functions (Hanning, Hamming, Blackman-Harris, etc.) can be applied to the ADC output data. The resulting signal is now coherent with a stable noise floor in the FFT spectrum.

3 Using Windowing

Windowing can be used to overcome the loss in coherency. Any of the common window functions (Hanning, Hamming, Blackman-Harris, etc.) can be applied to the ADC output data. The resulting signal is now coherent with a stable noise floor in the FFT spectrum.
The windowing has one side effect; the amplitude of all the frequency bins in the spectrum are reported smaller than their actual values. For a narrow-band signal, the fundamental signal and the rest of the spectrum are modified by different amounts as shown in Table 1.

### Table 1. Effect of Windowing

<table>
<thead>
<tr>
<th>Window Function</th>
<th>Delta SNR (dB)</th>
<th>Delta Signal Amplitude (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamming</td>
<td>4.01</td>
<td>5.35</td>
</tr>
<tr>
<td>Hanning</td>
<td>4.26</td>
<td>6.02</td>
</tr>
<tr>
<td>Blackman Harris</td>
<td>5.88</td>
<td>8.90</td>
</tr>
</tbody>
</table>

Use Table 1 to determine the SNR and signal amplitude of the original signal without windowing as follows:

\[
\text{SNR (pre-windowing)} = \text{SNR (post-windowing)} - \text{Delta SNR}
\]

\[
\text{Signal Amplitude (pre-windowing)} = \text{Signal Amplitude (post-windowing)} + \text{Delta Signal Amplitude}
\]

where SNR and Signal Amplitude are reported in dBFS values.

In this way, the correct SNR of the signal with SNRBoost³G enabled can be determined.

### 3.1 Example

With the preceding method, the improvement in SNR using SNRBoost³G technology in the ADS58C48 can be determined.

Consider a case where the sample clock is 184.32 MSPS and the input signal bandwidth is 60 MHz centered at 46 MHz. With SNRBoost³G disabled, the in-band SNR reported is 68.6 dBFS. After the SNRBoost³G technology is turned on, a Hamming window is applied to the ADC output data. The spectrum of the windowed signal reports an in-band SNR of 78.1 dBFS.

Using Table 1, find the in-band SNR (pre windowing) with SNRBoost³G ON:

\[
= 78.1 \text{ dBFS} - 4.01 \text{ dB}
\]

\[
= 74.09 \text{ dBFS}
\]

This means that the SNRBoost³G technology resulted in an in-band SNR improvement:

\[
= \text{SNR (pre-windowing)} \text{ with SNRBoost³G on} - \text{SNR with SNRBoost³G off}
\]

\[
= 74.01 \text{ dBFS} - 68.6 \text{ dBFS}
\]

\[
= 5.49 \text{ dB} \text{ within the band of 60 MHz.}
\]
Appendix A Theory

The aim of this section is to determine the change in spectral power after windowing. To find this, a single tone input signal is used that is coherent and has known power. The input signal is then multiplied by the window function. The difference in the power of the tone between the input signal and the windowed signal gives the desired result.

Consider two N-point sequences V(n) and W(n), that represent the ADC output data and the window function sequence, respectively.

\[ V(n) = A \cos \left( \frac{2\pi}{N} kn + \theta \right) \]  

\[ W(n) = m_1 + m_2 \cos \left( \frac{2\pi}{N-1} wn \right) \]  

(1)  

(2)

For simplicity of analysis, the ADC output sequence is assumed to be a single tone with all other tones having zero power. Because the spectrum from N/2 to N–1 is a mirror image of the spectrum from 0 to N/2–1, the spectrum is represented only in the range 0 to N/2–1, after adjusting the power of each tone.

\[ V(n) = A \cos \left( \frac{2\pi}{N} kn + \theta \right) \]

Similarly, the spectrum of the window function is shown in Figure 7.

Figure 6. Spectrum of ADC Output Sequence

Similarly, the spectrum of the window function is shown in Figure 7.
Windowing involves multiplication of the ADC output sequence by the window function. So,

\[ V_o(n) = V(n) \cdot W(n) \]

\[ \approx A \cos \left( \frac{2\pi}{N} kn + \theta \right) \cdot \left( m_1 + m_2 \cos \left( \frac{2\pi}{N} wn \right) \right) \]

\[ = m_1 A \cos \left( \frac{2\pi}{N} kn \right) + \frac{m_2 A}{2} \cos \left( \frac{2\pi}{N} (k - w)n + \theta \right) + \frac{m_2 A}{2} \cos \left( \frac{2\pi}{N} (k + w)n + \theta \right) \]

The spectrum of the signal after windowing is shown in Figure 8.

\[ V_o(n) = W(n) \cdot v(n) \]

Note that each tone with bin number 'k' in the input signal spreads into three tones.

- One main tone (at bin k) having amplitude \( m_1 A \) and
- Two side tones (at bins k ± w) having amplitudes \( \frac{m_2 A}{2} \).

The change in signal amplitude of the main tone is due to windowing \( = 20 \log (m_1) \).

Note that it is assumed that the input signal has zero amplitude noise components. With non-zero noise components, the input signal can be represented as:
$$V(n) = A \cos \left( \frac{2 \pi}{N} n + \theta \right) + \sum_{r=0}^{N-1} B_r \cos \left( \frac{2 \pi}{N} n + \phi_r \right)$$

(4)

Similar to the signal tone, each noise component is also spread into three tones having amplitudes $m_1 B_r$ and $\frac{m_2 B_r}{2}$.

(5)

The total power of all noise components after windowing is

$$= m_1^2 \sum_{r=0}^{N-1} B_r^2 + \frac{m_2^2}{4} \sum_{r=0}^{N-1} B_r^2 + \frac{m_2^2}{4} \sum_{r=0}^{N-1} B_r^2$$

$$= \left( m_1^2 + \frac{m_2^2}{2} \right) \sum_{r=0}^{N-1} B_r^2$$

(6)

Because $\sum_{r=0}^{N-1} B_r^2$ represents the noise power of the ADC output sequence, Equation 6 can be re-written as:

Noise power after windowing = $\left( m_1^2 + \frac{m_2^2}{2} \right) \times$ ADC output noise power.

(7)

So, the change in the SNR after windowing = $10 \log \left( m_1^2 + \frac{m_2^2}{2} \right)$

(8)

Table 2 lists the changes in signal amplitude and SNR after windowing for some common window functions.

<table>
<thead>
<tr>
<th>Window Function</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$w$</th>
<th>Delta SNR (dB)</th>
<th>Delta Signal Amplitude (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamming</td>
<td>0.54</td>
<td>-0.46</td>
<td>1</td>
<td>4.01</td>
<td>5.35</td>
</tr>
<tr>
<td>Hanning</td>
<td>0.5</td>
<td>-0.5</td>
<td>1</td>
<td>4.26</td>
<td>6.02</td>
</tr>
</tbody>
</table>
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