Sensitivity Analysis for Power Supply Design

Tahar Allag

ABSTRACT

In power supply design, tolerance in component values, and reference voltages and currents cause deviation to the desired system output. However, the deviation of the system output due to a variation of each input variation is different. The output is more sensitive to certain inputs than others. In this application note, sensitivity analysis is explored and a design example is provided.

Contents

Sensitivity Analysis Overview ............................................................................................................... 1
Sensitivity Analysis with the TPS3808G01 ........................................................................................... 2
Design Example: ................................................................................................................................... 4
Conclusion .............................................................................................................................................. 6
References ............................................................................................................................................... 6

Figures

Figure 1. Functional Block Diagram of the TPS3808G01 ................................................................. 2
Figure 2. Sense pin for the TPS3808G01 ......................................................................................... 3

Sensitivity Analysis Overview

Sensitivity analysis is the study of how a certain system reacts to a variation of its inputs. It is an analysis technique for systematically changing variables in a model to determine the effects of such changes to its output.

Because of tolerance in component values, such as resistors, capacitors, inductors, and even reference voltage and current levels, sensitivity analysis is a useful technique in power supply design. It helps determine the effect of each input tolerance in the system and its effect on the overall performance of the model. The classical sensitivity function is defined as:

\[ S_x^y = \lim_{\Delta x \to 0} \frac{\Delta y / y}{\Delta x / x} \]  

(1)

Thus

\[ S_x^y = \frac{\partial y / x}{\partial x / y} \] 

(2)
x represents the value of the components such as resistors, capacitors, inductors, and even reference voltages and currents of the system. y is the output parameter of interest such as frequency, phase, or output voltage and current. The sensitivity result determines the per-unit change in y due to a given per-unit change in x.

Equation 2 can be simplified further for small changes as follow:

\[ S_y = \frac{\Delta y}{\Delta x} \]  

(3)

Sensitivity Analysis with the TPS3808G01

The TPS3808G01 is a microprocessor supervisor circuit. It uses the SENSE pin to sense if the system voltage drops below the preset threshold (VIT). Figure 1 shows the SENSE pin for the TPS3808G01 part. The sense input provides a terminal at which any system voltage can be monitored by using a voltage divider as shown in Figure 2. The sense input pin is connected to a comparator that is referenced to 0.405 V.

Ideally, a comparator would have infinite input impedance that produces no current at the inputs. In practice; however, a real comparator has measurable input impedance and some degree of leakage current. These effects impact the accuracy of the trip point set by the resistive divider at the inputs, because this leakage current cannot be exactly determined and varies from device to device. For the TPS3808G01, it is called “Is” and its maximum and minimum values are provided in the data sheet as ±25 nA.
R1, R2, Is, and VREF are variables and have tolerances. V\textsubscript{i} is the target output voltage.

Sensitivity analysis determines how the tolerances on R1, R2, Is, and V\textsubscript{REF} affects the output V\textsubscript{i}.

Sensitivity of V\textsubscript{i} with respect to R1:

\[ S_{\text{R1}}^{V\textsubscript{i}} = \frac{\partial V\textsubscript{i}}{\partial R_1} \frac{R_1}{V\textsubscript{i}} \]  \hspace{1cm} (6)

\[ \frac{\partial V\textsubscript{i}}{\partial R_1} = \frac{V_S}{R_2} + I_S \]  \hspace{1cm} (7)

\[ S_{\text{R1}}^{V\textsubscript{i}} = \frac{(V_S + I_S R_2) R_1}{V_S (R_1 + R_2) + I_S R_1 R_2} \]  \hspace{1cm} (8)

Sensitivity of V\textsubscript{i} with respect to R2:

\[ S_{\text{R2}}^{V\textsubscript{i}} = \frac{\partial V\textsubscript{i}}{\partial R_2} \frac{R_2}{V\textsubscript{i}} \]  \hspace{1cm} (9)
\[
\frac{\partial V_I}{\partial R_2} = \frac{V_S R_2 - V_S (R_1 + R_2)}{R_2}
\]  
(10)

\[
S_{R_2}^{VI} = \frac{-V_S R_1}{V_S(R_1 + R_2) + I_S R_1 R_2}
\]  
(11)

Sensitivity of \(V_I\) with respect to \(I_s\):

\[
S_{I_s}^{VI} = \frac{\partial V_I}{\partial I_s} = \frac{L_s}{V_I}
\]  
(12)

\[
\frac{\partial V_I}{\partial I_s} = I_s
\]  
(13)

\[
S_{I_s}^{VI} = \frac{I_s R_2 R_1}{V_S(R_1 + R_2) + I_S R_1 R_2}
\]  
(14)

Sensitivity of \(V_I\) with respect to \(V_{REF}\):

\[
S_{V_{REF}}^{VI} = \frac{\partial V_{out}}{\partial V_{REF}} \frac{V_{REF}}{V_{out}}
\]  
(15)

\[
\frac{\partial V_{out}}{\partial V_{REF}} = \frac{R_1 + R_2}{R_2}
\]  
(16)

\[
S_{V_{REF}}^{VI} = \frac{V_{REF}(R_1 + R_2)}{V_{REF}(R_1 + R_2) + I_S R_1 R_2}
\]  
(17)

**Design example:**

The threshold voltage is set by the ratio of the two resistors \(R_1\) and \(R_2\) in the divider. Multiple combinations of \(R_1\) and \(R_2\) can keep the ratio constant. However, to select the actual resistor values, there are tradeoffs to consider. With higher resistances, the leakage current at the comparator input can affect the threshold voltage accuracy. With lower resistances, the current through the divider is increased and this lowers the efficiency of the whole system. Especially
for battery-powered applications, this current can be a significant drain on the battery life and the run time.

For selecting optimally-sized resistors, a detailed analysis has been done in the SLVA450 application report. In this application note, the following two equations were used to calculate R1 and R2:

\[
R_2 = \frac{V_{IT}}{I_{S}} \left( \frac{V_{IT}}{V_{REF}} - 1 \right) \tag{18}
\]

\[
R_1 = R_2 \left( \frac{V_{IT}}{V_{ref}} - 1 \right) \tag{19}
\]

\(V_{IT}\): The threshold input voltage
\(Acc\): Accuracy of the divider

In this design example, a 3.3 V input is used; anything below a 10% drop from that value is not desired \((V_{IT} \approx 2.97 \text{ V})\). For 1% desired accuracy \((Acc)\), R2 and R1 are calculated to be 187\(K\Omega\) and 1.18 M\(\Omega\) respectively. The reference voltage \((V_{REF})\) and absolute leakage current \((abs(I_S))\) are provided in the data sheet of the TPS3808G01 as 0.405 V and 25 nA respectively. Using the standard values of R1 and R2, the lower threshold of the input voltage is calculated as follows:

\[
V_L = (1 + \frac{R_1}{R_2}) V_{ref} = (1 + \frac{1.18 M\Omega}{187 K\Omega}) 0.405 = 2.974 V \tag{20}
\]

\[
S_{R1}^{VI} = \frac{0.405 + 25 \times 10^{-9} \times 187 \times 10^3}{0.405(1.18 \times 10^6 + 187 \times 10^3)} = 0.865 \tag{21}
\]

\[
S_{R2}^{VI} = \frac{-0.405 \times 1.18 \times 10^6}{0.405(1.18 \times 10^6 + 187 \times 10^3)} = -0.855 \tag{22}
\]

\[
S_{I_S}^{VI} = \frac{25 \times 10^{-9} \times 1.18 \times 10^6 \times 187 \times 10^3}{0.405 \times (1.18 \times 10^6 + 187 \times 10^3)} = 9.865 \times 10^{-3} \tag{23}
\]

\[
S_{V_{ref}}^{VI} = \frac{0.405(1.18 \times 10^6 + 187 \times 10^3)}{0.405(1.18 \times 10^6 + 187 \times 10^3) + 25 \times 10^{-9} \times 1.18 \times 10^6 \times 187 \times 10^3} = 0.99 \tag{24}
\]
The following table provides the best values for R1 and R2 based on the most used Vi and 10% voltage threshold \( V_{IT} \). Then the sensitivity numbers are provided accordingly.

<table>
<thead>
<tr>
<th>( V_i ) (V)</th>
<th>( V_{IT} ) (V)</th>
<th>( I_s ) (A)</th>
<th>( V_s ) (V)</th>
<th>( R1 ) (( \Omega ))</th>
<th>( R2 ) (( \Omega ))</th>
<th>( S_{V1}^{VI} )</th>
<th>( S_{R2}^{VI} )</th>
<th>( S_{V}^{VI} )</th>
<th>( S_{V_{REF}}^{VI} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5.40</td>
<td>2.50E-08</td>
<td>0.405</td>
<td>2.16E+06</td>
<td>1.75E+05</td>
<td>0.926</td>
<td>-0.916</td>
<td>9.90E-03</td>
<td>0.990</td>
</tr>
<tr>
<td>5</td>
<td>4.50</td>
<td>2.50E-08</td>
<td>0.405</td>
<td>1.80E+06</td>
<td>1.78E+05</td>
<td>0.911</td>
<td>-0.901</td>
<td>9.90E-03</td>
<td>0.990</td>
</tr>
<tr>
<td>3.3</td>
<td>2.97</td>
<td>2.50E-08</td>
<td>0.405</td>
<td>1.19E+06</td>
<td>1.88E+05</td>
<td>0.865</td>
<td>-0.855</td>
<td>9.90E-03</td>
<td>0.990</td>
</tr>
<tr>
<td>1.8</td>
<td>1.62</td>
<td>2.50E-08</td>
<td>0.405</td>
<td>6.48E+05</td>
<td>2.16E+05</td>
<td>0.752</td>
<td>-0.743</td>
<td>9.90E-03</td>
<td>0.990</td>
</tr>
<tr>
<td>1.2</td>
<td>1.08</td>
<td>2.50E-08</td>
<td>0.405</td>
<td>4.32E+05</td>
<td>2.59E+05</td>
<td>0.629</td>
<td>-0.619</td>
<td>9.90E-03</td>
<td>0.990</td>
</tr>
<tr>
<td>1</td>
<td>0.90</td>
<td>2.50E-08</td>
<td>0.405</td>
<td>3.60E+05</td>
<td>2.95E+05</td>
<td>0.554</td>
<td>-0.545</td>
<td>9.90E-03</td>
<td>0.990</td>
</tr>
</tbody>
</table>

Table 01: Different VI with 10% drop threshold voltages and the corresponding sensitivity calculations

The obtained results show that the \( V_i \) is not sensitive to the leakage current, less than 1%. However, it is sensitive to the reference voltage. Any change in \( V_{REF} \) affects \( V_i \) by 99%. For the resistors, R1 and R2, the sensitivity is changing as the resistor values changes. Both sensitivities, with respect to R1 and R2, are almost the same in value but in opposite signs.

**Conclusion**

Sensitivity analysis is a very useful method to determine how the tolerance of inputs can affect the overall performance of the system. Based on the outcome of this analysis, the analog power designer can choose his variables while keeping the tolerance of the output of his system within the desired range. This helps in the component selection during the power supply design process.

**References**

1. TPS3808 Data sheet ([SBVS050J](#))
2. Optimizing Resistor Dividers at a Comparator Input ([SLVA450](#))
3. Ballast Resistors Allow Load Sharing Between Two Paralleled DC/DC Converters ([SLVA250](#)).
4. Effect of Resistor Tolerances on Power Supply Accuracy ([SLVA423](#))
IMPORTANT NOTICE

Texas Instruments Incorporated and its subsidiaries (TI) reserve the right to make corrections, modifications, enhancements, improvements, and other changes to its products and services at any time and to discontinue any product or service without notice. Customers should obtain the latest relevant information before placing orders and should verify that such information is current and complete. All products are sold subject to TI's terms and conditions of sale supplied at the time of order acknowledgment.

TI warrants performance of its hardware products to the specifications applicable at the time of sale in accordance with TI's standard warranty. Testing and other quality control techniques are used to the extent TI deems necessary to support this warranty. Except where mandated by government requirements, testing of all parameters of each product is not necessarily performed.

TI assumes no liability for applications assistance or customer product design. Customers are responsible for their products and applications using TI components. To minimize the risks associated with customer products and applications, customers should provide adequate design and operating safeguards.

TI does not warrant or represent that any license, either express or implied, is granted under any TI patent right, copyright, mask work right, or other TI intellectual property right relating to any combination, machine, or process in which TI products or services are used. Information published by TI regarding third-party products or services does not constitute a license from TI to use such products or services or a warranty or endorsement thereof. Use of such information may require a license from a third party under the patents or other intellectual property of the third party, or a license from TI under the patents or other intellectual property of TI.

Reproduction of TI information in TI data books or data sheets is permissible only if reproduction is without alteration and is accompanied by all associated warranties, conditions, limitations, and notices. Reproduction of this information with alteration is an unfair and deceptive business practice. TI is not responsible or liable for such altered documentation. Information of third parties may be subject to additional restrictions.

Resale of TI products or services with statements different from or beyond the parameters stated by TI for that product or service voids all express and implied warranties for the associated TI product or service and is an unfair and deceptive business practice. TI is not responsible or liable for any such statements.

TI products are neither designed nor intended for use in automotive applications or environments unless the specific TI products are specifically designated by TI as military-grade or "enhanced plastic." Only products designated by TI as military-grade meet military specifications. Buyers acknowledge and agree that they are solely responsible for all legal and regulatory requirements in connection with such use. Buyers represent that they have all necessary expertise in the safety and regulatory ramifications of their applications, and acknowledge and agree that they are solely responsible for all legal, regulatory and safety-related requirements concerning their products and any use of TI products in such safety-critical applications, notwithstanding any applications-related information or support that may be provided by TI. Further, Buyers must fully indemnify TI and its representatives against any damages arising out of the use of TI products in such safety-critical applications.

Resale of TI products or services with statements different from or beyond the parameters stated by TI for that product or service voids all express and implied warranties for the associated TI product or service and is an unfair and deceptive business practice. TI is not responsible or liable for any such statements.

TI products are not authorized for use in safety-critical applications (such as life support) where a failure of the TI product would reasonably be expected to cause severe personal injury or death, unless officers of the parties have executed an agreement specifically governing such use. Buyers represent that they have all necessary expertise in the safety and regulatory ramifications of their applications, and acknowledge and agree that they are solely responsible for all legal, regulatory and safety-related requirements concerning their products and any use of TI products in such safety-critical applications, notwithstanding any applications-related information or support that may be provided by TI. Further, Buyers must fully indemnify TI and its representatives against any damages arising out of the use of TI products in such safety-critical applications.

Resale of TI products or services with statements different from or beyond the parameters stated by TI for that product or service voids all express and implied warranties for the associated TI product or service and is an unfair and deceptive business practice. TI is not responsible or liable for any such statements.

TI products are neither designed nor intended for use in military/aerospace applications or environments unless the TI products are specifically designated by TI as military-grade or "enhanced plastic." Only products designated by TI as military-grade meet military specifications. Buyers acknowledge and agree that any such use of TI products which TI has not designated as military-grade is solely at the Buyer's risk, and that they are solely responsible for compliance with all legal and regulatory requirements in connection with such use.

TI products are neither designed nor intended for use in automotive applications or environments unless the specific TI products are designated by TI as compliant with ISO/TS 16949 requirements. Buyers acknowledge and agree that, if they use any non-designated products in automotive applications, TI will not be responsible for any failure to meet such requirements.

Following are URLs where you can obtain information on other Texas Instruments products and application solutions:

<table>
<thead>
<tr>
<th>Products</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Audio</td>
<td>Communications and Telecom</td>
</tr>
<tr>
<td>Amplifiers</td>
<td>Computers and Peripherals</td>
</tr>
<tr>
<td>Data Converters</td>
<td>Consumer Electronics</td>
</tr>
<tr>
<td>DLP® Products</td>
<td>Energy and Lighting</td>
</tr>
<tr>
<td>DSP</td>
<td>Industrial</td>
</tr>
<tr>
<td>Clocks and Timers</td>
<td>Medical</td>
</tr>
<tr>
<td>Interface</td>
<td>Security</td>
</tr>
<tr>
<td>Logic</td>
<td>Space, Avionics and Defense</td>
</tr>
<tr>
<td>Power Mgmt</td>
<td>Transportation and Automotive</td>
</tr>
<tr>
<td>Microcontrollers</td>
<td>Video and Imaging</td>
</tr>
<tr>
<td>RFID</td>
<td></td>
</tr>
<tr>
<td>OMAP Mobile Processors</td>
<td><a href="http://www.ti.com/omap">www.ti.com/omap</a></td>
</tr>
<tr>
<td>Wireless Connectivity</td>
<td><a href="http://www.ti.com/wirelessconnectivity">www.ti.com/wirelessconnectivity</a></td>
</tr>
</tbody>
</table>

TI E2E Community Home Page e2e.ti.com

Mailing Address: Texas Instruments, Post Office Box 655303, Dallas, Texas 75265
Copyright © 2011, Texas Instruments Incorporated