

AC Modeling of Power Stage in Flyback Converter

Rushil K K

PWR-DCDC Controllers

ABSTRACT

This application report provides a step-by-step procedure for constructing the AC model for the power stage of a Flyback DC/DC Converter in Continuous Conduction Mode. The model is used in the Tina simulator to plot and verify the control-to-output transfer function.

Contents

1	Introduction	1
	1.1 How it Works	2
2	Circuit Manipulations	3
3	Construction of the Mathematical Model.....	5
4	Simulation and Results.....	8
5	Conclusion:	11

Figures

Figure 1.	Basic Topology of Flyback Converter	2
Figure 2.	Separating Out the Non-Linear Switch Elements	3
Figure 3.	Voltage Waveforms for the 2-Port Network.....	4
Figure 4.	Current Waveforms for the 2-Port Network.....	4
Figure 5.	Replacing the Non-Linear Switch Network with Equivalent Linear Network.....	7
Figure 6.	Complete Linear Circuit for Flyback Converter	8
Figure 7.	Implementation of Example Case in Tina Simulator	10
Figure 8.	Control-to-Output Transfer Function Plotted using Tina.....	11

1 Introduction

The flyback DC-DC converter is derived from the basic buck-boost converter. The conversion ratio for this converter is given by,

$$\frac{V_{out}}{V_{in}} = n \cdot \frac{D}{1 - D}$$

As seen from this equation, the conversion ratio is similar to that of the buck-boost converter with an included factor of n , which is the turns ratio of the flyback transformer.

1.1 How it Works

In a flyback converter, when the current ramps up in the primary, energy is built up in the transformer core. When the FET switches off, this energy is dumped into the secondary, allowing current to flow in the secondary, so the primary current collapses immediately to zero. This causes an immediate rise in current in the secondary, after which, the current in the secondary ramps down linearly as it discharges, supplying energy to the load. The current in the secondary when the FET switches off, is proportional to the peak in the primary, and is determined by the turns ratio.

The circuit topology of the flyback converter is as shown in [Figure 1](#).

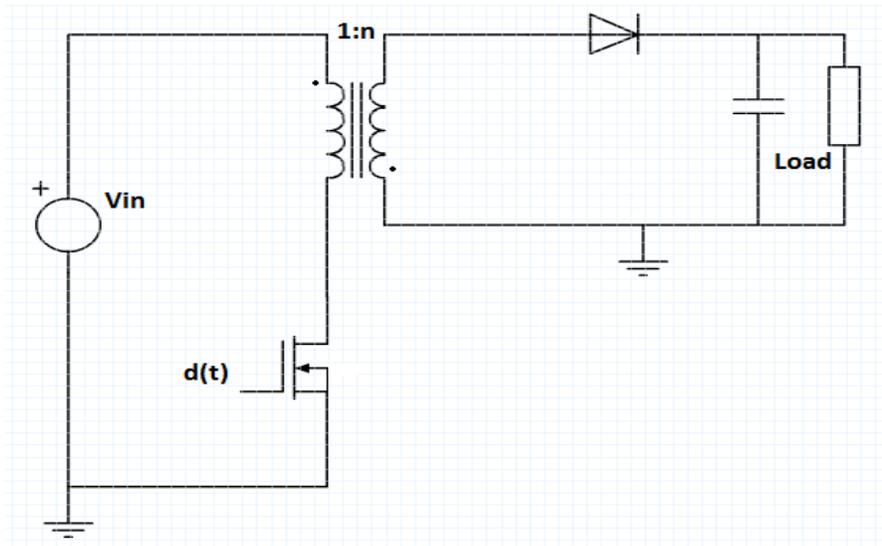


Figure 1. Basic Topology of a Flyback Converter

2 Circuit Manipulations

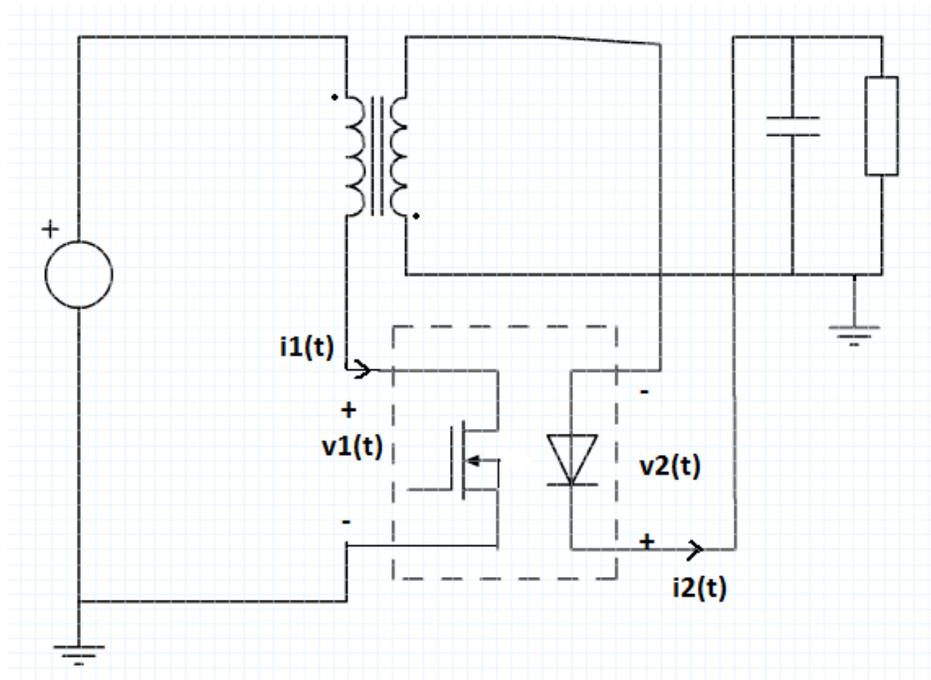
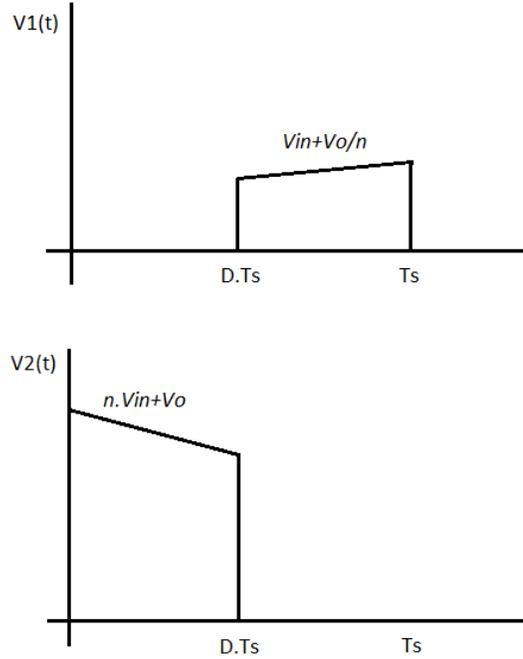


Figure 2. Separating Out the Non-Linear Switch Elements

The switching elements (MOSFET and Diode) are shown in the dashed rectangular box. These are non-linear switching elements. The box is treated as a 2-port network with input voltage $v_1(t)$, input current $i_1(t)$, output voltage $v_2(t)$, and output current $i_2(t)$.

The waveforms for $v_1(t)$ and $v_2(t)$ over one switching period, T_s , are shown in [Figure 3](#). A simplifying assumption of zero MOSFET and diode conduction drop has been made for obtaining the waveform shown below.



Note: The slope of the voltage waveform in the figure is exaggerated for the sake of clarity.

Figure 3. Voltage Waveforms for the 2-Port Network

Similarly, the waveforms for $i_1(t)$ and $i_2(t)$ are shown in [Figure 4](#).

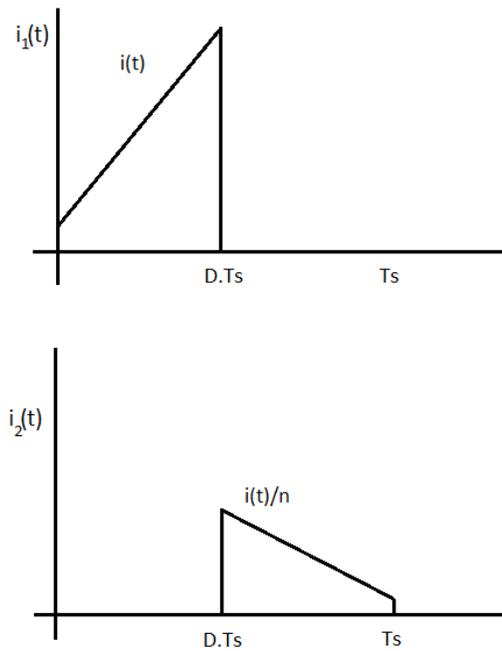


Figure 4. Current Waveforms for the 2-Port Network

3 Construction of the Mathematical Model

The waveforms in Figure 3 and Figure 4 are averaged over one switching period, which effectively removes the switching harmonics. The switching harmonics have no role of consequence in the development of the AC model. Observing the waveform, it can be seen that:

$$\begin{aligned} \langle V_1(t) \rangle_{T_s} &= d'(t) * \{V_{in} + V_o/n\} \\ \langle V_2(t) \rangle_{T_s} &= d(t) * \{nV_{in} + V_o\} \end{aligned}$$

Dividing the above equations, we get the following relation:

$$\langle V_1(t) \rangle_{T_s} / \langle V_2(t) \rangle_{T_s} = \frac{1}{n} * \frac{d'(t)}{d(t)}$$

Hence, we have:

$$\langle V_1(t) \rangle_{T_s} = \frac{1}{n} * \frac{d'(t)}{d(t)} * \langle V_2(t) \rangle_{T_s} \quad (1)$$

Equation 1 is a non-linear one, since it contains products of time-dependent variables. To obtain a linear relation, the variables are perturbed about the quiescent operating point.

$$\begin{aligned} d(t) &= D + \hat{d} \\ \langle V_1(t) \rangle_{T_s} &= V_1 + \hat{v}_1 \\ \langle V_2(t) \rangle_{T_s} &= V_2 + \hat{v}_2 \end{aligned}$$

Using the preceding equations in (1):

$$\begin{aligned} V_1 + \hat{v}_1 &= \frac{1}{n} * \frac{D' - \hat{d}}{D + \hat{d}} * (V_2 + \hat{v}_2) \\ V_1 * D + V_1 * \hat{d} + \hat{v}_1 * D + \hat{v}_1 * \hat{d} &= \frac{1}{n} * \{V_2 * D' - V_2 * \hat{d} + \hat{v}_2 * D' - \hat{v}_2 * \hat{d}\} \end{aligned}$$

Since the perturbations are small, the non-linear terms $\hat{v}_1 * \hat{d}$ and $\hat{v}_2 * \hat{d}$ are small enough to be ignored. Upon rearranging and grouping terms appropriately:

$$(V_1 + \hat{v}_1)D + V_1 * \hat{d} = \frac{1}{n} [(V_2 + \hat{v}_2) * D' - V_2 * \hat{d}] \quad (2)$$

The equation is re-arranged into the below form,

$$V_1 + \hat{v}_1 = \frac{1}{nD} [(V_2 + \hat{v}_2)D' - \hat{d}(V_2 + nV_1)] \quad (3)$$

Note: Equating the quiescent terms in Equation 2, we have $V_1 * D = \frac{1}{n} V_2 * D'$.

Substituting this relation for V_2 in Equation 3,

$$V_1 + \hat{v}_1 = \frac{1}{nD} [(V_2 + \hat{v}_2)D' - \hat{d}(\frac{nD}{D'} V_1 + nV_1)]$$

$$V_1 + \widehat{v}_1 = \frac{1}{n} [(V_2 + \widehat{v}_2) \frac{D'}{D} - \frac{\widehat{d}}{D} (\frac{D+D'}{D'}) n V_1]$$

Now, $D+D'=1$.

So the above equation finally reduces to:

$$V_1 + \widehat{v}_1 = \frac{1}{n} [(V_2 + \widehat{v}_2) \frac{D'}{D} - \frac{\widehat{d}}{DD'} * n V_1] \quad (A)$$

Using the same procedure for the 2-port network currents $i_1(t)$ and $i_2(t)$, Equation B is obtained:

$$I_2 + \widehat{i}_2 = \frac{1}{n} [(I_1 + \widehat{i}_1) \frac{D'}{D} - \frac{\widehat{d}}{DD'} * n I_2] \quad (B)$$

Equations (A) and (B) are the relations for the input voltage and output current of the 2-port network and are used to obtain the linear model for the averaged switch network.

These relations can be implemented by a network consisting of an ideal transformer and an independent voltage and current source, both driven by duty cycle variations, $d(t)$, as shown in [Figure 5](#).

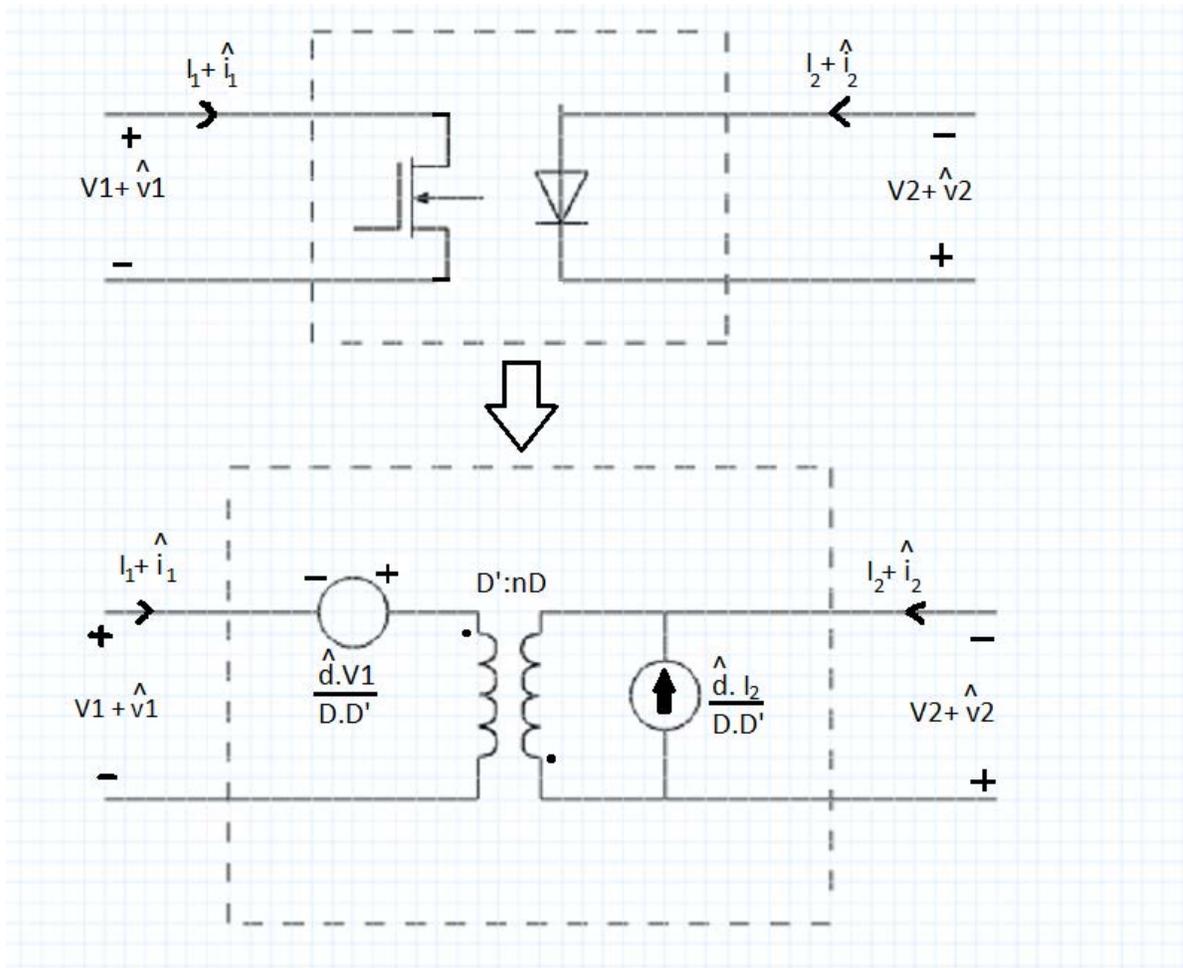


Figure 5. Replacing the Non-Linear Switch Network with an Equivalent Linear Network

The linear model for the switching element thus obtained is used in place of the MOSFET and diode in the rectangular dashed box of [Figure 2](#) to obtain the circuit shown in [Figure 6](#).

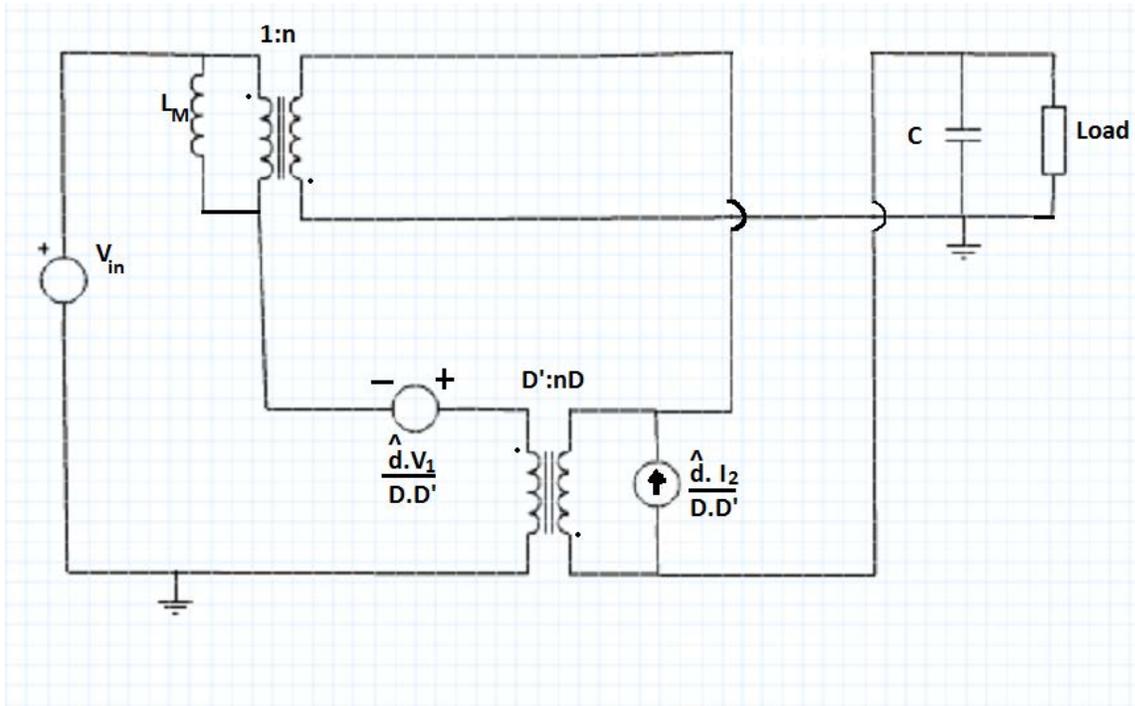


Figure 6. Complete Linear Circuit for Flyback Converter

The inductance L_M indicates the magnetizing inductance, and is used with an ideal transformer of turns ratio 1:n.

4 Simulation and Results

Example:

Consider a flyback converter designed for the following specifications: $V_{in} = 5V$, $V_{out} = 10V$. A load resistance of $10\ \Omega$ is connected at the output, so the load current drawn is 1 A. The magnetizing inductance of the flyback transformer is $6\ \mu H$, and load capacitance used is $500\ \mu F$.

The turns ratio $n=4$, hence, the duty cycle, D , required to produce a 10-V output is $D = 0.33$.

The quiescent value for $v_1(t)$, is $V_1 = D' \cdot (V_{in} + V_o/n)$, as seen from waveform.

Similarly, $I_2 = I_{load}$

Hence, $V_1 = 4.95\ V$.

Also, $I_2 = 1\ A$.

Now, for modeling in Tlna, the voltage and current sources controlled by $d(t)$ are implemented using the VCVS and VCCS sources respectively, as shown in [Figure 7](#).

The gain of the VCVS source is set to be $\frac{V_1}{D.D'}$ (since the voltage source obtained in the derived ac model is $\frac{V_1}{D.D'} * \hat{d}$)

so, the gain of VCVS source is 22.31.

The gain of VCCS source must be $\frac{I_2}{D.D'}$ (again, according to derived model).

The gain of VCCS source is calculated to be 4.5.

The turns ratio of the transformer (of the switch model) is $nD/D' = 2$.

Verification:

For a flyback converter, the location of RHPZ and LC resonant pole, is at⁽¹⁾:

$$f_{RHPZ} = \frac{(R_o * D'^2)}{(n^2 * 2\pi * L_M * D)}$$

$$f_{LC} = \frac{D'}{(n * 2\pi * \sqrt{L_M C})}$$

Also, the gain at DC is, $G_{d0} = V_0/DD'$

Using the example values in the above equations, we should have the following:

$$f_{RHPZ} = 21.88 \text{ kHz}$$

$$f_{LC} = 883.3 \text{ Hz}$$

$$G_{d0} = 45.09$$

$$G_{d0}|dB = 33.2 \text{ dB}$$

The circuit is constructed in TINA Simulator as shown in [Figure 7](#).

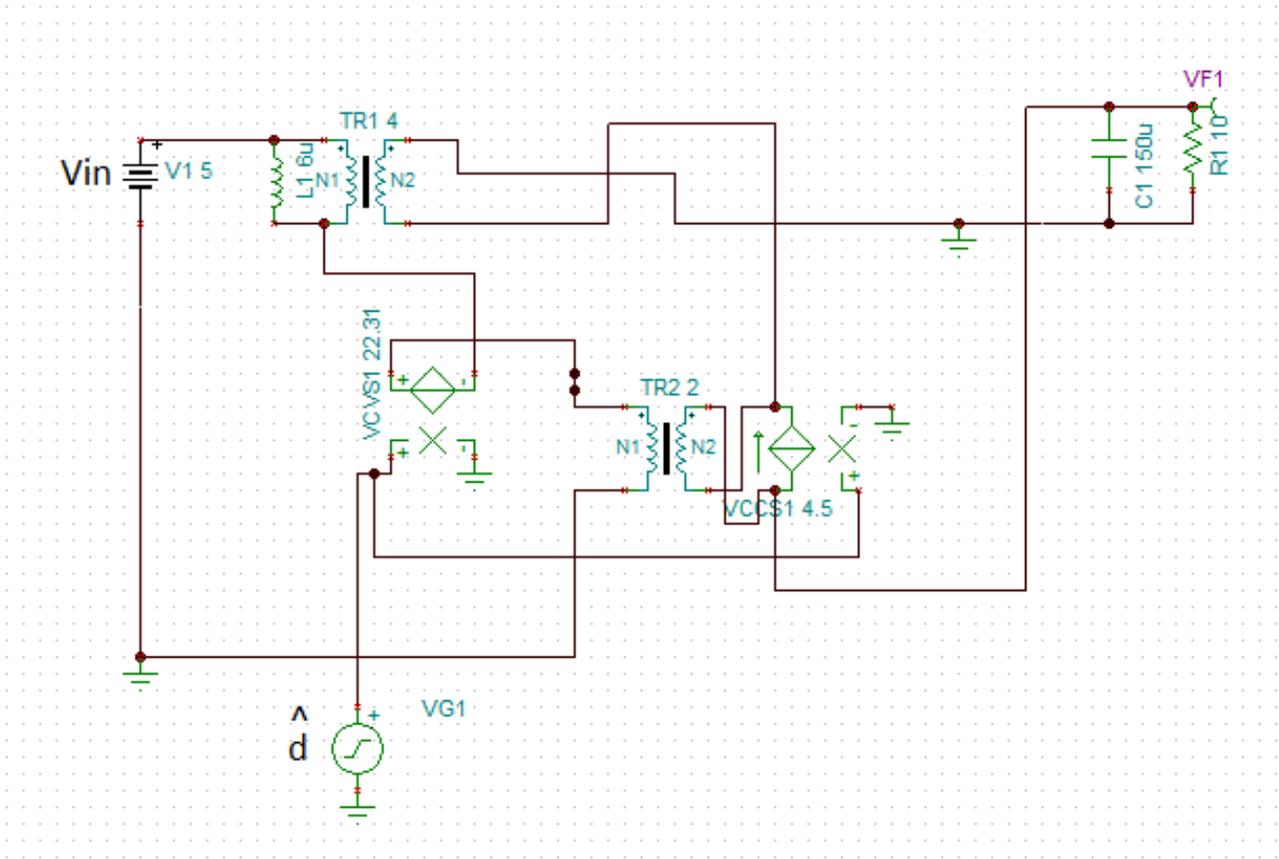


Figure 7. Implementation of the Example Case in TINA Simulator

Running the AC analysis, the bode plot for the control-to-output transfer function is obtained as shown in [Figure 8](#).

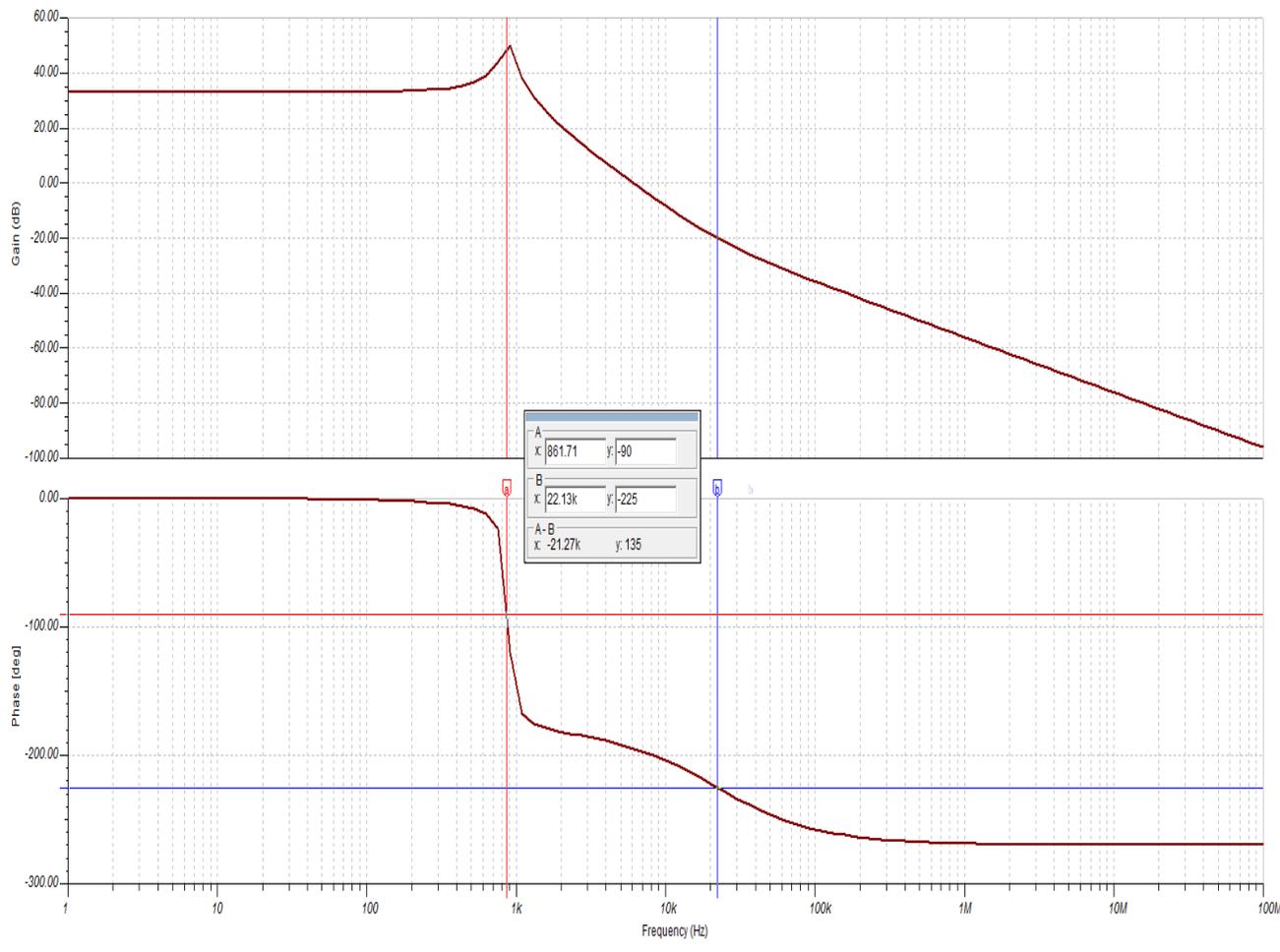


Figure 8. Control-to-Output Transfer Function Plotted using Tina

As seen from [Figure 8](#), the LC resonant pole appears at around 862 Hz.

The RHPZ is located at around 22.13 kHz.

Gain at DC is 33 dB. Hence, it is seen that the response obtained from the model matches closely with the theoretical calculations.

5 Conclusion

The AC model of the power stage for a flyback converter can be constructed following the steps shown in this application report. The response of the model, implemented in a Tina Simulator, is found to be in alignment with what is theoretically expected.

6 References

1. Basso, C., "How to Keep a FLYBACK Switch Mode Supply Stable with a Critical-Mode Controller," ON Semiconductor™, *AN1681/D Application Note*, September 2000 – Rev. 0

IMPORTANT NOTICE

Texas Instruments Incorporated and its subsidiaries (TI) reserve the right to make corrections, enhancements, improvements and other changes to its semiconductor products and services per JESD46, latest issue, and to discontinue any product or service per JESD48, latest issue. Buyers should obtain the latest relevant information before placing orders and should verify that such information is current and complete. All semiconductor products (also referred to herein as "components") are sold subject to TI's terms and conditions of sale supplied at the time of order acknowledgment.

TI warrants performance of its components to the specifications applicable at the time of sale, in accordance with the warranty in TI's terms and conditions of sale of semiconductor products. Testing and other quality control techniques are used to the extent TI deems necessary to support this warranty. Except where mandated by applicable law, testing of all parameters of each component is not necessarily performed.

TI assumes no liability for applications assistance or the design of Buyers' products. Buyers are responsible for their products and applications using TI components. To minimize the risks associated with Buyers' products and applications, Buyers should provide adequate design and operating safeguards.

TI does not warrant or represent that any license, either express or implied, is granted under any patent right, copyright, mask work right, or other intellectual property right relating to any combination, machine, or process in which TI components or services are used. Information published by TI regarding third-party products or services does not constitute a license to use such products or services or a warranty or endorsement thereof. Use of such information may require a license from a third party under the patents or other intellectual property of the third party, or a license from TI under the patents or other intellectual property of TI.

Reproduction of significant portions of TI information in TI data books or data sheets is permissible only if reproduction is without alteration and is accompanied by all associated warranties, conditions, limitations, and notices. TI is not responsible or liable for such altered documentation. Information of third parties may be subject to additional restrictions.

Resale of TI components or services with statements different from or beyond the parameters stated by TI for that component or service voids all express and any implied warranties for the associated TI component or service and is an unfair and deceptive business practice. TI is not responsible or liable for any such statements.

Buyer acknowledges and agrees that it is solely responsible for compliance with all legal, regulatory and safety-related requirements concerning its products, and any use of TI components in its applications, notwithstanding any applications-related information or support that may be provided by TI. Buyer represents and agrees that it has all the necessary expertise to create and implement safeguards which anticipate dangerous consequences of failures, monitor failures and their consequences, lessen the likelihood of failures that might cause harm and take appropriate remedial actions. Buyer will fully indemnify TI and its representatives against any damages arising out of the use of any TI components in safety-critical applications.

In some cases, TI components may be promoted specifically to facilitate safety-related applications. With such components, TI's goal is to help enable customers to design and create their own end-product solutions that meet applicable functional safety standards and requirements. Nonetheless, such components are subject to these terms.

No TI components are authorized for use in FDA Class III (or similar life-critical medical equipment) unless authorized officers of the parties have executed a special agreement specifically governing such use.

Only those TI components which TI has specifically designated as military grade or "enhanced plastic" are designed and intended for use in military/aerospace applications or environments. Buyer acknowledges and agrees that any military or aerospace use of TI components which have **not** been so designated is solely at the Buyer's risk, and that Buyer is solely responsible for compliance with all legal and regulatory requirements in connection with such use.

TI has specifically designated certain components as meeting ISO/TS16949 requirements, mainly for automotive use. In any case of use of non-designated products, TI will not be responsible for any failure to meet ISO/TS16949.

Products

Audio	www.ti.com/audio
Amplifiers	amplifier.ti.com
Data Converters	dataconverter.ti.com
DLP® Products	www.dlp.com
DSP	dsp.ti.com
Clocks and Timers	www.ti.com/clocks
Interface	interface.ti.com
Logic	logic.ti.com
Power Mgmt	power.ti.com
Microcontrollers	microcontroller.ti.com
RFID	www.ti-rfid.com
OMAP Applications Processors	www.ti.com/omap
Wireless Connectivity	www.ti.com/wirelessconnectivity

Applications

Automotive and Transportation	www.ti.com/automotive
Communications and Telecom	www.ti.com/communications
Computers and Peripherals	www.ti.com/computers
Consumer Electronics	www.ti.com/consumer-apps
Energy and Lighting	www.ti.com/energy
Industrial	www.ti.com/industrial
Medical	www.ti.com/medical
Security	www.ti.com/security
Space, Avionics and Defense	www.ti.com/space-avionics-defense
Video and Imaging	www.ti.com/video

TI E2E Community

e2e.ti.com