Switched-mode power supplies (SMPSs) are used to regulate voltage to a certain level. SMPSs have an inherent switching action, which causes the currents and voltages in the circuit to switch and fluctuate. The output voltage also has ripple on top of the regulated steady-state DC value. Designers of power systems consider the output voltage ripple to be both a key parameter for design considerations and a key figure of merit. The online WEBENCH® Power Designer recognizes the key importance of peak-to-peak voltage output ripple voltage—the ripple voltage is calculated and reported in the visualizer [1]. This application report presents a closed-form analytical formulation for the output voltage ripple waveform and the peak-to-peak ripple voltage. This formulation is accurate over all regions of operation and harmonizes the peak-to-peak ripple voltage calculation over all regions of operation. The new analytical formulation presented in this application report gives an accurate evaluation of the output ripple as compared to the simplified linear or root-mean square (RMS) approximations often used.

In this application report, the analytical model for output voltage waveform and peak-to-peak ripple voltage for buck is derived. This model is validated against SPICE TINA-TI simulations. This report presents the behavior of ripple peak-to-voltage for various input conditions and choices of output capacitor and compare it against SPICE TINA-TI results. This report analyzes and presents the validity of the linear and the RMS approximation. Examples from the TI portfolio are presented that compare the experimental ripple waveforms in different regimes.
1 Introduction

The system under consideration is a typical buck SMPS circuit (see Figure 1). The switching action of the synchronized MOSFET causes the current in the inductor to have a triangular waveform [2]. The DC component of the inductor current flows through the output load, and the AC portion of the inductor flows through the output capacitor, as shown in Figure 2. The time-varying current through the capacitor causes the voltage across the capacitor to be perturbed. This application report aims to present an accurate yet easy-to-implement formulation for the output voltage ripple.

![Figure 1. Buck SMPS](image)

![Figure 2. Currents in the Buck SMPS: Inductor Current, Average Output Current, and Current Through the Output Capacitor](image)
2 Output Voltage Ripple Waveform Derivation

To derive the output voltage waveform, assume a simple schematic of the output capacitor (see Figure 3). The inductor current is determined by design parameters like input and output voltage, switching frequency, and inductance. It is important to note the inductor current is not dependent on the choice of the capacitor, and hence can be considered an independent current source. The output capacitor has capacitance, $C$, and is assumed to have an equivalent series resistance (ESR), $R$.

![Figure 3. Schematic for Analytical Derivation of Output Voltage Waveform](image)

The current flowing through the inductor, as shown in Figure 2 and Figure 3, can be written as a sum of the average output current in the load and the ripple current [2].

$$i(t) = I_0 + i_{\text{ripple}}(t)$$  \hspace{1cm} (1)

The ripple current has an average value of 0 due to the charge-second balance principle [2]. The ripple current is composed of two piece-wise linear analytical forms, as shown in Figure 2. The first region is when the high-side MOSFET is turned on for time $T_{on}$, and the second region occurs when it is off for time $T_{off} = T_{sw} - T_{on}$.

$$i_{\text{ripple}}(t) = \begin{cases} i_h(t), & t \in [0, T_{on}] \\ i_l(t), & t \in [T_{on}, T_{sw}] \end{cases}$$  \hspace{1cm} (2)

The total switching time is $T_{sw} = 1 / F_{sw}$. Piece-wise linear current during the two portions can be written in terms of the given peak-to-peak inductor current, $I_{p2p}$:

$$i_h(t) = \frac{L_{p2p}}{2} \left( 1 + \frac{t}{T_{on}} \right)$$

$$i_l(t') = \frac{L_{p2p}}{2} \left( 1 - \frac{t'}{T_{off}} \right)$$  \hspace{1cm} (3)

The origin of the second time segment for analytical is changed for convenience: $t' = t - T_{on}$.

Voltage across the output capacitor is the sum of voltage dropped across the capacitance, $C$, and equivalent series resistance, $R$. According to the Kirchhoff’s Voltage Law:

$$v(t) = v_R(t) + v_C(t)$$  \hspace{1cm} (4)

Using the relationship of current to voltage across the capacitor:

$$v(t) = i(t)R + \int_0^t \frac{i(t)}{C} dt + v_C(t = 0)$$  \hspace{1cm} (5)
With no loss of generality, assume the initial voltage on the capacitor is 0. Use the values from Equation 3 in Equation 5 to get:

\[ v_i(t) = i(t)R + \int_0^t \frac{i(t) \, dt}{C} \]

\[ = R \left( \frac{b_{2p}}{2} \left( -1 + 2 \frac{t}{T_{on}} \right) + \int_0^1 \frac{b_{2p}}{2C} \left( -1 + 2 \frac{t}{T_{on}} \right) \right) \]

\[ = R \left( \frac{b_{2p}}{2} \left( -1 + 2 \frac{t}{T_{on}} \right) + \frac{b_{2p}}{2} \left( -t + \frac{t^2}{T_{on}} \right) \right) \]

Equation 6 is quadratic in time; hence, the output voltage waveform has a parabolic shape.

The voltage reaches its minimum at \( t = t_{min} \), which is obtained by solving the following equation:

\[ \frac{dv_i(t_{min})}{dt} = 0 \]

Solving for \( t_{min} \) by using Equation 6 and Equation 7 we obtain:

\[ t_{min} = \frac{T_{on}}{2} - RC \]

Equation 8 states that the minimum is shifted from center of the on time period \( T_{on} \) by the RC time constant. Because the minimum time cannot be < 0, it implies that \( t_{min} \geq 0 \).

The minimum voltage at \( t_{min} \) is:

\[ v_{out,min} = v_i(t_{min}) = \frac{b_{2p}R^2C}{T_{on}} + \frac{b_{2p}}{2CT_{on}} \left( \frac{T_{on}}{2} - (RC)^2 \right) \]

Similarly, for the second time segment, output voltage is:

\[ v_{ii}(t') = R \left( \frac{b_{2p}}{2} \left( 1 - 2 \frac{t'}{T_{off}} \right) + \frac{b_{2p}}{2} \left( t' - \frac{t'^2}{T_{off}} \right) \right) \]

Whose maximum voltage occurs at:

\[ t_{max}' = \frac{T_{off}}{2} - RC \]

Also, because this time cannot be negative, \( t_{max}' \geq 0 \). The maximum voltage is:

\[ v_{out,max} = v_{ii}(t_{max}') = \frac{b_{2p}R^2C}{T_{off}} + \frac{b_{2p}}{2CT_{off}} \left( \frac{T_{off}}{2} - (RC)^2 \right) \]

This section established several relationships. First, the waveform of the output ripple is given by Equation 6 and Equation 10. Second, the maximum output voltage is given by Equation 12, and the minimum output voltage is given by Equation 9. Now, the peak-to-peak ripple voltage in various regimes can be evaluated.
3 Output Peak-to-Peak Ripple Voltage

The output voltage peak-to-peak ripple is the difference between the maximum and minimum of the waveform:

\[ V_{\text{out,p2p}} = v_l(t_{\text{max}}) - v_l(t_{\text{min}}) \]  

The analytical expressions are given in three possible regimes for the output capacitor resistance-capacitance time constant, \( RC \): small, intermediate, and large. The following subsections examine these regimes.

3.1 Small Resistance-Capacitance Case

When the capacitor RC product is small:

\[ t_{\text{min}} = \frac{T_{\text{on}}}{2} - RC > 0, \text{ and } t_{\text{max}}' = \frac{T_{\text{off}}}{2} - RC > 0 \]  

Applying Equation 13, the peak-to-peak output voltage is:

\[ V_{\text{out,p2p}} = \frac{l_{\text{p2p}}}{T_{\text{off}}} R^2 C + \frac{l_{\text{p2p}}}{2C T_{\text{off}}} \left( \frac{T_{\text{off}}}{2} - (RC)^2 \right) + \frac{l_{\text{p2p}}}{T_{\text{on}}} R^2 C + \frac{l_{\text{p2p}}}{2C T_{\text{on}}} \left( \frac{T_{\text{on}}}{2} - (RC)^2 \right) \]  

Noting that:

\[ T_{\text{on}} = D T_s w = \frac{D}{F_{\text{sw}}} \]
\[ T_{\text{off}} = (1-D)T_s w = \frac{1-D}{F_{\text{sw}}} \]
\[ T_{\text{on}} + T_{\text{off}} = T_s w = \frac{1}{F_{\text{sw}}} \]
\[ \frac{1}{T_{\text{on}}} + \frac{1}{T_{\text{off}}} = \frac{F_{\text{sw}}}{D(1-D)} \]

Arrive at the expression for output peak-to-peak ripple voltage:

\[ V_{\text{out,p2p}} = \frac{l_{\text{p2p}}}{8C F_{\text{sw}}} + \frac{l_{\text{p2p}}}{2} \frac{R^2 C}{F_{\text{sw}}} \frac{D}{(1-D)} \]  

Often, engineers think in terms of contribution to output ripple due to the capacitor only, or due to resistance only. The capacitor-only portion is obtained by setting equivalent series resistance to 0, \( R = 0 \), which gives the familiar expression \( V_C = \frac{l_{\text{p2p}}}{8C F_{\text{sw}}} \). If the resistor was alone on the output circuit, then the resistor-only portion is \( V_R = \frac{l_{\text{p2p}}}{R} \). Equation 17 can be written as:

\[ V_{\text{out,p2p}} = V_C + V_R \left( \frac{RC F_{\text{sw}}}{2D(1-D)} \right) \]  

Comparing Equation 18 with Equation 17 reveals the total output ripple cannot be written in terms of a simple sum of ripple due to the capacitor or resistor alone; other factors such as duty cycle, switching frequency, and the values of resistor and capacitor, also play a role. More details are provided in the discussion of linear approximation in Section 6.
3.2 **Large Resistance-Capacitance Case**

When the \( RC \) product is large:

\[
RC > \frac{T_{on}}{2}, \text{ and } RC > \frac{T_{off}}{2}
\]

Use \( t_{min} = 0 \) and \( t'_{max} = 0 \) in Equation 6 and Equation 10 to calculate the peak-to-peak output voltage:

\[
V_{out,p2p} = v_{il} (t'_{max} = 0) - v_{i} (t_{min} = 0) = \frac{I_{p2p} R}{2} - \left( -\frac{I_{p2p} R}{2} \right) = I_{p2p} R
\]

Equation 19 implies that for a large ESR output, voltage ripple is independent of capacitance. Voltage ripple is also independent of other variables such as duty cycle and switching frequency.

3.3 **Intermediate Resistance-Capacitance Case**

When the \( RC \) product is of an intermediate value, it is either:

\[
RC > \frac{T_{on}}{2}, \text{ and } RC < \frac{T_{off}}{2}
\]

Or

\[
RC < \frac{T_{on}}{2}, \text{ and } RC > \frac{T_{off}}{2}
\]

One extreme of the waveform, either the maximum or minimum voltage, is dependent only on peak-to-peak current and resistance. The other side must be estimated from either Equation 9 or Equation 12.

When \( RC > \frac{T_{on}}{2}, \text{ and } RC < \frac{T_{off}}{2} \):

\[
V_{out,p2p} = \frac{I_{p2p} R^2 C}{T_{off}} + \frac{I_{p2p} R^2}{2CT_{off}} \left( \frac{T_{off}}{2} - (RC)^2 \right) + \frac{I_{p2p} R}{2}
\]

When \( RC < \frac{T_{on}}{2}, \text{ and } RC > \frac{T_{off}}{2} \):

\[
V_{out,p2p} = \frac{I_{p2p} R^2 C}{T_{on}} + \frac{I_{p2p} R^2}{2CT_{on}} \left( \frac{T_{on}}{2} - (RC)^2 \right)
\]
4 Validation Against SPICE

The analytical formulation is validated against SPICE TINA-TI simulations [3]. Compare for three cases. Figure 4, shows the simple case of a duty cycle of 0.5 and an ESR of 0. SPICE waveform and the analytical formulation agree with each other. The waveform is quadratic in time in both of the segments. This quadratic shape is sometimes mistaken as sinusoidal. The nonsinusoidal quadratic nature is evident for a duty cycle less than or greater than 0.5, as shown in Figure 5, where we compare the analytical formulation against SPICE TINA-TI, for a duty cycle of 0.25. ESR is set to 0, so the curves show a fully quadratic formulation in both segments. Figure 6 uses the same conditions except that ESR is 0.25 Ω, which makes $RC > T_{on}$. The waveform shape has a sharp triangular-like shape during the on time. The minimum of ripple waveform occurs at time 0. The analytical formulation agrees with SPICE TINA-TI for all cases.

**Figure 4. Comparison of the Analytical Formulation to TINA-TI SPICE Simulation, Duty Cycle 0.5**

| Switching frequency is 125 kHz at duty cycle of 0.5 with Iout peak-to-peak of 2 A. | Output capacitance is 10 µF with an ESR of 0 Ω.

**Figure 5. Comparison of the Analytical Formulation to TINA-TI SPICE Simulation, Duty Cycle 0.25**

| Switching frequency is 125 kHz at duty cycle of 0.25 with Iout peak-to-peak of 2 A. | Output capacitance is 10 µF with an ESR of 0 Ω. |
A Switching frequency is 125 kHz at duty cycle of 0.25 with I_{out} peak-to-peak of 2 A.
B Output capacitance is 10 µF with an ESR resistance of 0.25 Ω.

Figure 6. Comparison of the Analytical Formulation to TINA-TI SPICE Simulation, ESR of 0.25 Ω

5 Results

To show the effect of changing capacitance and resistance, Figure 7 shows the effect of changing equivalent series resistance. Resistance value varies from 0, to small, to intermediate, to large.

The shape of the waveform evolves from quadratic to more triangular. The peak-to-peak value increases in the process.

A As the resistance increases, it first hits the RC ≥ T_{on} for R = 0.15 Ω; RC ≥ T_{off} for R = 0.35 Ω. With the increasing value of resistance, the shape of the waveform becomes more triangular and the ripple peak-to-peak value keeps increasing.

Figure 7. Analytical Output Waveform for Varying Resistance Values
Figure 8 shows the effect of changing the capacitance. Capacitance of the circuit is increased, which causes the peak-to-peak voltage to drop initially. Increasing capacitance makes the RC product become larger than $T_{on}$, and then larger than $T_{off}$. This increased capacitance causes the waveform to become more triangular. Peak-to-peak value ceases to be dependent on capacitance, becoming a function of resistance only at that point.

![Analytical Output Waveform for Varying Capacitance Values](image)

As the capacitance increases, peak-to-peak ripple decreases. As capacitance becomes closer to 80 µF, peak-to-peak ripple voltage ceases to change because it is determined by resistance only.

Figure 8. Analytical Output Waveform for Varying Capacitance Values

Figure 9 shows the impact of changing ESR on the peak-to-peak voltage. Figure 10 shows the impact of increasing capacitance. With the increasing value of capacitance, peak-to-peak ripple continues to decrease. Figure 11 shows the impact of increasing switching frequency, while keeping other variables constant. For higher frequencies, peak-to-peak voltage is lower. In all of the cases, the analytical formulation agrees with results from SPICE TINA-TI.

![Figure 9. Effect of ESR on Peak-to-Peak Ripple Voltage](image)
6 Approximations

For quick calculations, often a simpler model of the output voltage ripple is assumed. The two common models are the linear model and the RMS model.

The linear model assumes that the total output ripple is due to the linear sum of the voltage ripple due to the capacitor alone and the voltage ripple due to resistor alone:

\[ V_{\text{out,p2p,Linear}} = \frac{I_{\text{p2p}}}{8CF_{\text{sw}}} + I_{\text{p2p}}R \]  

Some TI data sheets, assume this ripple model, such as the TPS62290 device [4].

The RMS model assumes that the total voltage ripple is an RMS sum of the voltage ripple due to the capacitor alone and the voltage ripple due to the resistor alone.

\[ V_{\text{out,p2p,RMS}} = \sqrt{\left(\frac{I_{\text{p2p}}}{8CF_{\text{sw}}}\right)^2 + \left(I_{\text{p2p}}R\right)^2} \]  

Neither the linear model nor the RMS model has any dependence on duty cycle.
Neither of the two implementations can be derived from the analytical model presented. They are acceptable approximations for hand calculations, but the full model should be used for exact calculations. The linear and RMS approximations do not have direct duty-cycle dependence, as shown in Figure 12. Both approximations tend to be in error. The linear approximation can have an error of almost 60%; the RMS approximation has an error of 15%, near the duty cycle of 0.5.

Figure 13 shows the peak-to-peak with varying ESR and also shows that the error for the RMS approximation can be as high as 15%.

A  Both the linear and RMS approximations have no dependence on duty cycle.
B  The linear approximation has an error of 60%; the RMS approximation is closer, but still has an error of 15% at duty cycle of 0.5.

Figure 12. Comparison of the Linear and RMS Approximation Against the Analytical Formulation, Analytical versus Spice

A  The RMS approximation has a maximum error of 15%.

Figure 13. Comparison of the Linear and RMS Approximation Against the Analytical Formulation
The three illustrative types of waveforms are visible on many TI products and EVMs. With ceramic capacitors of very low ESR, the waveform is quadratic in shape and dominated by capacitance. This output waveform ripple is evident in the EVM for the TPS54320 device [5], as shown in Figure 14.

For the intermediate ESR case, the illustrative example is the ripple shown on the data sheet of the TPS53819A device [6], as shown in Figure 15.

The high-ESR electrolytic output capacitor for the TPS5420 device [7] causes the output voltage ripple to be dominated by the ESR, as shown in Figure 16.
8 Practical Implementation

For practical calculation of the output voltage ripple peak-to-peak value, follow this procedure:

1. Compute the on time and off time: $T_{on} = D / F_{sw}$ and $T_{off} = (1 - D) / F_{sw}$.
2. Compute the location of minimum of ripple voltage: $T_{min} = \max(0, T_{on} / 2 - RC)$.
3. Compute the location of maximum of ripple voltage: $T_{max} = \max(0, T_{off} / 2 - RC)$.
5. Compute $V_{out,max}$ using Equation 10.
6. Compute the ripple voltage: $V_{out,p2p} = V_{out,max} - V_{out,min}$. After algebraic manipulation, this reduces to the following form:

$$V_{out,p2p} = V_{out,max} - V_{out,min} = I_{p2p} R \left( 1 - \frac{T_{max}}{T_{on}} + \frac{T_{min}}{T_{off}} \right) + I_{p2p} \frac{T_{max} + T_{min}}{2C} \left( \frac{T_{max}^2}{T_{off}} + \frac{T_{min}^2}{T_{on}} \right)$$

This fully accurate expression can be used instead of the linear or the RMS approximation.

9 Conclusion

This application report presents the analytical model for buck output voltage peak-to-peak ripple voltage. This model is simple and accurate over all ranges of operation and capacitor parameters. This model is validated against SPICE TINA-TI simulations. The behavior of ripple peak-to-voltage for various input conditions was presented and compared against SPICE TINA-TI results. This model can be implemented in code to calculate output peak-to-peak ripple voltage.
10 Bibliography


## Revision History

### Changes from Original (January 2014) to A Revision

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<td>• Updated Equation 8, replaced ‘R’ with ‘RC’</td>
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