Application Report

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Conditional Stability in Feedback Systems

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ABSTRACT

Conditional stability can make a device appear unstable at first glance. This application report attempts to give insight about why a conditional stability system could be stable, as well as provide a mathematical approach to proving the stability. These methods are applied to the part TPS7H1101A-SP, because its response is consistent with being conditionally stable.

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1 Introduction

Traditional stability measurements involve measuring the loop gain of a part. This measurement is accomplished by breaking the loop at the feedback resistor divider and injecting a signal. The gain and phase of the signal are measured from one side of the injected signal to the other. This measurement is done over a wide range of frequencies. A Bode plot, the frequency response of the part, is created from the measurement. Traditional stability says that the phase margin of the part is the phase where the gain of the part goes to zero, and the gain margin is the negative of the gain when the phase crosses over to zero. If the gain margin is large and positive and the phase margin is large and positive, then the part is stable. While this way of doing stability works well, it only takes into account simplistic systems. More specifically, the process does not take into account systems where the phase crossovers occur before the gain crossover, but increase to a positive value before that gain crossover occurs. The TPS7H1101A-SP device has a frequency response similar to what has been described and is used as an example.

2 Closed Loop Gain

The first step in showing how a system with a frequency response like the TPS7H1101A-SP is stable is obtaining an understanding of why it is stable. To obtain this understanding, users must look at the following control loop structure (see Figure 1), where X is the input and Y is the output.

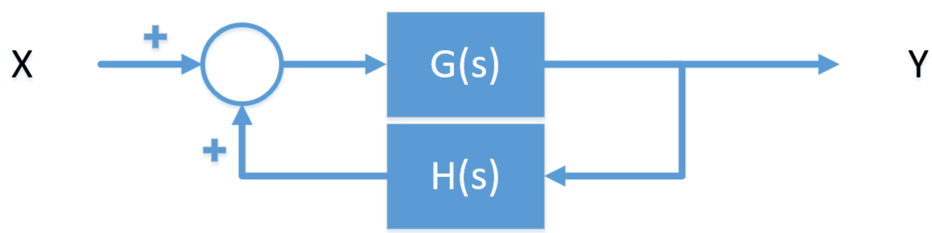

Figure 1. Simple Block Diagram of Control Loop

G(s) and H(s) are transfer functions that describe the feedback loop structure. When H(s) is positive the loop is in negative feedback, and when H(s) is negative the loop is in positive feedback. When the loop structure is in positive feedback, the function that describes the complete control loop is given in Equation 1.

$$\frac{G(s)}{1 - G(s)H(s)} \tag{1}$$

Equation 1 is unstable when the loop gain, G(s)H(s), is equal to 1, so this behavior is avoided. In negative feedback, Equation 2 describes the closed loop gain.

$$\frac{G(s)}{1 + G(s)H(s)} \tag{2}$$

Similar to positive feedback, the equation for negative feedback is unstable when the loop gain is equal to –1. An important point to take from these two equations is that positive and negative feedback are very similar to one another. For large values of the loop gain, the equations for closed-loop gain are almost just out-of-phase with each other. For small values of the loop gain, the equations are almost exactly the same. It is only for values very close to 1 that the equations deviate drastically.

If the loop gain is 100, in negative feedback the closed loop gain (Y/X) would be G(s)/101, while in positive feedback the closed loop gain would be –G(s)/99. In either case, the gain is finite and thus the closed control loop is not necessarily unstable. This is similar to how when designing a closed-loop control system, the desired loop gain when in positive feedback (or the frequency of the gain margin and beyond) is small. This occurrence is due to the previously mentioned idea that positive and negative feedback are almost the same for small loop gains. If the loop gain is 0.01, then the closed loop gain is G(s)/1.01 in negative feedback and G(s)/0.99 in positive feedback. What really matters is the phase around the crossover frequency of the device, which is where loop gains near 1 occur. A device is stable as long as it is in negative feedback at the crossover frequency. Experiments were conducted on control loops with positive feedback before the crossover frequency, where the phase of the crossover frequency was controlled to be positive and negative [1]. These experiments showed the control loop as stable when the phase margin was positive and unstable when the phase margin was negative.

This finding is not to say that having a dip of positive feedback is desired before the crossover frequency. If the crossover frequency were to move to an area where there was positive feedback, it would cause the device to become unstable. Intuitively, negative gain margin of a device does not necessarily mean that a device is unstable. Gain margin is meant to show that for changes in the gain of a device over process, there is a buffer between the gain dropping to 0 and where the phase crosses over into positive feedback. It is more useful to look at the absolute value of that gain margin, because the gain of a device is stable during positive feedback for both very small gains and very large gains.

3 TPS7H1101A-SP Stability

The transient response of the TPS7H1101A-SP device was throughly tested, and the device is known to be stable from these transients. Because the transients are stable, the frequency response of the device is also be stable.

The frequency response of the TPS7H1101A-SP device was found using the TI EVM, with V_{IN} = 3.3 V, V_{OUT} = 2.5 V, and I_{OUT} = 2.3 A, measured in Figure 2.

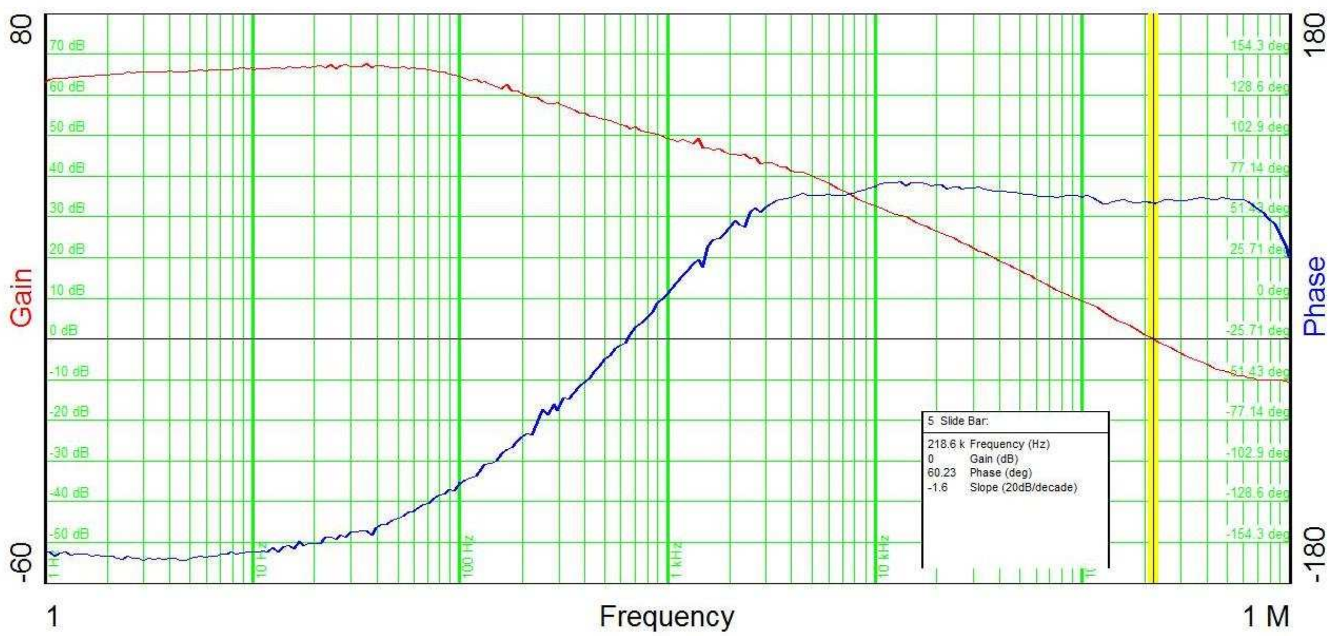

Figure 2. Frequency Response of TPS7H1101A-SP

The main difference between this frequency response and commonly seen responses is the fact that the phase increases for frequencies lower than 1 kHz, but higher than DC. As explained in Section 2, this finding does not mean the device is unstable. The fact that the gain is so high means the device is stable, unless there is a very large, negative change in the gain of the device. The negative phase is a result of the fact that the device is sometimes in positive feedback for very low frequencies.

The device remains in negative feedback for its DC gain and frequencies very near it. This fact is important because the phase of the DC gain determines the DC operating point of the device. While Bode measurements will not go down to DC, remember that the device acts the same as any other LDO when it comes to its DC operating point, and the phase trains off the Bode plot coming down to 180 degrees. It is because of these factors that it is safe to say the device is in negative feedback for frequencies at and near DC.

Connecting the previous ideas to gain margin and phase margin, the phase margin of the device is the same as any other device. The phase at the crossover frequency is the phase margin of the device. The gain margin of the device is a little different. The device, instead of having one gain margin, has two. These gain margins are the absolute value of the gain at each point of phase crossover. The high-frequency phase crossover happens at higher frequencies than the graph shows, and the low frequency gain margin happens around 700 Hz.

When it comes to gain margin, the more gain there is at the point of phase crossover the better. It does not matter if it is in the positive or negative direction. This occurrence is due to what was previously discussed, where for very large and small gains positive and negative feedback is similar and stable. What does matter is that the phase being negative is no where near the crossover frequency.

The TPS7H1101A-SP device, with the Bode plot shown, is stable for all frequencies. The fact that at low frequencies the phase is different than traditional LDOs does not effect the stability of the part.

Two models were created similar to the TPS7H1101A-SP device, where one model included the loop that added the positive feedback into the part and one model took that loop out. Figure 3 shows the frequency response of the two parts. The model that includes the positive feedback loop shows the gain in pink and the phase in red. The model without the positive feedback loop shows the gain in yellow and the phase in green.

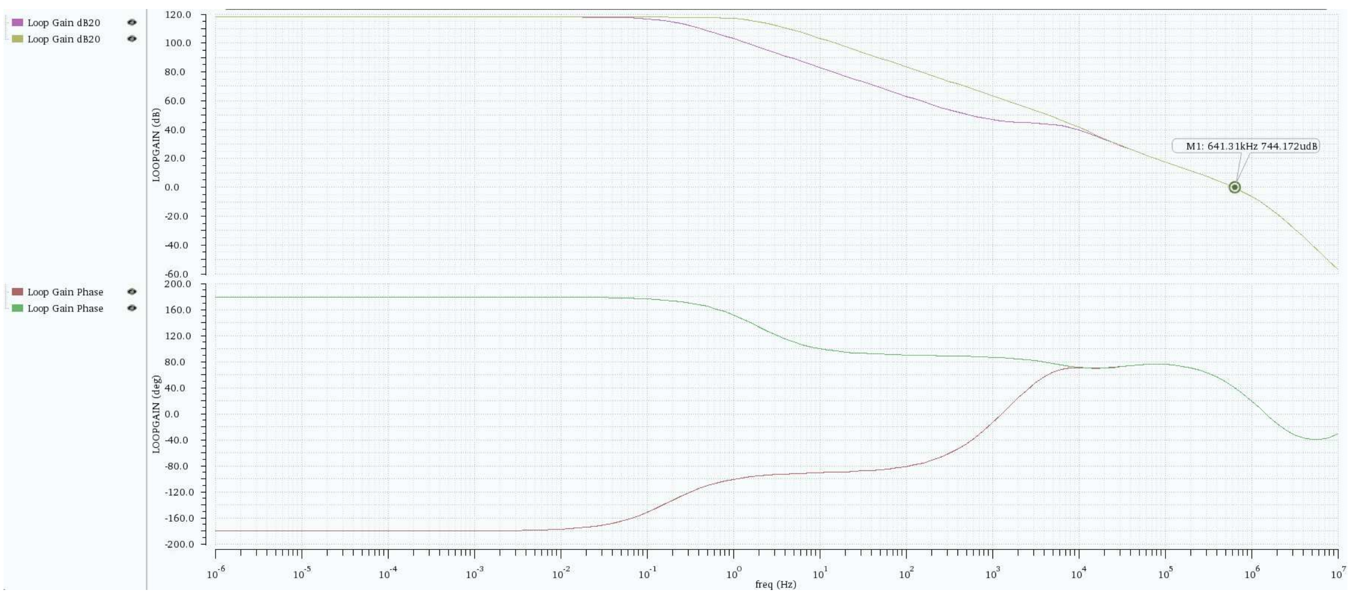

Figure 3. Frequency Responses of Models

Notice that the models become the same for frequencies after about 1 kHz. This shows the positive-feedback loop only changes the response for frequencies before the crossover frequency, and thus the two models have the same crossover frequency and phase margin. Crossover frequency and phase margin are the two major contributors that determine the transient response of a control loop, and this was supported when the models had their transient responses measured, shown in Figure 4.

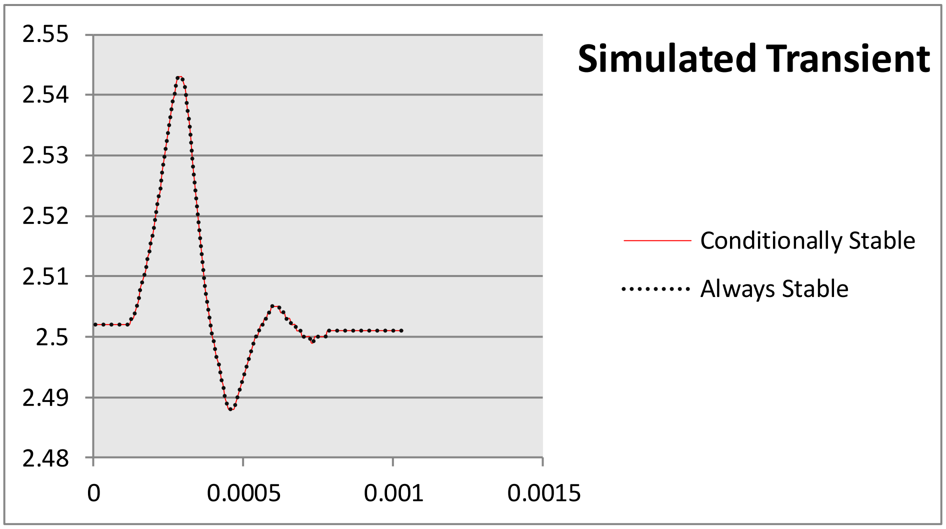

Figure 4. Transient Responses of Models

The models, despite one having positive feedback, have the same transient response. This supports the idea that positive feedback is only an issue for frequencies near the crossover frequency. Parts like the TPS7H1101A-SP that have positive feedback for some frequencies not near their crossover frequency can be just as stable as parts that do not include positive feedback. The models are supported when looking at a load step from 2.75 A to 0 A for the TPS7H1101A-SP, as shown in Figure 5.

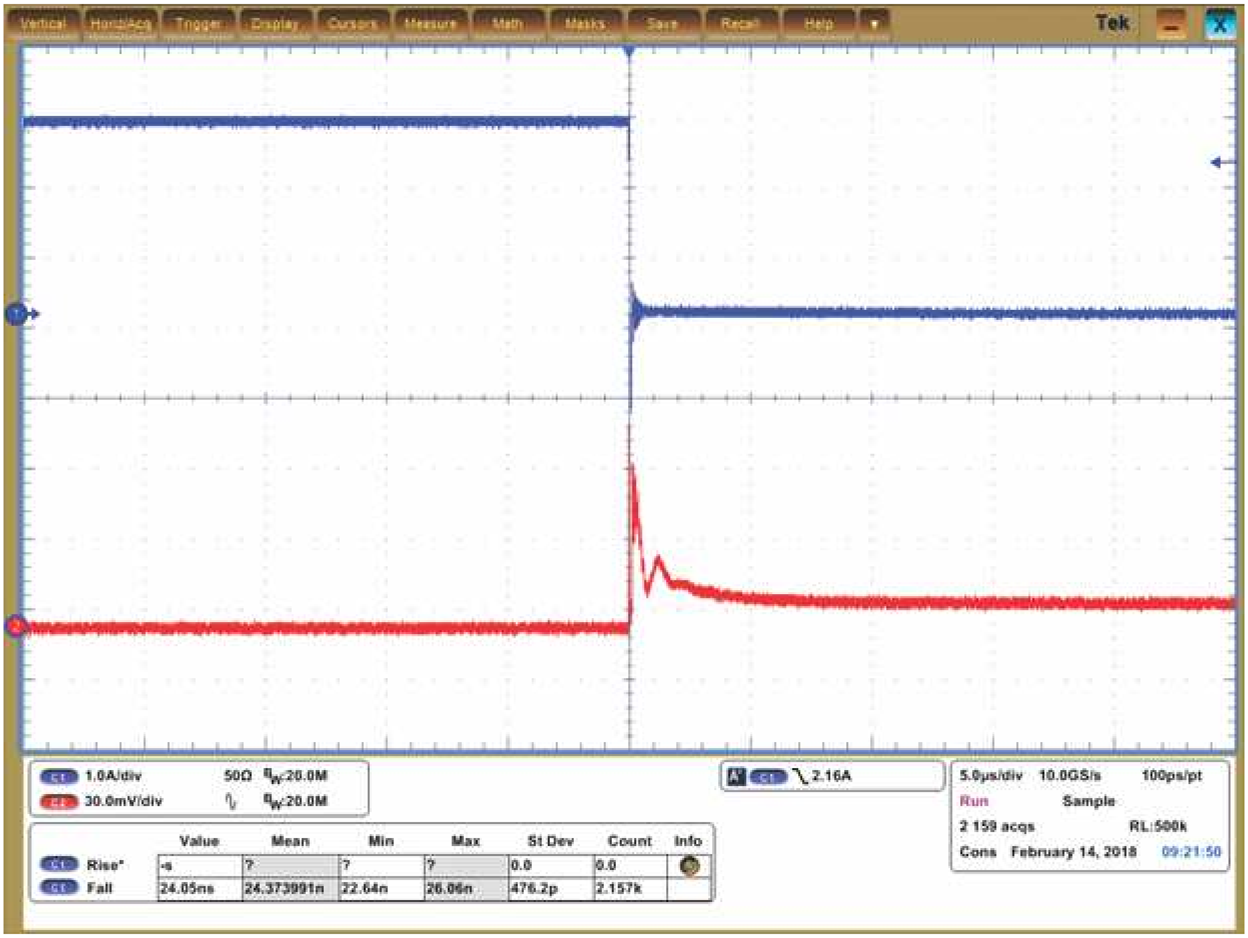

Figure 5. Transient Response of TPS7H1101A-SP

4 Nyquist Stability

Nyquist stability is a stability method that was used as the basis for the traditional Bode plot stability that is used today. The stability method focuses on the denominator of the negative-feedback closed-control loop. Nyquist diagrams is a transform of the transfer function of the control system that plots it on real and imaginary axises. Figure 6 shows an example of what a Nyquist diagram may look like.

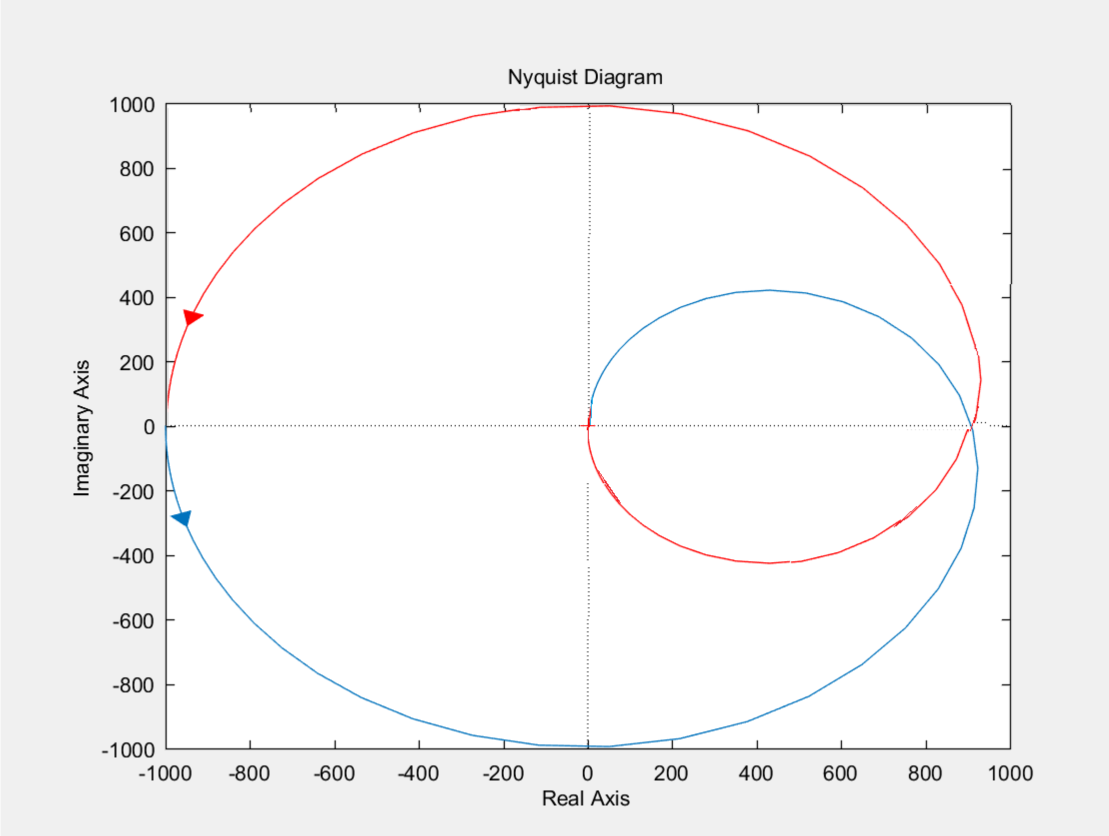

Figure 6. Nyquist Plot Example

The diagrams are also always symmetrical because half of the frequencies the diagram plots are positive and half of them are negative. In the graph, the blue line is the positive frequencies and the red line is the negative frequencies. The arrows in the diagram point toward higher frequencies for positive frequencies and lower frequencies for negative frequencies. These facts are important for later when the Nyquist diagram of the TPS7H1101A-SP device is found on bench and it looks like only half of the diagram is there. Additionally, no arrows are present in the diagram. Part of the reason only half of the graph is there is that the positive frequencies are measured, and the negative frequencies are assumed to be symmetrical around the x-axis.

To determine if a system is stable using Nyquist stability, use Equation 3 [2].

$$Z = P + N$$

where

- Z = The number of right half plane (RHP) poles of the closed-loop system
- P = The number of RHP poles in the loop gain system
- N = The number of clockwise rotations around –1 (a counterclockwise rotation around –1 is counted as a negative rotation) (3)

The closed-loop system is stable when there are no RHP poles in the system. This fact is because an RHP pole in a transfer function becomes an exponential with a positive exponent in the time domain, as given in Figure 7.

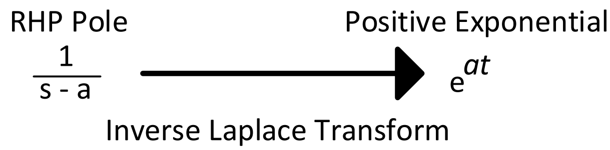

Figure 7. Right Half Plane Pole

RHP poles can be identified in a Bode plot as a pole where instead of the phase decreasing, the phase increases. RHP poles correspond to an unstable equation in time, therefore if there are any in the closed loop function, the control loop is unstable. Thus for Equation 3, the closed-loop transfer function of a system is stable when $Z \leq 0$.

5 Nyquist Analysis of TPS7H1101A-SP

The different closed-loop response of the TPS7H1101A-SP device changes the equation that the Nyquist plot must consider. While most applications of Nyquist plots have the encirclements around the point (–1,0), the TPS7H1101A-SP device has the encirclements around the point (1,0).

This difference is because the TPS7H1101A-SP, and other parts, create a different denominator than the denominator that traditional Nyquist stability considers. While Nyquist stability has a denominator that looks like 1 + G(s)H(s), the TPS7H1101A-SP device has a denominator that looks more like –1 + G(s)H(s). This discrepancy is due to the instability point and how Bode plots relate to Nyquist plots.

To obtain an understanding of why the point changes from (–1,0) to (1,0), the time at which a control loop is considered unstable must be examined. The transform that creates Nyquist diagrams plots the magnitude and phase of the data in the Bode plot on a circular plot, with the magnitude as the radius and the phase as the angle from the left-hand side x-axis. This plotting is very similar to polar graphs. In traditional bode plots for power control loops, the point at which a device is considered unstable is at 0 dB and 0 degrees of phase margin. Because 0 dB corresponds to 1 in amplitude this makes the point at which the device is unstable at (1,0). While traditional bode plots for power control loops start at 180 degrees and decrease to 0 degrees, this is not true of traditional Nyquist stability. Traditional Nyquist stability starts at 0 degrees and decreases to –180 degrees [2]. For traditional Nyquist stability, the angle would therefore be –180 and the magnitude is 0 dB, which is the point (–1,0). This idea has a basis in the mathematics behind Nyquist stability.

Nyquist stability can be built from Cauchy's argument principle [2], also known as the principle of variation of the argument. Cauchy's argument principle is used to prove Nyquist stabilityReference [2]. The source shows how Nyquist stability can be built from Cauchy's principle by setting the function f(s) = 1 + L(s), where L(s) is the loop-transfer function. Where f(s) would have encirclements around the origin, the 1 in the equation simply shifts the origin over to (–1,0). With the TPS7H1101A-SP device, the denominator 1 – L(s), can be found from the bode plot frequency response, starting at 180 degrees. This act changes the equation that plots the Nyquist stability from f(s) = 1 + L(s) to f(s) = 1 - L(s). This act rotates the point (–1,0) to (1,0) as the negative rotates the function by 180 degrees. The direction of the arrow is conserved from this rotation.

The first step to determining the stability of the TPS7H1101A-SP device is looking at the Bode plot of the frequency response and looking for any RHP poles. Figure 8 shows the frequency response of the TPS7H1101A-SP device for convenience.

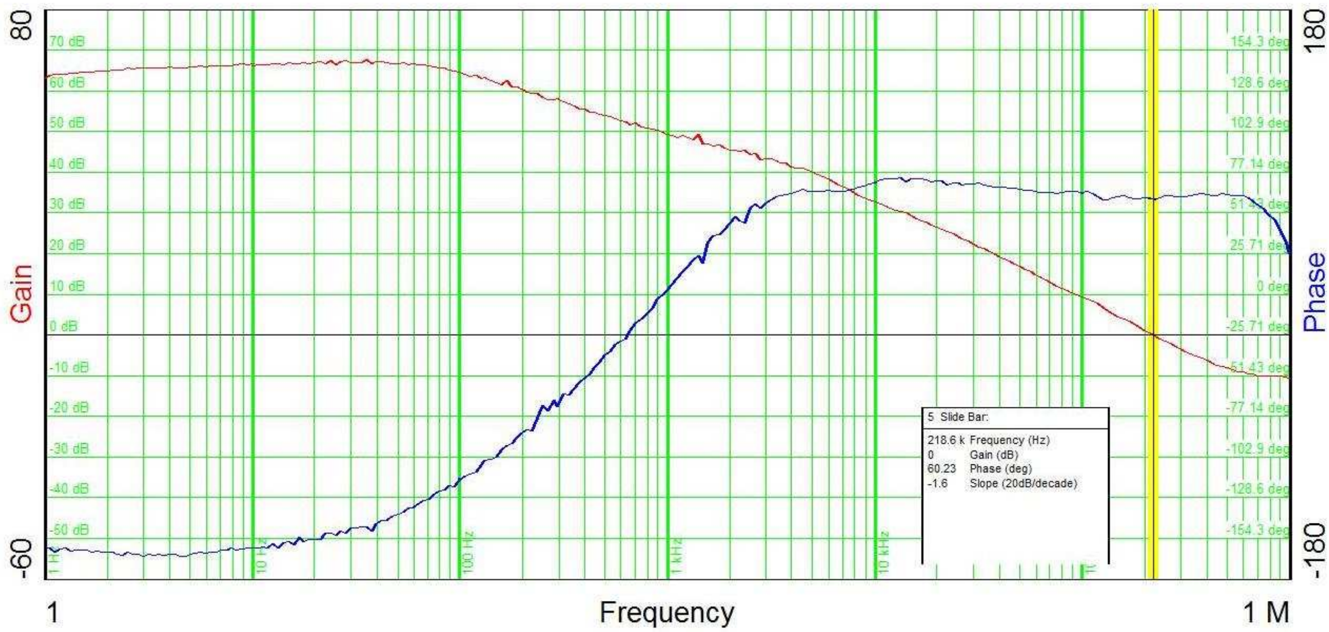

Figure 8. Frequency Response of TPS7H1101A-SP

Looking at the Bode plot there is one instance of the gain decreasing and the phase increasing. There is what looks like a RHP pole in the region of 10 to 100 Hz, but while the phase generally increases by only 90 degrees, the phase ends up increasing by around 250 degrees. Given the phase change, it looks like there may be three RHP poles in the response, however given that the gain only decreases by around 20 dB/decade this seems unlikely. An explanation that fits this behavior is that there are two RHP poles and one zero. Adding these poles and zeros together around the location gives a slope of –20 dB/decade and still has that very large phase change. This occurrence means that for the Nyquist stability equation P = 2.

The next step in determining the stability of the device is finding the number of encirclements around 1 (instead of –1, as previously explained in this section). Figure 9 shows a Nyquist plot that was taken using the same machine and setup that the Bode plot used.

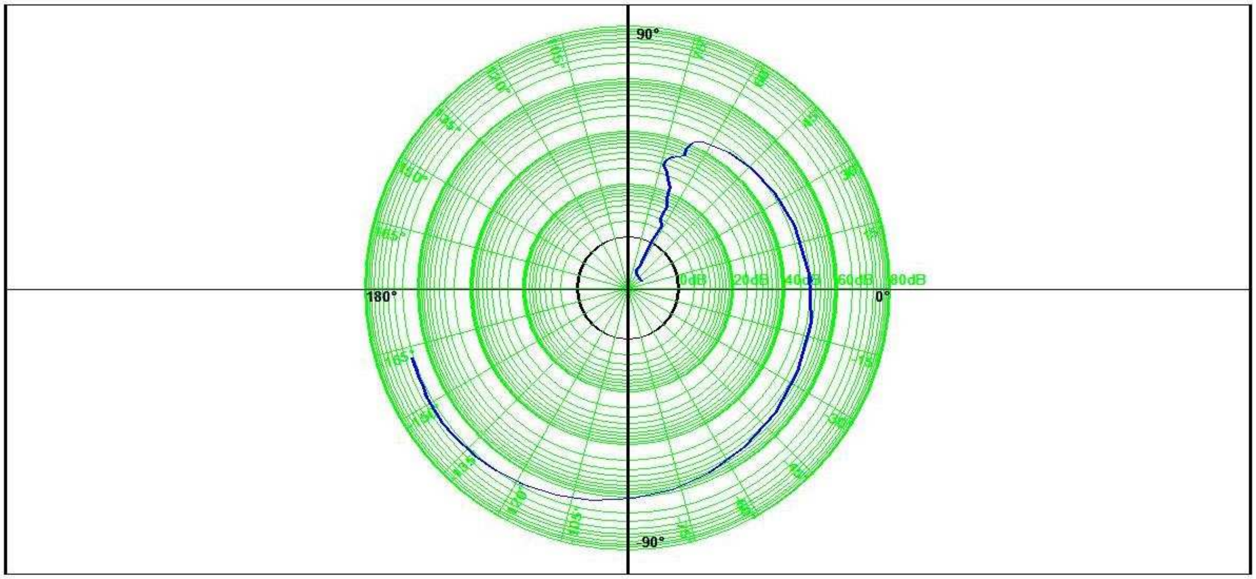

Figure 9. Nyquist Plot of TPS7H1101A-SP

Remember that as explained in Section 4, the results must be mirrored about the x-axis to get the full picture, because negative frequencies are not measured. What is not shown on the plot is that the direction of the plot is counter clockwise. This occurrence is because the low frequencies measured start on the far left of the graph and then follow the blue line as the frequencies increase. The 1 on this plot is the right most point of the 0 dB circle. If the results are mirrored about the x-axis, as shown in Section 4, there are –2 encirclements (or 2 counterclockwise encirclements) around point 1.

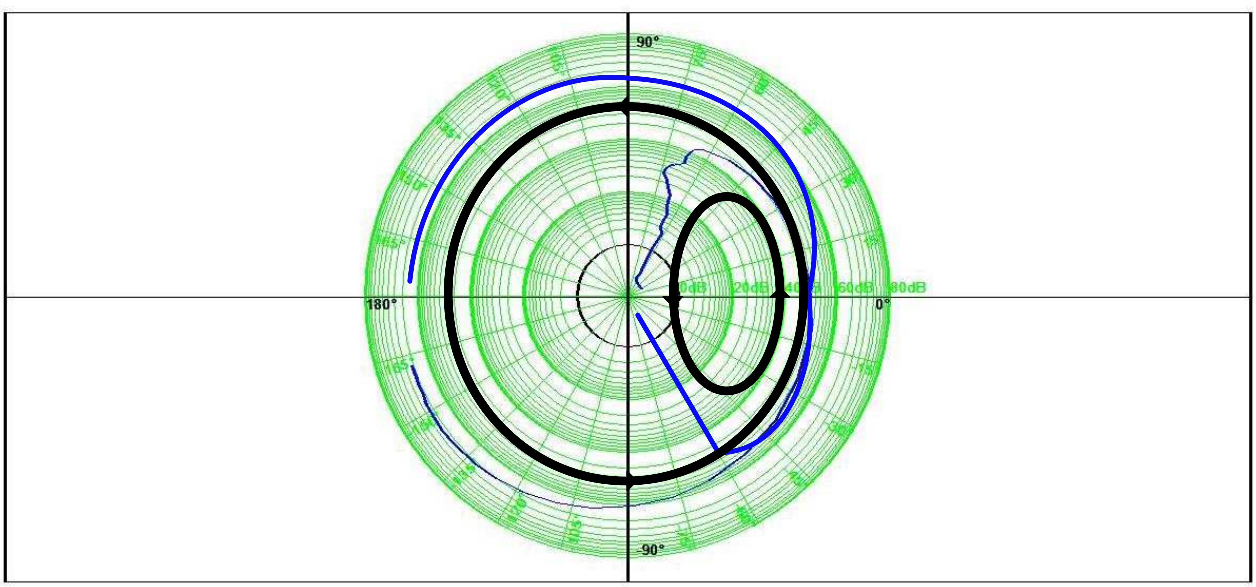

Figure 10. Nyquist Plot of Mirrored Response

One encirclement is caused by the half circle at the bottom of the plot, and the other encirclement is caused by the half circle at the top of the plot, which means that N = -2.

Using Equation 3 for Nyquist stability, Z = P + N becomes Z = 2 + (–2) and thus Z = 0. Because Z ≤ 0, the closed-loop system for the device is stable.

The phase margin of the device can also be concluded from the Nyquist graph. The angle from the positive x-axis, to the point at which the line intercepts the 0 dB circle, is considered the phase margin. Looking at the Nyquist graph, the point at which the line intercepts the circle is at a little above 60 degrees from the positive x-axis. This correlates well to the 60.23 degrees of phase margin that were found in the Bode plot for the control loop.

6 Conclusion

Control loops that have phase crossovers before their crossover frequency are not always unstable. The equations for closed loop gain show how the phase that really matters is the phase of the crossover frequency. This fact is because for very large and very small gains, positive and negative feedback are very similar. As long as the phase at the crossover frequency indicates the device is in negative feedback, the device is stable. A minor tweak to Nyquist stability can be used to verify the stability of the device. Both Nyquist stability and Bode plot stability can be used to find the phase margin of the device.

7 References

1. Roberge, James. *Lecture 17: Conditional Stability*. Video. MIT OpenCourseWare, Massachusetts Institute of Technology. 2017.
2. Åström, Karl Johan and Murray, Richard M. Frequency Domain Analysis. In *Feedback Systems: an Introduction for Scientists and Engineers*, 267–292. Princeton University Press, 2011.

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