Calculating noise figure in op amps

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Introduction
Noise figure is commonly used in communications systems because it provides a simple method to determine the impact of system noise on sensitivity.

Today, the performance of wide-band op amps is making them viable alternatives to more traditional open-loop amplifiers like monolithic microwave integrated circuits (MMICs) and discrete transistors in communications design.

Recognizing the need to specify wide-band op amps in RF engineering terminology, some manufacturers do provide noise figure, but they seem to be the exception rather than the rule.

Op amp manufacturers typically specify noise performance by giving the input-referred voltage and current noise. The noise figure depends on these parameters, the circuit topology, and the value of external components. If you have all this information, noise figure can be calculated.

Review of noise figure
Noise figure (NF) is the decibel equivalent of noise factor (F): NF (dB) = 10log(F).

Noise factor of a device is the power ratio of the signal-to-noise ratio (SNR) at the input (SNR_I) divided by the SNR at the output (SNR_O):

F = \frac{\text{SNR}_I}{\text{SNR}_O} \quad (1)

The output signal (S_O) is equal to the input signal (S_I) times the gain: S_O = S_I \times G. The output noise is equal to the noise delivered to the input (N_I) from the source plus the input noise of the device (N_A) times the gain: N_O = (N_I + N_A) \times G. Substituting into Equation 1 and simplifying, we get

F = \frac{\frac{S_I}{N_I}}{\frac{G \times S_I}{G(N_I + N_A)}} = 1 + \frac{N_A}{N_I} \quad (2)

Assuming that the input is terminated in the same impedance as the source, N_I = kT = -174 dBm/Hz, where k is Boltzman's constant and T = 300 Kelvin). Once we find the input noise spectral density of the device, it is a simple matter to plug it into Equation 2 and calculate F.

NF in op amps
Op amps specify input-referred voltage and current noise. Using these two parameters, adding the noise of the external resistors, and calculating the total input-referred noise based on the circuit topology, we can calculate the input spectral density and use it in Equation 2.

In this discussion, the terms “op amp” and “amplifier” mean different things. “Op amp” refers to only the active device itself, whereas “amplifier” includes the op amp and associated passive resistors that make it work as a usable amplifier stage. In other words, the amplifier is everything shown in Figures 1–3 except R_S, and the op amp is only the components within the dashed triangles. In this way, the plane marked N_A and N_I is the input to the amplifier. This is the point to which the noise sources must be referred so that Equation 2 can be used.
The noise from the source and the input noise of the amplifier are referred to the same point. Because the impedance is the same, expressing the ratio between \( N_A \) and \( N_I \) as a voltage ratio squared is equivalent to the power ratio. An op amp is a voltage-driven device, so using voltage-squared terms makes the calculations easier. In the following discussion, voltage-squared terms are used for \( N_A \) and \( N_I \).

Op amps use negative feedback to control the gain of the amplifier. One result is that the voltage across the input terminals is driven to zero. This is often referred to as a “virtual short.” It is used in the following analysis* and referred to as “amplifier action,” since it is a by-product of the op amp doing its job as an amplifier.

Superposition is used throughout the analysis, wherein all sources except the one under consideration are defeated—voltage sources are shorted and current sources are opened.

**Non-inverting amplifier**

Of the three basic op amp circuits, it is easiest to find the input-referred noise for the non-inverting op amp amplifier, so it will be discussed first. Figure 1 shows a noise analysis diagram for a non-inverting op amp amplifier with the noise sources identified.

The source resistance \( R_S \) generates a noise voltage equal to \( \sqrt{4kTR_S} \). The noise voltage delivered to the amplifier input from the source is divided by the resistors \( R_S \) and \( R_T \). Therefore,

\[
N_I = 4kTR_S \left( \frac{R_T}{R_S + R_T} \right)^2.
\]

\( R_T \) is typically used to terminate the input so that \( R_T = R_S \), in which case \( N_I = kTR_S \).

The amplifier’s voltage noise is a combination of \( i_{ni} \), \( i_{ni} \), and \( i_{ni} \) with associated impedances \( e_F \), \( e_G \), and \( e_F \). These are all referred

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*The virtual-short concept simplifies the analysis. Much more work is required to obtain the same results by other means such as nodal analysis.
to the input by their respective scaling factors and summed to find $N_A$; i.e.,

$$N_A = c_1e_1^2 + c_2e_2^2 + c_3e_3^2 + c_4e_4^2 + c_5e_5^2 + c_6e_6^2,$$  \hspace{1cm} (3)

where $c_1$ through $c_6$ are the scaling factors.

The op amp's input voltage noise is $e_{ii}$. It appears directly at the amplifier's input and its scaling factor is 1 or unity, so that $c_1e_{1i}^2 = e_{1i}^2$.

The op amp's non-inverting input current noise is $i_{ii}$. It develops a voltage through the parallel combination of $R_S$ and $R_T$, which appears directly at the amplifier's input, so that

$$c_{2i}^2_{ii} = i_{ii}^2 \left( \frac{R_S}{R_S + R_T} \right)^2,$$

The op amp's inverting input current noise is $i_{ii}$. It develops a voltage through $RT$ that appears directly at the amplifier's input, so that

$$c_{3i}^2_{ii} = i_{ii}^2 \left( \frac{R_T}{R_T + R_G} \right)^2.$$

The noise voltage term $e_p$ associated with $R_T$ is equal to $\sqrt{4kTR_T}$. It is divided by the resistors $R_S$ and $R_T$, so that

$$c_{4p}^2 = 4kTR_T \left( \frac{R_S}{R_S + R_T} \right)^2.$$

If $R_T = R_S$, then $c_{4i}^2 = kTR_T$.

The noise voltage term $e_G$ associated with $R_G$ is equal to $\sqrt{4kTR_G}$. This noise is divided by the resistors $R_F$ and $R_G$ and applied to the op amp's inverting input. Again by amplifier action, noise from $R_G$ appears at the amplifier's input, so that

$$c_{5G}^2 = 4kTR_G \left( \frac{R_F}{R_F + R_G} \right)^2.$$

The noise voltage term $e_F$ associated with $R_F$ is equal to $\sqrt{4kTR_F}$ and appears at the amplifier's output. Dividing by the signal gain gives us

$$c_{6e}^2_F = 4kTR_F \left( \frac{R_G}{R_F + R_G} \right)^2.$$

With all the terms in Equation 3 quantified, we can take the sum to find $N_A$ and use $N_A$ along with $N_i$ in Equation 2 to find $F$.

**Inverting amplifier**

Finding the input-referred noise of an inverting op amp amplifier is more cumbersome than finding that of a non-inverting op amp amplifier. The main problem is that the signal gain of the amplifier and the noise gain are different.

Figure 2 shows a noise analysis diagram for an inverting op amp amplifier with the noise sources identified.

To find the input-referred noise, it is easiest in some cases to find the output noise and then divide by the signal gain of the amplifier.

The noise voltage delivered to the input from the source is divided by the resistors $R_S$ and $R_M$ in parallel with $R_G$. Therefore,

$$N_i = 4kTR_S \left[ \frac{R_M}{R_S(R_M + R_G) + (R_M R_G)} \right]^2,$$

$R_M$ is typically selected so that $R_M \parallel R_G = R_S$, in which case $N_i = kTR_S$.

The amplifier’s input-referred voltage noise is a combination of $e_{ii}$ and $i_{ii}$ with associated impedances $e_T$, $e_G$, and $e_M$. These are all referred to the input by their respective scaling factors and summed to find $N_f$, i.e.,

$$N_f = c_{1i}e_{1i}^2 + c_{2i}e_{2i}^2 + c_3e_3^2 + c_4e_4^2 + c_5e_5^2 + c_6e_6^2.$$

The op amp’s input voltage noise, $e_{ii}$ at the op amp’s non-inverting input appears at the amplifier output as a function of the amplifier noise gain,

$$1 + \frac{R_F}{R_G + \frac{R_G R_M}{R_S + R_M}},$$

and is then referred back to the amplifier input as a function of the signal gain, $R_F/R_G$. Thus,

$$c_{2i}^2_{iii} = c_{2i}^2_{ii} \left( \frac{R_G}{R_F} + \frac{R_G R_M}{R_S + R_M} \right)^2.$$

The op amp’s non-inverting input current noise is $i_{ii}$. It develops a voltage through $R_i$ that appears directly at the amplifier’s input, so that

$$c_{2i}^2_{ii} = i_{ii}^2 \left( \frac{R_G R_M}{R_F + R_G} + \frac{R_F R_M}{R_S + R_M} \right)^2.$$

It is hard to see how to calculate the op amp’s inverting input current noise, $i_{ii}$. Basically, due to amplifier action, the inverting node is at ground so that no current is drawn through the input resistor $R_G$. The noise current flows through $R_F$, producing a voltage at the output equal to $i_{ii} R_F$. Referring to the amplifier’s input results in $c_{2i}^2_{ii} = i_{ii}^2 (R_F)^2$.

The noise voltage term $e_p$ associated with $R_F$ is equal to $\sqrt{4kTR_F}$. Just like $e_{ii}$ it appears at the output as a function
of the amplifier noise gain and is then referred back to the amplifier input as a function of the signal gain, so that

\[
\frac{4kT}{R_M} = kT R^2 \left( \frac{R_G}{R_F} \right),
\]

The noise voltage term \( e_G \) associated with \( R_G \) is equal to \( \sqrt{4kT R^2} \). It is divided by the resistors \( R_G \) and \( R_S \) in parallel with \( R_M \), so that

\[
\frac{4kT}{R_M} = kT R^2 \left( \frac{R_G}{R_F} \right) \left( \frac{R_S}{R_S + R_M} \right) \]

The noise voltage term \( e_F \) associated with each \( R_F \) is equal to \( \sqrt{4kT R^2} \) and appears directly at the amplifier's output. Dividing by the signal gain gives us

\[
\frac{4kT}{R_F} = kT R^2 \left( \frac{R_G}{R_F} \right) \left( \frac{R_S}{R_S + R_M} \right) \]

The noise source \( e_M \) associated with the input termination matching resistor \( R_M \) is equal to \( \sqrt{4kT R^2} \). It is divided by the resistors \( R_M \) and \( R_S \) in parallel with \( R_G \), so that

\[
\frac{4kT}{R_M} = kT R^2 \left( \frac{R_G}{R_F} \right) \left( \frac{R_S}{R_S + R_M} \right) \]

With all the terms in Equation 4 quantified, we can take the sum to find \( N_A \) and use \( N_A \) along with \( N_f \) in Equation 2 to find \( F \).

### Fully differential amplifier

Fully differential op amp amplifiers are very similar to inverting op amp amplifiers, and the analysis follows very closely. Figure 3 shows the noise analysis diagram.

The source resistance generates thermal noise equal to \( \sqrt{4kT R^2} \). The noise voltage delivered to the input from the source is divided by the resistors \( R_S \) and \( R_M \) in parallel with \( 2R_G \). Therefore,

\[
N_f = 4kT R^2 \left( \frac{2R_G}{R_M + 2R_G} \right) \]

\[R_M \text{ is typically selected so that } R_M = 2R_G = R_S, \text{ in which case } N_f = kT R^2.\]

The amplifier's input-referred voltage noise is a combination of \( i_{in} \) and \( i_i \) with associated impedances \( e_G, e_F, \) and \( e_M \). These are all referred to the input by their respective scaling factors and summed to find \( N_A \); i.e.,

\[
N_A = c_4 e_i^2 + c_2 e_{in}^2 + c_3 e_{in}^2 + c_4 e_M^2 + c_6 e_M^2,
\]

where \( c_1 \) through \( c_6 \) are the scaling factors.

In this analysis it is assumed that the two input resistors \( R_G \) are equal and that the two feedback resistors \( R_F \) are equal.

The op amp's input voltage noise, \( e_{in} \), at the op amp's input appears at the amplifier output as a function of the amplifier noise gain,

\[
1 + \frac{R_F}{R_G} + \frac{R_S R_M}{2(R_S + R_M)}
\]

and is then referred back to the amplifier input as a function of the signal gain, \( R_F/R_G \). Thus,

\[
\frac{4kT}{R_M} = kT R^2 \left( \frac{R_G}{R_F} \right) \left( \frac{R_S}{R_S + R_M} \right) \]

Since the input resistors are equal and the feedback resistors are equal, the op amp's non-inverting input current noise, \( i_{in} \), and inverting input current noise, \( i_i \), have the same scaling factors. Due to amplifier action, the input nodes of the op amp are ac grounds so that no current is drawn through the input resistors \( R_G \). All the noise current flows through \( R_S \) producing a voltage at the output equal to \( i_{in} R_F \) or \( i_i R_F \). Referring to the amplifier's input results in \( c_{1/2} e_{in} = i_{in}^2 (R_F)^2 \) and \( c_{3/4} e_i = i_i^2 (R_F)^2 \).

The noise voltage term \( e_G \) associated with each \( R_G \) is equal to \( \sqrt{4kT R^2} \). It is divided by the resistors \( R_G \) and one-half \( R_S \) in parallel with \( R_M \), so that

\[
\frac{4kT}{R_M} = kT R^2 \left( \frac{R_G}{R_F} \right) \left( \frac{R_S}{R_S + R_M} \right) \]

The noise voltage term \( e_F \) associated with each \( R_F \) is equal to \( \sqrt{4kT R^2} \) and appears directly at the amplifier's output. Dividing by the signal gain gives us

\[
\frac{4kT}{R_M} = kT R^2 \left( \frac{R_G}{R_F} \right) \left( \frac{R_S}{R_S + R_M} \right) \]

The noise source \( e_M \) associated with the input termination matching resistor \( R_M \) is equal to \( \sqrt{4kT R^2} \). It is divided by the resistors \( R_M \) and \( R_S \) in parallel with \( 2R_G \), so that

\[
\frac{4kT}{R_M} = kT R^2 \left( \frac{R_G}{R_F} \right) \left( \frac{R_S}{R_S + R_M} \right) \]

As before, with all the terms in Equation 5 quantified, \( N_A \) can be calculated and used with \( N_f \) in Equation 2 to find the noise factor.
**Conclusion**

The input-referred voltage noise and current noise, along with the circuit configuration and component values, can be used to calculate noise figure. This is a tedious task at best. Setting up a spreadsheet for each topology where component values and op amp specs can be entered is recommended. In this way, various scenarios can be quickly tested. Verification by testing the circuit with a noise figure analyzer is always suggested.

As an example of how well the theory outlined in this article matches test results, the noise figure of three op amp amplifiers configured as previously detailed were measured with an Agilent N8973A noise figure analyzer. Table 1 shows that the results are good, with the input current and voltage noise specifications given as typical values.

**Related Web sites**

- [analog.ti.com](http://analog.ti.com)
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**Appendix—Summary of noise terms in op amp amplifiers**

**Signal input noise \( (N_i) \) terms**

<table>
<thead>
<tr>
<th>AMPLIFIER CONFIGURATION</th>
<th>NOISE SOURCE</th>
<th>NOISE CONTRIBUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-inverting</td>
<td>Source thermal noise</td>
<td>[4kT_R S \left( \frac{R_T}{R_S + R_T} \right)^2 ]</td>
</tr>
<tr>
<td>Inverting</td>
<td>Source thermal noise</td>
<td>[N_i = 4kT_R S \left( \frac{R_M R_G}{R_S (R_M + R_G) + (R_M R_G)} \right)^2 ]</td>
</tr>
<tr>
<td>Fully differential</td>
<td>Source thermal noise</td>
<td>[4kT_R S \left( \frac{2R_M R_G}{R_S + 2R_G R_M + 2R_G} \right)^2 ]</td>
</tr>
</tbody>
</table>
## Appendix—Summary of noise terms in op amp amplifiers (Continued)

### Device input noise ($N_A$) terms

<table>
<thead>
<tr>
<th>AMPLIFIER CONFIGURATION</th>
<th>NOISE SOURCE</th>
<th>NOISE CONTRIBUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non-inverting</strong></td>
<td>Op amp input-referred voltage noise</td>
<td>$e_{in}^2$</td>
</tr>
<tr>
<td></td>
<td>Op amp non-inverting input-referred current noise</td>
<td>$i_{in}^2 \left( \frac{R_T R_F}{R_S + R_T} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Op amp inverting input-referred current noise</td>
<td>$i_{in}^2 \left( \frac{R_F R_G}{R_F + R_G} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Termination resistor thermal noise voltage</td>
<td>$4kT R_T \left( \frac{R_S}{R_S + R_T} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Gain resistor thermal noise voltage</td>
<td>$4kT R_G \left( \frac{R_F}{R_F + R_G} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Feedback resistor thermal noise voltage</td>
<td>$4kT R_F \left( \frac{R_G}{R_F + R_G} \right)^2$</td>
</tr>
<tr>
<td><strong>Inverting</strong></td>
<td>Op amp input-referred voltage noise</td>
<td>$e_{in}^2 \left( \frac{R_G}{R_F} + \frac{R_G R_M}{R_S + R_M} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Op amp non-inverting input-referred current noise</td>
<td>$i_{in}^2 \left( \frac{R_F R_G}{R_F + R_G} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Op amp inverting input-referred current noise</td>
<td>$i_{in}^2 (R_G)^2$</td>
</tr>
<tr>
<td></td>
<td>Non-inverting bias matching resistor thermal noise voltage</td>
<td>$4kT R_T \left( \frac{R_G}{R_F} + \frac{R_G R_M}{R_S + R_M} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Gain resistor thermal noise voltage</td>
<td>$4kT R_G \left( \frac{R_G}{R_G + \frac{R_S R_M}{R_S + R_M}} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Feedback resistor thermal noise voltage</td>
<td>$4kT R_F \left( \frac{R_G}{R_F} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Inverting termination matching resistor thermal noise voltage</td>
<td>$4kT R_M \left[ \frac{R_S R_G}{R_M (R_S + R_G) + R_S R_G} \right]^2$</td>
</tr>
</tbody>
</table>
### Appendix—Summary of noise terms in op amp amplifiers (Continued)

#### Device input noise ($N_A$) terms (Continued)

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<tbody>
<tr>
<td></td>
<td>Op amp input-referred voltage noise</td>
<td>$e_{n_{in}}^2 \left( \frac{R_G}{R_F} + \frac{R_G}{R_G + \frac{R_S R_M}{2(R_S + R_M)}} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Op amp non-inverting input-referred current noise</td>
<td>$\frac{i_{n_{in}}^2}{R_G} (R_G)^2$</td>
</tr>
<tr>
<td></td>
<td>Op amp inverting input-referred current noise</td>
<td>$\frac{i_{i_{in}}^2}{R_G} (R_G)^2$</td>
</tr>
<tr>
<td>Fully differential</td>
<td>Gain resistor thermal noise voltage</td>
<td>$2 \times 4kT R_G \left( \frac{R_G}{R_G + \frac{R_S R_M}{2(R_S + R_M)}} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Feedback resistor thermal noise voltage</td>
<td>$2 \times 4kT R_F \left( \frac{R_G}{R_F} \right)^2$</td>
</tr>
<tr>
<td></td>
<td>Termination matching resistor thermal noise voltage</td>
<td>$4kT R_M \left( \frac{2R_S R_G}{R_S + 2R_G} + \frac{R_S + 2R_G}{2R_S R_G} \right)^2$</td>
</tr>
</tbody>
</table>
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