Operational amplifier gain stability, Part 2: DC gain-error analysis

By Henry Surtihadi, Analog Design Engineer, and Miroslav Oljaca, Senior Applications Engineer

Introduction
The goal of this three-part series of articles is to provide readers with an in-depth understanding of gain accuracy in closed-loop circuits using two of the most common operational amplifier (op amp) configurations: non-inverting and inverting. Often, the effects of various op amp parameters on the accuracy of the circuit's closed-loop gain are overlooked and cause an unexpected gain error both in the DC and AC domains.

This article, Part 2, focuses on DC gain error, which is primarily caused by the finite DC open-loop gain of the op amp as well as its temperature dependency. This article builds upon the results obtained in Part 1 (see Reference 1), in which two separate equations were derived for calculating the transfer functions of non-inverting and inverting op amps. Part 2 presents a step-by-step example of how to calculate the worst-case gain error, starting with finding the pertinent data from the product data sheet. It then shows how to use the data in conjunction with the two aforementioned equations to perform the gain-error calculation.

In Part 3, the gain error for AC input signals will be calculated. In the AC domain, the closed-loop gain error is affected by the AC open-loop response of the op amp. Part 3 will discuss one of the most common mistakes that occur when the AC gain response is calculated.

Transfer functions of non-inverting and inverting op amps
In Part 1 (Reference 1), the closed-loop transfer function of the non-inverting op amp configuration in the frequency domain was calculated. Specifically, the transfer function was derived with the assumption that the op amp had a first-order open-loop response. For calculating gain error, the magnitude response is of interest. For convenience, the result is repeated in Equation 1:

$$|A_{CL}(f)|_{dB} = 20 \log \frac{\frac{1}{A_{OL_{DC}}}}{1 + \beta \times \frac{1}{A_{OL_{DC}}}},$$  \hspace{1cm} (1)

where $\beta$ is defined as

$$\beta = \frac{V_{FB}}{V_{OUT}} = \frac{R_F}{R_1 + R_F}. \hspace{1cm} (2)$$

Also derived in the same article was the equation for calculating the magnitude of the inverting configuration's closed-loop gain. The result is repeated in Equation 3:

$$|A_{CL}(f)|_{dB} = 20 \log \frac{\frac{\alpha}{1 + \beta \times A_{OL_{DC}}}}{\sqrt{\frac{1 + \frac{f^2}{f_0^2} \times \frac{1}{(1 + \beta \times A_{OL_{DC}})^2}}}}$$  \hspace{1cm} (3)

Equation 3 uses the same variable $\beta$ defined by Equation 2. Additionally, the variable $\alpha$ is defined by Equation 4:

$$\alpha = \frac{V_{FB}}{V_{IN}} = \frac{R_F}{R_1 + R_F}. \hspace{1cm} (4)$$

At this point, the closed-loop gain for non-inverting and inverting amplifiers is represented by Equations 1 and 3, respectively. These equations will be used for subsequent analysis. The analysis of DC closed-loop circuits has been treated in slightly different ways in References 2 to 7; however, the results agree with this analysis.

DC gain error for non-inverting configuration
To illustrate the impact of an op amp’s finite open-loop gain on the accuracy of DC closed-loop gain in a non-inverting configuration, a step-by-step example will be presented on how to calculate the gain error when the op amp is set in an ideal closed-loop gain. An ideal closed-loop gain of 200 (1/\beta = 200), as shown in Figure 1, will be used. This example focuses on using only the Texas Instruments (TI)
OPA211 op amp, but circuit designers can repeat the calculation with similar values from the data sheet of any other op amp they choose.

To calculate the DC closed-loop-gain error of a non-inverting op amp, Equation 1 is evaluated for zero frequency ($f = 0$ Hz):

$$A_{CL\_DC} = A_{CL}(0 \text{ Hz}) = \frac{A_{OL\_DC}}{1 + \beta \times A_{OL\_DC}} \quad (5)$$

In the case of an ideal op amp with infinite open-loop gain, the DC closed-loop gain of the non-inverting configuration is reduced to

$$A_{CL\_DC(ideal)} = \lim_{A_{OL\_DC} \to \infty} \frac{A_{OL\_DC}}{1 + \beta \times A_{OL\_DC}} = \frac{1}{\beta} \quad (6)$$

In other words, the DC closed-loop gain is entirely determined by the external feedback network.

From the closed-loop models of non-inverting and inverting amplifiers in Figures 3 and 6, respectively, in Part 1 (see Reference 1), it can be seen that the open-loop gain of the op amp is the ratio of $V_{OUT}$ to the input-error voltage, $V_{ERR}$. $V_{ERR}$ is the voltage difference between the inverting and non-inverting op amp inputs. It can also be seen as input offset voltage. In a product data sheet, the open-loop gain is typically expressed in decibels. In this case, the number represents the ratio of $V_{OUT}$ to $V_{ERR}$ in the logarithmic domain. For future calculation, $A_{OL\_DC}$ must always be converted from decibels to $V/V$. As an example, an op amp with an open-loop gain of 106 dB can be written in terms of $V/V$ as

$$A_{OL\_DC(UV)} = 10^{106 \text{dB}} = 10^{10/20} = 10^{6} \frac{V}{V} = 199,526 \frac{V}{V} \quad (7)$$

Figure 2 shows the simplified open-loop gain of the OPA211 along with the closed-loop gain in a non-inverting configuration. The difference between these two curves is the loop gain, $\beta \times A_{OL}$. Because the focus of this example is DC gain error, only the loop gain at low frequency ($\beta \times A_{OL\_DC}$) is of interest.

When using the data from the typical curves, designers should consider possible variations. To calculate worst-case values, the open-loop-gain data provided in the product data sheet should be used. Such data are shown in Table 1 for the TI OPA211/2211 op amps. As the table shows, when the output signal is more than 200 mV from the supply rails and has a 10-kΩ load, the typical value for the DC open-loop gain is 130 dB, while the minimum ensured gain is 114 dB. To calculate the typical and the worst-case DC gain

Table 1. Excerpt from TI OPA211/2211 data sheet

| ELECTRICAL CHARACTERISTICS: $V_S = \pm 2.25$V to $\pm 18$V
| BOLDFACE limits apply over the specified temperature range, $T_A = -40^\circ$C to $+125^\circ$C.

At $T_A = +25^\circ$C, $R_L = 10k\Omega$ connected to midsupply, $V_{CM} = V_{OUT} = $ midsupply, unless otherwise noted.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>CONDITIONS</th>
<th>Standard Grade OPA211A, OPA2211A</th>
<th>High Grade OPA211</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPEN-LOOP GAIN</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Open-Loop Voltage Gain</td>
<td>$A_{OL}(V-) + 0.2V \leq V_O \leq (V+) - 0.2V$, $R_L = 10k\Omega$</td>
<td>114</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td>$A_{OL}(V-) + 0.6V \leq V_O \leq (V+) - 0.6V$, $R_L = 600\Omega$</td>
<td>110</td>
<td>114</td>
</tr>
<tr>
<td>Over Temperature</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OPA211</td>
<td>$A_{OL}(V-) + 0.6V \leq V_O \leq (V+) - 0.6V$, $I_O \leq 15mA$</td>
<td>110</td>
<td>110</td>
</tr>
<tr>
<td>OPA211</td>
<td>$A_{OL}(V-) + 0.6V \leq V_O \leq (V+) - 0.6V$, $15mA \leq I_O \leq 30mA$</td>
<td>103</td>
<td>103</td>
</tr>
<tr>
<td>OPA2211 (per channel)</td>
<td>$A_{OL}(V-) + 0.6V \leq V_O \leq (V+) - 0.6V$, $I_O \leq 15mA$</td>
<td>100</td>
<td>dB</td>
</tr>
</tbody>
</table>
errors at room temperature, the minimum $A_{OL, DC}$ from the data sheet should be substituted into Equation 5. Note that in the OPA211 data sheet, “$A_{OL, DC}$” is written as “$A_{OL}$.”

The first step in this process is to convert $A_{OL, DC}$ from decibels to V/V:

$$A_{OL, DC}^{130 \text{ dB}} = 10^{130 \text{ dB}/20} = 3,162,278 \frac{V}{V}$$

(8)

$$A_{OL, DC}^{114 \text{ dB}} = 10^{114 \text{ dB}/20} = 501,187 \frac{V}{V}$$

(9)

A value for $\beta$ of 1/200 (the ideal closed-loop gain of 200) can be used in Equation 5 to find the typical DC gain:

$$A_{CL, DC}^{130 \text{ dB}} = \frac{A_{OL, DC}}{1 + \beta \times A_{OL, DC}} = \frac{3,162,278}{1 + \frac{1}{200} \times 3,162,278} = 199.98735$$

(10)

The actual minimum ensured DC gain can be found in the same manner:

$$A_{CL, DC}^{114 \text{ dB}} = \frac{501,187}{1 + \frac{1}{200} \times 501,187} = 199.92022$$

(11)

The DC gain error caused by the open-loop-gain value of the op amp can then be calculated:

$$\epsilon_{typ} = \frac{A_{CL, DC(\text{ideal})} - A_{CL, DC}}{A_{CL, DC(\text{ideal})}} \times 100$$

$$= \frac{200 - 199.98735}{200} \times 100 = 0.00632\%$$

$$\epsilon_{max} = \frac{200 - 199.92022}{200} \times 100 = 0.0399\%$$

(12)

(13)

The actual DC closed-loop gain of 199.92 has an error of 0.0399% compared to the desired ideal gain of 200.

Over temperature, the OPA211 is characterized to ensure that $A_{OL, DC}$ is higher than 110 dB over the specified temperature range and when loaded with less than 15-mA output current, which is the absolute worst case. For this value, in terms of V/V, 110 dB is equivalent to

$$A_{OL, DC}^{110 \text{ dB}} = 10^{110 \text{ dB}/20} = 316,228 \frac{V}{V}.$$ 

(14)

This number can be substituted into Equation 5 to find the absolute worst-case condition for the DC closed-loop gain:

$$A_{CL, DC}^{110 \text{ dB}} = \frac{316,228}{{1 + \frac{1}{200}}} = 199.8736$$

(15)

The gain error for this result, 0.063%, represents a slight degradation from the room-temperature case of 0.0399% previously calculated in Equation 13.

**DC gain error for inverting configuration**

To illustrate the impact of the op amp’s finite open-loop gain on the accuracy of DC closed-loop gain in an inverting configuration, another step-by-step example will be presented of calculating the gain error when the op amp is set in an ideal closed-loop gain. This example will use an ideal closed-loop gain of $-200$ ($\alpha/\beta = -200$), as shown in Figure 3. So that results can be properly compared, the same op amp, OPA211, will be used.

Similar to the non-inverting case, to calculate the DC closed-loop-gain error of the inverting op amp, Equation 3 is first evaluated for zero frequency ($f = 0$ Hz):

$$A_{CL, DC} = A_{CL}(0 \text{ Hz}) = -\frac{\alpha}{1 + \beta \times A_{OL, DC}}$$

(16)

The negative sign indicates the inverting configuration.

In the case of an ideal op amp with infinite open-loop gain, the DC closed-loop gain of the inverting configuration is reduced to

$$A_{CL, DC(\text{ideal})} = \lim_{A_{OL, DC} \to \infty} -\frac{\alpha}{1 + \beta \times A_{OL, DC}} = -\frac{\alpha}{\beta}.$$ 

(17)
As in the non-inverting configuration, the DC closed-loop gain is entirely determined by the external feedback network.

With the same open-loop-gain specifications of 130 dB (typical) and 114 dB (minimum) at room temperature, and 110 dB (minimum) across the specified temperature range—i.e., the worst case—the same calculations can be done for the inverting configuration as were done for the non-inverting configuration. For an inverting amplifier with an ideal closed-loop gain of –200 (–a/b = –200), the coefficients \( a = 200/201 \) and \( b = 1/201 \) can be used for the following three gain calculations.

- **Typical DC gain:**
  \[
  A_{CL_{DC}\{130 dB\}} = \frac{-a}{1+b} \times A_{OL_{DC}}
  \]
  \[
  = \frac{-200}{201} \times \frac{3,162,278}{1 + \frac{1}{201} \times 3,162,278}
  \]
  \[
  = -199.98729
  \]

- **Minimum ensured DC gain at room temperature:**
  \[
  A_{CL_{DC}\{114 dB\}} = \frac{-200}{201} \times \frac{501,187}{1 + \frac{1}{201} \times 501,187}
  \]
  \[
  = -199.9198
  \]

- **Worst-case DC closed-loop gain over temperature:**
  \[
  A_{CL_{DC}\{110 dB\}} = \frac{-200}{201} \times \frac{316,228}{1 + \frac{1}{201} \times 316,228}
  \]
  \[
  = -199.87296
  \]

The DC gain error caused by the variation of the open-loop-gain value of the op amp can then be calculated:

- **Typical:**
  \[
  \varepsilon_{\text{typ}} = \frac{A_{CL_{DC}\{\text{ideal}\}} - A_{CL_{DC}}}{A_{CL_{DC}\{\text{ideal}\}}} \times 100
  \]
  \[
  = \frac{200 - 199.98729}{200} \times 100 = 0.00636\%
  \]

- **Maximum:**
  \[
  \varepsilon_{\text{max}} = \frac{200 - 199.9198}{200} \times 100 = 0.0401\%
  \]

The calculated absolute worst-case condition over temperature for the DC closed-loop gain for the inverting configuration is 0.0635%, compared to 0.0632% for the non-inverting configuration. This example shows that the difference between the non-inverting and inverting configurations is minimal and in many cases can be ignored.

**Normalized open-loop gain versus temperature**

It should be clear at this point that the DC closed-loop gain is determined by the DC open-loop gain \( A_{OL_{DC}} \) of the op amp. Thus, the stability of the DC open-loop gain determines the stability of the DC closed-loop gain. The stability of the open-loop DC gain is determined by many factors, such as the power-supply rejection ratio (PSRR), the temperature, and process variations.

Figure 4 shows the OPA211’s normalized DC open-loop gain versus temperature. Note that the changes in open-loop gain are shown in \( \mu V/V \). As an alternative to representing changes in \( A_{OL_{DC}} \) with decibels as before, \( A_{OL_{DC}} \) can also be represented in terms of \( \mu V/V \). This representation shows the ratio of the op amp’s change in input voltage (error or offset) to the change in its output voltage. In
other words, the μV/V values have an inverse correlation to the decibel values. As an example, an op amp with an open-loop gain of 199,526 V/V can be written in terms of decibels as

\[ A_{OL, DC}^{\text{dB}} = 20 \log \frac{V_{\text{OUT}}}{V_{\text{ERR}}} = 20 \log(199,526) = 106 \text{ dB.} \quad (24) \]

In terms of μV/V, the same gain is written as

\[ A_{OL, DC}^{\mu\text{V/V}} = \frac{V_{\text{ERR}}}{V_{\text{OUT}}} = \frac{1}{199,526} = 5.012 \mu\text{V/V.} \quad (25) \]

Figure 4 shows how the OPA211’s \(A_{OL, DC}\) (in terms of μV/V) may change over temperature. For a device with a given \(A_{OL, DC}\) at room temperature (25ºC), \(A_{OL, DC}\) will typically change less than 0.25 μV/V in the specified temperature range (–40ºC to 125ºC). For example, if the typical \(A_{OL, DC}\) performance is 130 dB, or 0.32 μV/V, at room temperature, then over the specified temperature range, \(A_{OL, DC}\) may typically vary between 0.32 μV/V and 0.57 μV/V. To ensure stable operation over temperature, the minimum gain is as follows:

\[ A_{OL, DC}^{\mu\text{V/V}} = \frac{V_{\text{OUT}}}{V_{\text{ERR}}} = \frac{1}{1,754,386} = 570 \mu\text{V/V.} \quad (26) \]

This is equivalent to an \(A_{OL, DC}\) ranging from 124.88 dB to 130 dB. Keep in mind that these are typical data. It is suggested that, during the circuit-design process, the designer not use typical values but instead use minimum ensured values published by the op amp’s manufacturer.

Note that none of the calculations in this article include other factors that also affect \(A_{OL, DC}\), such as the PSRR or the common-mode rejection ratio. The procedure to include these types of errors is similar. Simply add the additional error to the \(A_{OL, DC}\) term and recalculate the closed-loop DC gain.

**Conclusion**

Part 1 of this article series explored general feedback-control-system analysis and synthesis as they apply to first-order transfer functions. The analysis technique was applied to both non-inverting and inverting op amp circuits, resulting in a frequency-domain transfer function for each configuration. Part 2 has shown how to use these two transfer functions and manufacturer data-sheet specifications to analyze the DC gain error of a closed-loop op amp circuit. This analysis also took into consideration the temperature dependency of the open-loop gain as well as its finite value. Part 3 will explore the frequency dependency of the closed-loop gain, which will help designers avoid the common mistake of using DC gain calculations for AC-domain analysis.

**References**

For more information related to this article, you can download an Acrobat® Reader® file at www.ti.com/lit and replace “litnumber” with the TI Lit. # for the materials listed below.

**Document Title**

TI Worldwide Technical Support

Internet

TI Semiconductor Product Information Center
Home Page
support.ti.com

TI E2E™ Community Home Page
e2e.ti.com

Product Information Centers

Americas
Phone  +1(972) 644-5580
Brazil
Phone  0800-891-2616
Mexico
Phone  0800-670-7544
Fax  +1(972) 927-6377
Internet/Email support.ti.com/sc/pic/americas.htm

Europe, Middle East, and Africa

Phone
European Free Call  00800-ASK-TEXAS (00800 275 83927)
International  +49 (0) 8161 80 2121
Russian Support  +7 (4) 95 98 10 701

Fax  +(49) (0) 8161 80 2045
Internet support.ti.com/sc/pic/euro.htm

Japan
Phone  Domestic  0120-92-3326
Fax  International  +81-3-3344-5317
Domestic  0120-81-0036
Internet/Email International support.ti.com/sc/pic/japan.htm
Domestic www.tij.co.jp/pic

Asia
Phone
International  +91-80-41381665
Domestic  Toll-Free Number
Australia  1-800-999-084
China  800-820-8682
Hong Kong  800-96-5941
India  1-800-425-7888
Indonesia  001-803-8861-1006
Korea  080-551-2804
Malaysia  1-800-80-3973
New Zealand  0800-446-934
Philippines  1-800-765-7404
Singapore  800-886-1028
Taiwan  0800-006800
Thailand  001-800-886-0010
Fax  +886-2-2378-6808
Email tiasia@ti.com or ti-china@ti.com
Internet support.ti.com/sc/pic/asia.htm

Important Notice: The products and services of Texas Instruments Incorporated and its subsidiaries described herein are sold subject to TI's standard terms and conditions of sale. Customers are advised to obtain the most current and complete information about TI products and services before placing orders. TI assumes no liability for applications assistance, customer's applications or product designs, software performance, or infringement of patents. The publication of information regarding any other company's products or services does not constitute TI's approval, warranty or endorsement thereof.

E2E is a trademark of Texas Instruments. All other trademarks are the property of their respective owners.
IMPORTANT NOTICE

Texas Instruments Incorporated and its subsidiaries (TI) reserve the right to make corrections, modifications, enhancements, improvements, and other changes to its products and services at any time and to discontinue any product or service without notice. Customers should obtain the latest relevant information before placing orders and should verify that such information is current and complete. All products are sold subject to TI's terms and conditions of sale supplied at the time of order acknowledgment.

TI warrants performance of its hardware products to the specifications applicable at the time of sale in accordance with TI's standard warranty. Testing and other quality control techniques are used to the extent TI deems necessary to support this warranty. Except where mandated by government requirements, testing of all parameters of each product is not necessarily performed.

TI assumes no liability for applications assistance or customer product design. Customers are responsible for their products and applications using TI components. To minimize the risks associated with customer products and applications, customers should provide adequate design and operating safeguards.

TI does not warrant or represent that any license, either express or implied, is granted under any TI patent right, copyright, mask work right, or other TI intellectual property right relating to any combination, machine, or process in which TI products or services are used. Information published by TI regarding third-party products or services does not constitute a license from TI to use such products or services or a warranty or endorsement thereof. Use of such information may require a license from a third party under the patents or other intellectual property of TI.

Reproduction of TI information in TI data books or data sheets is permissible only if reproduction is without alteration and is accompanied by all associated warranties, conditions, limitations, and notices. Reproduction of this information with alteration is an unfair and deceptive business practice. TI is not responsible or liable for such altered documentation. Information of third parties may be subject to additional restrictions.

Resale of TI products or services with statements different from or beyond the parameters stated by TI for that product or service voids all express and any implied warranties for the associated TI product or service and is an unfair and deceptive business practice. TI is not responsible or liable for such any statements.

TI products are neither designed nor intended for use in safety-critical applications (such as life support) where a failure of the TI product would reasonably be expected to cause severe personal injury or death, unless officers of the parties have executed an agreement specifically governing such use. Buyers represent that they have all necessary expertise in the safety and regulatory ramifications of their applications, and acknowledge and agree that they are solely responsible for all legal, regulatory and safety-related requirements concerning their products and any use of TI products in such safety-critical applications, notwithstanding any applications-related information or support that may be provided by TI. Further, Buyers must fully indemnify TI and its representatives against any damages arising out of the use of TI products in such safety-critical applications.

TI products are not authorized for use in military/aerospace applications or environments unless the TI products are specifically designated by TI as military-grade or "enhanced plastic." Only products designated by TI as military-grade meet military specifications. Buyers acknowledge and agree that any such use of TI products which TI has not designated as military-grade is solely at the Buyer's risk, and that they are solely responsible for compliance with all legal and regulatory requirements in connection with such use.

TI products are neither designed nor intended for use in automotive applications or environments unless the specific TI products are designated by TI as compliant with ISO/TS 16949 requirements. Buyers acknowledge and agree that, if they use any non-designated TI products in automotive applications, TI will not be responsible for any failure to meet such requirements.

Following are URLs where you can obtain information on other Texas Instruments products and application solutions:

<table>
<thead>
<tr>
<th>Products</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplifiers</td>
<td>Audio</td>
</tr>
<tr>
<td>Data Converters</td>
<td>Automotive</td>
</tr>
<tr>
<td>DLP® Products</td>
<td>Communications and Telecom</td>
</tr>
<tr>
<td>DSP</td>
<td>Computers and Peripherals</td>
</tr>
<tr>
<td>Clocks and Timers</td>
<td>Consumer Electronics</td>
</tr>
<tr>
<td>Interface</td>
<td>Energy</td>
</tr>
<tr>
<td>Logic</td>
<td>Industrial</td>
</tr>
<tr>
<td>Power Mgmt</td>
<td>Medical</td>
</tr>
<tr>
<td>Microcontrollers</td>
<td>Security</td>
</tr>
<tr>
<td>RFID</td>
<td>Space, Avionics &amp; Defense</td>
</tr>
<tr>
<td>RF/IF and ZigBee® Solutions</td>
<td>Video and Imaging</td>
</tr>
<tr>
<td></td>
<td>Wireless</td>
</tr>
</tbody>
</table>

Mailing Address: Texas Instruments, Post Office Box 655303, Dallas, Texas 75265
Copyright © 2010, Texas Instruments Incorporated