How to compare your circuit requirements to active-filter approximations

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Introduction
Numerous filter approximations, such as Butterworth, Bessel, and Chebyshev, are available in popular filter software applications; however, it can be time consuming to select the right option for your system. So how do you focus in on what type of filter you need in your circuit? This article defines the differences between Bessel, Butterworth, Chebyshev, Linear Phase, and traditional Gaussian low-pass filters. A typical Butterworth low-pass filter is shown in Figure 1.

Generic low-pass filter frequency and time response
Figure 2 illustrates the frequency response of a generic low-pass filter. In this diagram, the x-axis shows the frequency in hertz (Hz) and the y-axis shows the circuit gain in volts/volts (V/V) or decibels (dB).

The low-pass filter has two frequency areas of operation: the passband region and the transition region. In the passband region, the input signal passes through with minimum modifications. The level of modification depends on the filter approximation type. For example, the amount of modification for the Butterworth filter is minimal because the transfer function is often described as maximally flat. In contrast, the Chebyshev filter will have ripple in the passband region.

In the transition region, the filter attenuates the input signal with increasing frequency. This attenuation rate is generally dependent on the filter order or number of poles. A first-order filter (single pole) performs with a 20-dB/decade attenuation after the 3-dB frequency. A fifth-order filter (five poles) generally produces a $20 \times 5$-dB/decade attenuation, or 100 dB/decade. Again, this attenuation rate has minimal variations between the approximation types.
In Figure 2, the DC gain of the filter is defined with the A parameter. Some filters exhibit a ripple on the DC gain curve as the frequency increases. This ripple magnitude is defined as $R_p$.

The corner frequency of the low-pass filter is at the 3-dB point below the DC gain level. Beyond the corner frequency, $A_{SB}$ in combination with $A$ and $f_s$, define the rate of attenuation.

Figure 3 shows the generic time response of a low-pass filter. In order to generate this filter response, a step response is applied to the input of the filter. In this diagram, the units for the x-axis is time, and the y-axis represents the responses amplitude in volts. There are five parameters that define the step-response curve: propagation delay, slew rate, overshoot, settling time, and settling-time error.

The propagation delay of a filter’s response is a result of the magnitude of the frequency domain phase. By definition, the propagation delay is the time delay between the filter’s input step and the resulting change at the output. Following the propagation delay period, the output of the filter attempts to match the input signal. During this time, the output rises and the measurement of the slew occurrence is from 10 to 90% of the entire span. This catch-up activity causes a signal overshoot. The overshoot is typically measured as a percentage.

The amplifier filter continues to attempt to match the input signal. During this time, there is a settling activity. The settling time combines the slew time and the time it takes for the filter circuit to settle to a final value. The definition of this final value is the settling-time error band. For 12-bit systems, this error band is usually equal to a half of the least significant bit (LSB), or 0.01%. For a 16-bit system, this error band is usually equal to 0.001%.

**Bessel low-pass active filter**

W. E. Thomson applied the Bessel function, which Friedrich Bessel developed in the mid-1800s, to filter a design in 1949. Figure 4 shows the frequency, phase and time response of a fifth-order, low-pass Bessel filter.

The frequency behavior has a maximally flat group/phase delay. This type of phase delay retains the wave shape in the passband region and exhibits very little ringing or overshoot in the transition region. This benefits the time-domain step response because there is very little overshoot or ringing compared to the Butterworth and Chebyshev filters. The rate of attenuation in the transition band is the slowest of the filters examined, so if a “brick-wall” filter is expected, the Bessel filter is not a good choice.

Where does this filter come in handy? It is found in audio cross-overs because of the minimal overshoot responses in the time domain.
**Butterworth low-pass active filter**

Stephen Butterworth described the Butterworth filter in his 1930 paper titled, “On the Theory of Filter Amplifiers.” As Figure 5 shows, the frequency behavior of the Butterworth filter has a maximally flat magnitude response in the passband.

The rate of attenuation in the transition band is an improvement over the Bessel filter. However, the step-response overshoot and ringing of the Bessel filter can be lower.

The uses of this middle-of-the-road filter are extensive. This filter is found across the entire range of applications. The maximally-flat characteristic in the passband region is beneficial.

**Chebyshev low-pass active filter**

In the frequency domain, there is a magnitude of ripple in the passband. The ripple magnitude in the gain response shown in Figure 6a is 3 dB. The rate of attenuation in the transition band is steeper than the Butterworth and Bessel filters, but there is a price to pay for this faster frequency fall-off. As Figure 6 shows, the step response has a fair degree of overshoot and ringing.

The attenuation rate can be faster in the transition region by increasing the magnitude of the ripple in the passband. However, be aware that this action will increase the ringing with the step response.

The Chebyshev filter would be used in systems where the higher-frequency noise requires more aggressive attenuation.
Linear-phase low-pass active filter
As the name of this filter indicates, the linear-phase filter has a linear-phase response in the passband (Figure 7). This is similar to the Bessel filter, but the linear range in this filter exceeds the wide range of the Bessel.

Because of ripples in the phase response, this filter also has faster attenuation after the 3-dB frequency. As the phase ripple increases, the constant delay region extends further into the transition region. The step response shows slightly more overshoot than the Bessel, but not as much as the Butterworth and Chebyshev.

Traditional Gaussian low-pass active filter
The traditional Gaussian 6-dB filter (Figure 8) is a compromise between a Chebyshev and a Bessel filter. The traditional Gaussian filter is similar to a Bessel filter, in that it has nearly linear phase shift and smooth, monotonic roll-off into the transition region.

The Gaussian 6-dB step response has less overshoot and is between the Bessel and Butterworth filter.
Conclusion

This article addressed the difference between Bessel, Butterworth, Chebyshev, linear phase, and traditional Gaussian low-pass filters. As shown in Table 1, these filters present variations in passband flatness, phase response, and step-response ringing and overshoot.

At this point, the decision now lies with you and your circuit requirements. If a well-behaved step response with minimal ringing is required, consider the linear phase filter. In contrast, if a filter to attenuate quickly in the transition region is required, the Chebyshev may be the best alternative.

The TI Filter Designer tool is a quick and easy way to compare these filters with your parameters. In this software, you can select a filter and then run simulations to take a closer look.

Table 1. Comparison between active fifth-order, low-pass filters

<table>
<thead>
<tr>
<th>Filter Approximation</th>
<th>Passband Region</th>
<th>Transition Region</th>
<th>Step Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bessel</td>
<td>Flat magnitude response in pass-band</td>
<td>Slower than the Butterworth or Chebyshev filters</td>
<td>Very little overshoot or ringing as compared to the Butterworth and Chebyshev filters</td>
</tr>
<tr>
<td>Butterworth</td>
<td>Maximally flat magnitude response in pass-band</td>
<td>Steeper than Bessel, not as good as Chebyshev filter</td>
<td>Some overshoot and ringing, but less than the Chebyshev filter</td>
</tr>
<tr>
<td>Chebyshev</td>
<td>Ripple in the pass-band</td>
<td>Steeper than Butterworth and Bessel filters</td>
<td>Fair degree of overshoot and ringing</td>
</tr>
<tr>
<td>Linear Phase</td>
<td>Linear phase response, which exceeds the bandwidth of the Bessel</td>
<td>Faster attenuation than Bessel due to ripples in phase response</td>
<td>Slightly more overshoot than Bessel, but not as much as the Butterworth</td>
</tr>
<tr>
<td>Traditional Gaussian</td>
<td>As with the Bessel, has a linear phase response</td>
<td>Smooth roll-off</td>
<td>Overshoot less than the Butterworth, with some ringing</td>
</tr>
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