Analog Active Audio Filters

Stephen Crump
Audio Products

ABSTRACT

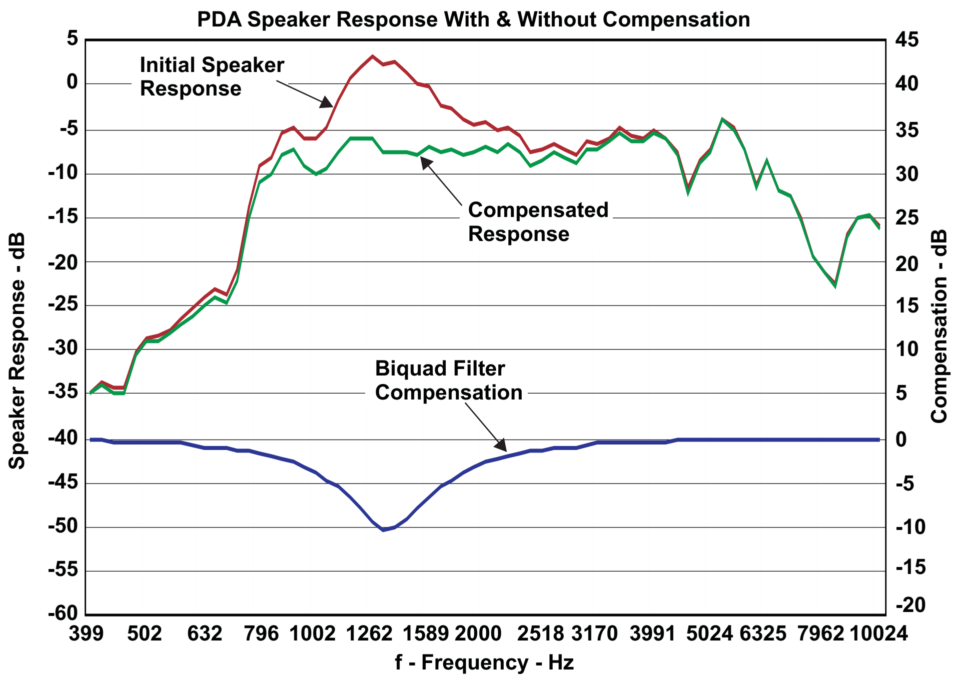
Analog active audio filters can be used to compensate frequency response problems in a variety of systems. Their responses are examined here to simplify the filter design task for design engineers.

Contents

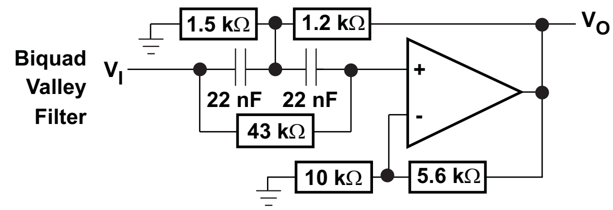
1 Introduction

Active audio filters may be used to compensate problems in frequency response of audio systems and loudspeakers. This paper deals with analog filters. These filters can produce a response that is approximately the inverse of a system response or a loudspeaker acoustic response so that when the two are summed the result is nearly flat. They may also be used to produce the difference of a target response minus the system or loudspeaker response, so that when the two are summed the result is nearly the target. In either case the final response is more accurate or more pleasing.

Cell phone and PDA speaker responses like the one shown below often have annoying peaks that reduce intelligibility. This is compensated with the response of a biquad filter, also shown. The sum, the compensated response, is much more pleasing and far more intelligible than the original.



The schematic for the biquad filter is at right. This and other filters are discussed in detail in this paper.



This paper begins with the relationship between filter singularities and their responses. Then it considers a number of possible filter implementations. (It's typically easier to adjust a set of filter singularities to achieve the best result than to adjust the numerous component values in a circuit. Once the singularities are decided the filter usually can be implemented relatively easily.) It provides equations for responses and the parameters in them and discusses optimizing component choices.

Audio filters may be first, second or higher order. First and second order analog filters are generally well understood and their audio uses are somewhat limited, so they are examined briefly. The paper also examines biquadratic filters, or biquads, in more depth because they are more powerful tools for response compensation or EQ. Bridged-T filters will be added in a later version.

2 First Order Filters

First-order filters implement responses with single poles as their denominators. They have limited response bands, either low-pass or high-pass, which are described below.

- Low pass: $H(s) = H_o\ \omega_o / (s + \omega_o)$.
  For small s, this is H_o, flat low-frequency response; for large s, it is $H_o\ \omega_o / s$, a first-order rolloff.
- High pass: $H(s) = H_o\ s / (s + \omega_o)$.
  For large s, this is H_o, flat high-frequency response; for small s, it is $H_o\ s / \omega_o$, a first-order rollup.

3 Second Order Filters

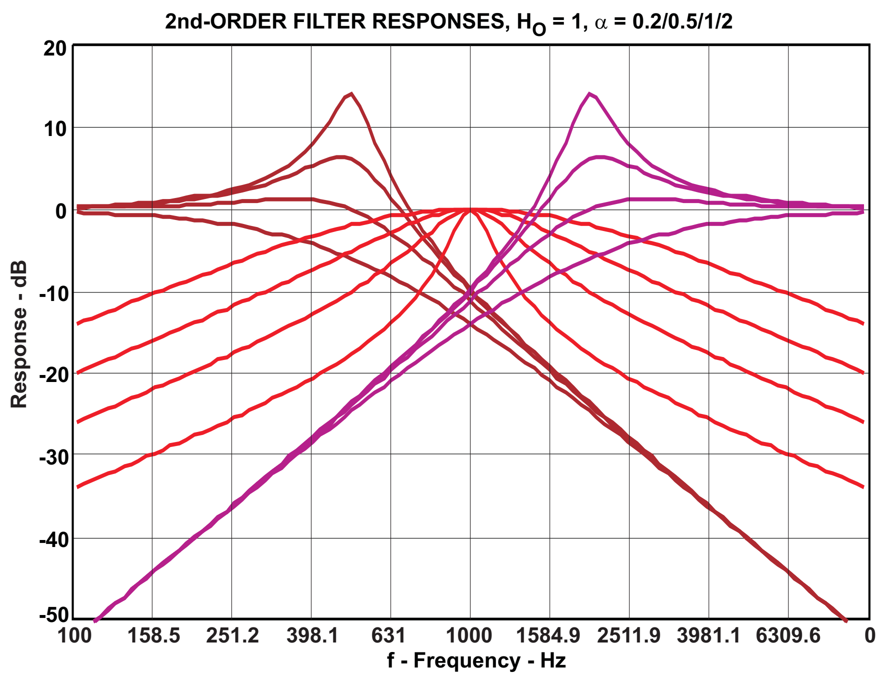
Second-order filters implement responses with quadratic terms as their denominators. They have limited response bands, either low-pass, band-pass or high-pass, which are described below.(1)

- Low pass: $H(s) = H_o \omega_o^2 / (s^2 + \alpha \omega_o s + \omega_o^2)$.
  For small s, this is H_o, flat low-frequency response; for large s, it is $H_o \omega_o^2 / s^2$, a second-order rolloff.
  Response magnitude is $G(\omega) = \mathrm{sqrt}(H_o^2 \omega_o^4 / (\omega^4 + \omega^2 \omega_o^2 (\alpha^2 - 2) + \omega_o^4))$.
  Phase is $\Phi(\omega) = \pi/2 - \arctan(\alpha \omega_o \omega / (\omega_o^2 - \omega^2))$.

- Band pass: $H(s) = H_o \alpha \omega_o s / (s^2 + \alpha \omega_o s + \omega_o^2)$.
  For small s, this is $H_o \alpha s / \omega_o$, an increasing first-order response or rollup; for large s, it is $H_o \alpha \omega_o / s$, a decreasing first-order response or rolloff. At $s = j\omega_o$, it is H_o, band-center response.
  Response magnitude is $G(\omega) = \mathrm{sqrt}(H_o^2 \alpha^2 \omega^2 \omega_o^2 / (\omega^4 + \omega^2 \omega_o^2 (\alpha^2 - 2) + \omega_o^4))$.
  Phase is $\Phi(\omega) - \arctan(\alpha \omega_o \omega / (\omega_o^2 - \omega^2))$.

- High pass: $H(s) = H_o s^2 / (s^2 + \alpha \omega_o s + \omega_o^2)$.
  For large s, this is H_o, flat high-frequency response; for small s, it is $H_o s^2 / \omega_o^2$, a second-order rollup.
  Response magnitude is $G(\omega) = \mathrm{sqrt}(H_o^2 \omega^4 / (\omega^4 + \omega^2 \omega_o^2 (\alpha^2 - 2) + \omega_o^4))$.
  Phase is $\Phi(\omega) = \pi - \arctan(\alpha \omega_o \omega / (\omega_o^2 - \omega^2))$.

H_o scales response magnitude, while ω_o sets the characteristic frequency, the frequency at which the filter operates. The variable α sets the sharpness of the peak the filter produces, which varies inversely with α (sharper peak with smaller α).



(1) Reference: Operational Amplifiers, Design and Applications, Graeme, Tobey and Huelsman, Burr-Brown, McGraw-Hill Book Company, 1971, ISBN 07-064917-0, pages 284-286.

4 Biquadratic Filters

Biquadratic or biquad filters implement responses with quadratic terms for both their numerators and their denominators. They have wide responses, with a single peak or valley or with low or high frequency boost with or without peaking. Their responses have the following form.

$$H(s) = H_o \, (s^2 + \alpha_Z \, \omega_Z \, s + \omega_Z{}^2) \, / \, (s^2 + \alpha_P \, \omega_P \, s + \omega_P{}^2).$$

Filter response is influenced by relationships between ω_Z and ω_P and between α_Z and α_P.

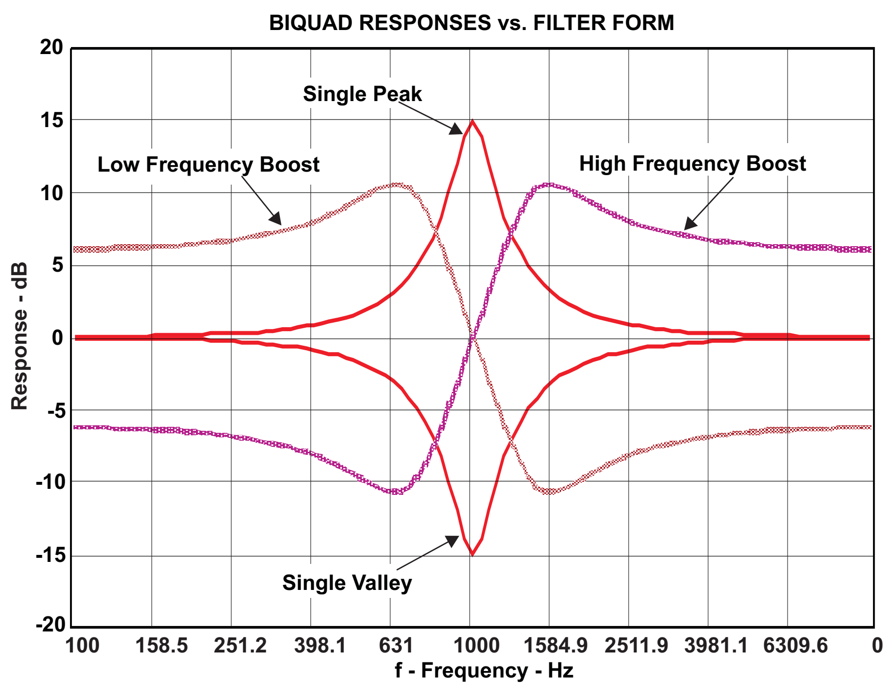
- Single peak or valley. The filter produces this response when ω_Z and ω_P are equal.
  If α_Z is greater than α_P the filter produces a peak. If α_P is greater than α_Z the filter produces a valley.

- Low frequency boost. The filter produces this response when ω_Z is greater than ω_P.
  If α_Z is less than about 1, the response includes a valley above the boost frequency. If α_P is less than about 1, the response includes a peak below the boost frequency.

- High frequency boost. The filter produces this response when ω_P is greater than ω_Z.
  If α_Z is less than about 1, the response includes a valley below the boost frequency. If α_P is less than about 1, the response includes a peak above the boost frequency.

Response magnitude is $G(\omega) = \mathrm{sqrt}((\omega^4 + \omega^2 \, \omega_z{}^2 \, (\alpha_z{}^2 - 2) + \omega_z{}^4) \, / \, (\omega^4 + \omega^2 \, \omega_p{}^2 \, (\alpha_p{}^2 - 2) + \omega_p{}^4))$.

Phase is $\Phi(\omega) = \arctan(\alpha_Z \, \omega_Z \, \omega \, / \, (\omega_Z{}^2 - \omega^2)) - \arctan(\alpha_P \, \omega_P \, \omega \, / \, (\omega_P{}^2 - \omega^2))$.

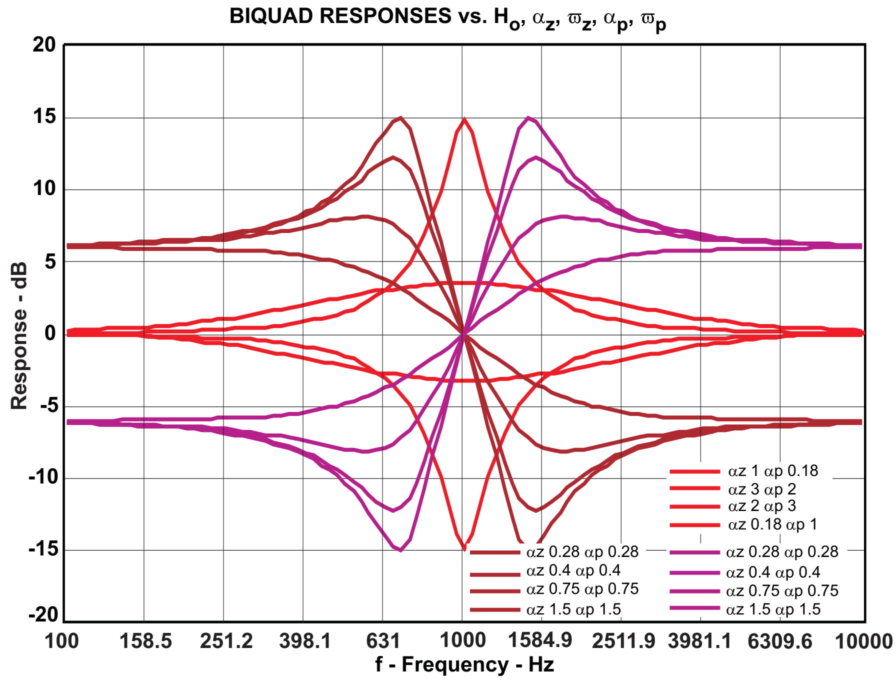


Traces in the graph that follows illustrate something of the range of responses a biquad filter can generate. The responses are arranged as follows for clarity.

- Single peak and valley responses are presented in the order of decreasing peaks, or decreasing α_Z with respect to α_P. For all these responses H_o is 1 and ω_Z and ω_P are 1 kHz.

- Low and high frequency boost responses are presented in the order of decreasing peaks and valleys, or increasing α_Z and α_P. Also, α_Z and α_P are made equal, ω_Z and ω_P are placed symmetrically around 1kHz, and H_o is set to 0.5 for low frequency boost and 2 for high frequency boost, to make the responses symmetrical around zero dB and 1kHz.

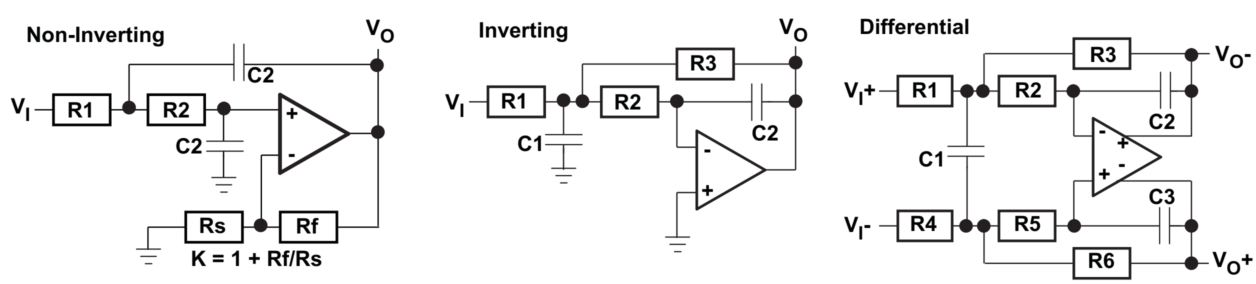
Note that α_Z and α_P do not have to be equal! Varying α_Z and α_P can create responses that range among and beyond the extremes in the graph.



5 Analog Filter Implementations: Second-Order Filters.

Second-order filters may be non-inverting or inverting. The schematics below show single-ended forms, both non-inverting and inverting, and an inverting, differential form, with equations for their H_o, α and ω_o.

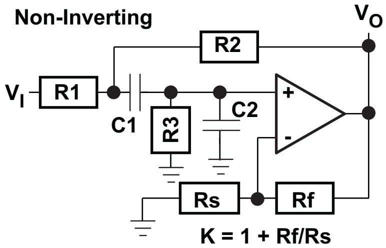
Low-Pass Filters

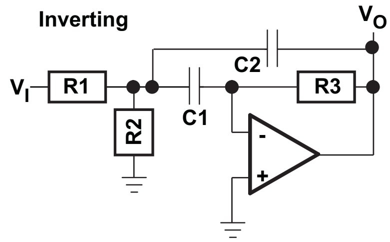


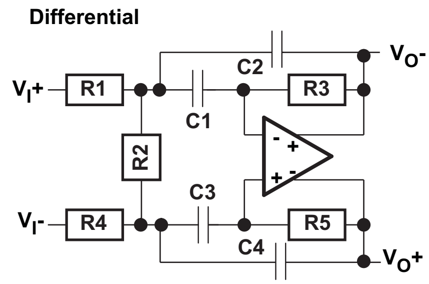
R4 = R1; R5 = R2; R6 = R3; C3 = C2

	Non-Inverting	Inverting	Differential Inverting
H_o	$K \left(= 1+\frac{R_f}{R_s}\right)$	$\frac{R3}{R1}$	$\frac{R3}{R1}$
ω_o	$\frac{1}{\sqrt{(R1 \times R2 \times C1 \times C2)}}$	$\frac{1}{\sqrt{(R2 \times R3 \times C1 \times C2)}}$	$\frac{1}{\sqrt{(R2 \times R3 \times (C1 \times 2) \times C2)}}$
α	$(1-K) \times \sqrt{\frac{R1\times C1}{R2\times C2}} + \sqrt{\frac{R2\times C2}{R1\times C1}} + \sqrt{\frac{R1\times C2}{R2\times C1}}$	$\sqrt{\frac{C2}{C1}} \times \left[\sqrt{\frac{R2}{R3}} + \sqrt{\frac{R3}{R2}} + \frac{\left(\sqrt{(R2\times R3)}\right)}{R1}\right]$	$\sqrt{\left(\frac{C2}{C1\times 2}\right)} \times \left[\sqrt{\frac{R2}{R3}} + \sqrt{\frac{R3}{R2}} + \frac{\left(\sqrt{(R2\times R3)}\right)}{R1}\right]$

Band-Pass Filters



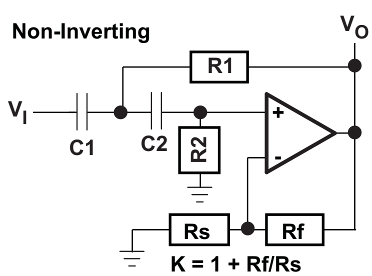


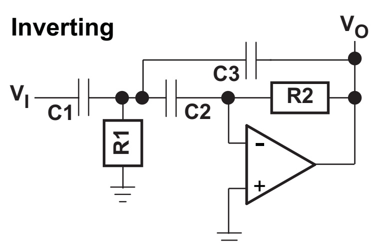


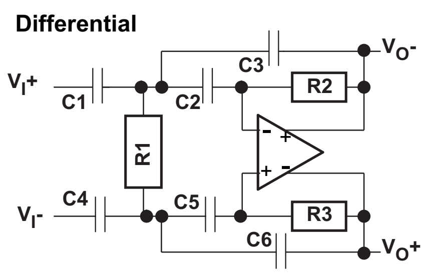
R4 = R1; R5 = R3; C3 = C1; C4 = C2

	Non-Inverting	Inverting	Differential Inverting
H_o	$K\left(=1+\frac{R_f}{R_s}\right)$	$\frac{R3/R1}{\left(1+\frac{C2}{C1}\right)}$	$\frac{R3/R1}{\left(1+\frac{C2}{C1}\right)}$
ω_o	$\frac{1}{\sqrt{R1\times R2\times C1\times C2}}$	$\frac{1}{\sqrt{\left(\frac{R3\times C1\times C2\times R1\times R2}{R1+R2}\right)}}$	$\frac{1}{\sqrt{\left(\frac{R3\times C1\times C2\times R1\times R2/2}{(R1+R2/2)}\right)}}$
α	$(1-K)\times\sqrt{\frac{R1\times C1}{R2\times C2}}+\sqrt{\frac{R2\times C2}{R1\times C1}}+\sqrt{\frac{R1\times C2}{R2\times C1}}$	$\left[\sqrt{\frac{C1}{C2}}+\sqrt{\frac{C2}{C1}}\right]\times\sqrt{\left(\frac{R1\times R2}{((R1+R2)\ R3)}\right)}$	$\left[\sqrt{\frac{C1}{C2}}+\sqrt{\frac{C2}{C1}}\right]\times\sqrt{\left(\frac{R1\times R2/2}{((R1+R2)\ R3)}\right)}$

High-Pass Filters







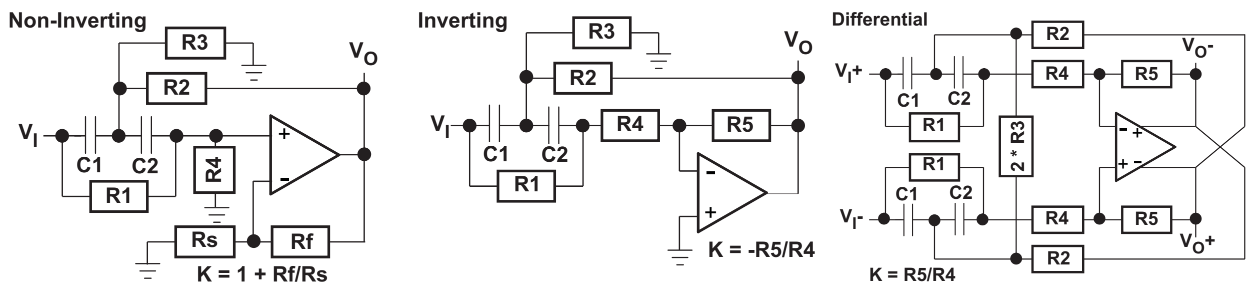
C4 = C1; C5 = C2; C6 = C3; R3 = R2

	Non-Inverting	Inverting	Differential Inverting
H_o	$K\left(=1+\frac{R_f}{R_s}\right)$	$\frac{C1}{C3}$	$\frac{R3/R1}{\left(1+\frac{C2}{C1}\right)}$
ω_o	$\frac{1}{\sqrt{(R1\times R2\times C1\times C2)}}$	$\frac{1}{\sqrt{(R1\times R2\times C2\times C3)}}$	$\frac{1}{\sqrt{\left(\frac{R1}{2}\right)\times R2\times C2\times C3}}$
α	$(1-K)\times\sqrt{\left(\frac{R1\times C1}{(R2\ C2)}\right)}+\sqrt{\left(\frac{R2\times C2}{(R1\ C1)}\right)}+\sqrt{\left(\frac{R1\times C2}{(R2\ C1)}\right)}$	$\left[\frac{C1}{\sqrt{C2xC3}}+\sqrt{\frac{C2}{C3}}+\sqrt{\frac{C3}{C2}}\right]\times\sqrt{\frac{R1}{R2}}$	$\left[\frac{C1}{\sqrt{C2\times C3}}+\sqrt{\frac{C2}{C3}}+\sqrt{\frac{C3}{C2}}\right]\times\sqrt{\left(\frac{(R1/2)}{R2}\right)}$

6 Analog Filter Implementations: Biquadratic Filters.

Biquadratic filters also may be non-inverting, inverting, or differential inverting. The schematics below show all these forms with equations for their H_o, α and ω_o. The equations begin with a factor K, the gain of the inner opamp circuit, for each of the filter forms. The remaining quantities, D, H_o, ω_Z, α_Z, ω_P and α_P, are common to all the filter forms. D is a multiplier used to simplify the following equations.

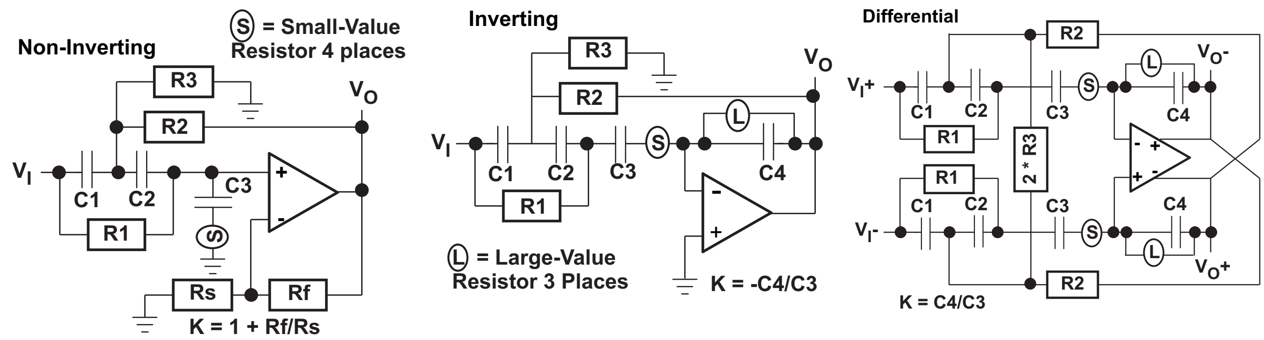
Biquad Filters – High Frequency Boost



	Non-Inverting	Inverting	Differential Inverting
K	$1+\frac{R_f}{R_s}$	$-\frac{R5}{R4}$(NEGATIVE)	$\frac{R5}{R4}$
D	$\frac{R3}{(R2 + R3)}$	Multplier for R2 and K	
H_o	K	Response at high frequency	
ω_z	$\frac{1}{\sqrt{(R1\ DR2\ C1\ C2)}}$	ω_P	$\frac{1}{\sqrt{\left(\frac{R1DR2\ R4\ C1C2}{R1+R4}\right)}}$
α_z	$\frac{(C1 + C2)\sqrt{DR2}}{\sqrt{(R1\ C1\ C2)}}$	α_p	$\frac{(R1 + R4)\ DR2\ (C1 + C2) + R1R4C2\ (1\text{-}KD)}{\sqrt{(R1\ DR2\ R4\ C1\ C2 \times (R1 + R4))}}$

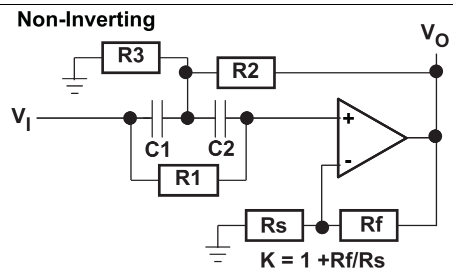
Schematics for low frequency boost filters follow. The input circuit in each filter includes a series chain of capacitors to ground or virtual ground. This load could destabilize either the opamp in the filter or an opamp driving the input, so a small value resistor, maybe 100Ω, is added in series with the final cap in the chain. Also, feedback elements in the inverting and differential filters are capacitors. These provide no path for DC bias, so a large value resistor, maybe 1 to 10 M, is added in parallel with the feedback cap. Of course, these resistances will have a small effect on filter responses, but they should not degrade them significantly.

Biquad Filters – Low Frequency Boost



	Non-Inverting	Inverting	Differential Inverting
K	$1+\frac{R_f}{R_s}$	$-\frac{C4}{C3}$(NEGATIVE)	$\frac{C4}{C3}$
D	$\frac{R3}{(R2 + R3)}$	Multplier for R2 and K	
H_o	$\frac{K \times C1C2}{(C1C2+C1C3+C1C3)}$	Response at low frequency	
ω_z	$\frac{1}{\sqrt{(R1\ DR2\ C1\ C2)}}$	ω_P	$\frac{1}{\sqrt{(R1DR2 \times (C1C2 + C1C3 + C1C3))}}$
α_z	$\frac{(C1 + C2)\sqrt{(DR2)}}{\sqrt{(R1\ C1\ C2)}}$	α_p	$\frac{DR2 \times (C1 + C2)+R1C3 + R1C2 \times (1\text{-}KD)}{\sqrt{(R1\ DR2 \times (C1\ C2 + C1C3+C2C3))}}$

Biquad Filters – Single Peak or Valley



There are NO Inverting or Differential versions because each of these loads the output of the input network and produces some low or high frequency shelf.

	Non-Inverting		
K	$1+\frac{R_f}{R_s}$		
D	$\frac{R3}{(R2 + R3)}$	Multiplier for R2 and K	
H_o	K	Response at very low and high frequencies	
ω_z	$\frac{1}{\sqrt{(R1\ DR2\ C1C2)}}$	ω_P (= ω_z)	$\frac{1}{\sqrt{(R1\ DR2\ C1\ C2)}}$
α_z	$\frac{(C1 + C2) \times \sqrt{DR2}}{\sqrt{(R1C1C2)}}$	α_p	$\frac{DR2 \times (C1 + C2) + R1\ C2 \times (1\text{-}KD)}{\sqrt{(R1\ DR2\ C1\ C2)}}$

The most complicated quantities are α_Z and α_P, so we will look at these in some detail. As we will see, it is more difficult to achieve low values of α_Z and α_P than high, so we will concentrate on reducing these quantities. We will consider how to produce values as small as about 0.5, a value that provides significant peaking.

α_Z is the same for all 3 filter configurations. The ratio (C1+C2) / $\sqrt{(C1C2)}$ in α_Z ranges from about 3.5 to 2 to about 3.5 again as (C1/C2) ranges from 0.1 to 1 to 10, so α_Z is reduced by making C1 and C2 different in value. So it is typically best to make C1 and C2 similar in value. α_Z can be controlled by varying the ratio $\sqrt{(DR2/R1)}$. If R1 = 20×DR2 and C1 = C2, α_Z = 2 / $\sqrt{(20)}$ = 0.45, probably close to the lowest value needed.

For α_P we face similar constraints with the first term or two in the numerators, but we have the advantage of the last term, which is negative in non-inverting and differential filters for any KD product greater than 1. So we can use this term to reduce α_P if we need to do so. (Beware, however: if KD is made large enough, the last term will cancel the rest of the numerator, α_P will equal zero and the filter will oscillate at ω_P!)

Note that we do NOT have this advantage in inverting biquads! In those, since K is negative, the sum (1-KD) is always greater than 1. As a result, it is likely to be very difficult to achieve low values of α_P. For this reason inverting biquads are not likely to be generally useful in EQ work.

Single peak or valley filters are a special case. In these, α_P = α_Z + R1C2×(1-KD) / $\sqrt{(R1DR2C1C2)}$. If C1 and C2 are similar in value, α_P ~= α_Z + (1-KD) $\sqrt{(R1/DR2)}$ ~= α_Z + 2×(1-KD)/α_Z. If KD is less than 1, (1-KD) is positive, α_P is greater than α_Z and the filter creates a valley. If KD equals 1, α_P equals α_Z and there is no peak or valley. If KD is greater than 1, the second term is negative, α_P is smaller than α_Z and the filter creates a peak.

Note that, for single peak or valley filters, reducing α_Z increases the magnitude of the second term in α_P. So in valley filters, with KD < 1, reducing α_Z tends to increase α_P. This tends to make the resulting valley broad and deep. Make (1-KD) smaller to narrow or reduce the valley. In peak filters, with KD > 1, reducing α_Z tends to reduce α_P by making the last term in α_P more negative. This tends to make the resulting peak narrow and high. Make (1-KD) less negative to broaden or reduce the peak.

IMPORTANT NOTICE

Texas Instruments Incorporated and its subsidiaries (TI) reserve the right to make corrections, modifications, enhancements, improvements, and other changes to its products and services at any time and to discontinue any product or service without notice. Customers should obtain the latest relevant information before placing orders and should verify that such information is current and complete. All products are sold subject to TI's terms and conditions of sale supplied at the time of order acknowledgment.

TI warrants performance of its hardware products to the specifications applicable at the time of sale in accordance with TI's standard warranty. Testing and other quality control techniques are used to the extent TI deems necessary to support this warranty. Except where mandated by government requirements, testing of all parameters of each product is not necessarily performed.

TI assumes no liability for applications assistance or customer product design. Customers are responsible for their products and applications using TI components. To minimize the risks associated with customer products and applications, customers should provide adequate design and operating safeguards.

TI does not warrant or represent that any license, either express or implied, is granted under any TI patent right, copyright, mask work right, or other TI intellectual property right relating to any combination, machine, or process in which TI products or services are used. Information published by TI regarding third-party products or services does not constitute a license from TI to use such products or services or a warranty or endorsement thereof. Use of such information may require a license from a third party under the patents or other intellectual property of the third party, or a license from TI under the patents or other intellectual property of TI.

Reproduction of TI information in TI data books or data sheets is permissible only if reproduction is without alteration and is accompanied by all associated warranties, conditions, limitations, and notices. Reproduction of this information with alteration is an unfair and deceptive business practice. TI is not responsible or liable for such altered documentation. Information of third parties may be subject to additional restrictions.

Resale of TI products or services with statements different from or beyond the parameters stated by TI for that product or service voids all express and any implied warranties for the associated TI product or service and is an unfair and deceptive business practice. TI is not responsible or liable for any such statements.

TI products are not authorized for use in safety-critical applications (such as life support) where a failure of the TI product would reasonably be expected to cause severe personal injury or death, unless officers of the parties have executed an agreement specifically governing such use. Buyers represent that they have all necessary expertise in the safety and regulatory ramifications of their applications, and acknowledge and agree that they are solely responsible for all legal, regulatory and safety-related requirements concerning their products and any use of TI products in such safety-critical applications, notwithstanding any applications-related information or support that may be provided by TI. Further, Buyers must fully indemnify TI and its representatives against any damages arising out of the use of TI products in such safety-critical applications.

TI products are neither designed nor intended for use in military/aerospace applications or environments unless the TI products are specifically designated by TI as military-grade or "enhanced plastic." Only products designated by TI as military-grade meet military specifications. Buyers acknowledge and agree that any such use of TI products which TI has not designated as military-grade is solely at the Buyer's risk, and that they are solely responsible for compliance with all legal and regulatory requirements in connection with such use.

TI products are neither designed nor intended for use in automotive applications or environments unless the specific TI products are designated by TI as compliant with ISO/TS 16949 requirements. Buyers acknowledge and agree that, if they use any non-designated products in automotive applications, TI will not be responsible for any failure to meet such requirements.

Following are URLs where you can obtain information on other Texas Instruments products and application solutions:

Products		**Applications**	
Amplifiers	amplifier.ti.com	Audio	www.ti.com/audio
Data Converters	dataconverter.ti.com	Automotive	www.ti.com/automotive
DLP® Products	www.dlp.com	Communications and Telecom	www.ti.com/communications
DSP	dsp.ti.com	Computers and Peripherals	www.ti.com/computers
Clocks and Timers	www.ti.com/clocks	Consumer Electronics	www.ti.com/consumer-apps
Interface	interface.ti.com	Energy	www.ti.com/energy
Logic	logic.ti.com	Industrial	www.ti.com/industrial
Power Mgmt	power.ti.com	Medical	www.ti.com/medical
Microcontrollers	microcontroller.ti.com	Security	www.ti.com/security
RFID	www.ti-rfid.com	Space, Avionics & Defense	www.ti.com/space-avionics-defense
RF/IF and ZigBee® Solutions	www.ti.com/lprf	Video and Imaging	www.ti.com/video
		Wireless	www.ti.com/wireless-apps

Mailing Address: Texas Instruments, Post Office Box 655303, Dallas, Texas 75265