

Optimizing Power Consumption and Power-Up Overshoot Using TPS54160-Q1 Family in Automotive Applications

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ABSTRACT

The TPS54160 family of switching buck converters can be used in many applications where less than 50 μ A of power is drained from the battery during power off. The advantages of these devices are due to Texas Instruments advanced process and design technologies.

The power consumption and minimal power-up overshoot advantages are exceptional in 12-V, 24-V, and 48-V commercial and industrial applications. These applications include: global positioning systems (GPS), video, entertainment, and other aftermarket automotive accessories.

This application report illustrates in-depth information and detailed calculations of the first switching cycle differential equations. It also provides solution to minimize overshoot and input inrush current during the power-up cycle.

Many of today's high-performance DSPs and ASICs require special care of their voltage rails during the power-up sequence. An overshoot at power up may violate the sequence required by the integrated circuit (IC), causing a latch-up condition and degrade the long-term reliability of the IC.

1 Introduction

In order to maintain low quiescent current during power off, TPS54160 and other devices have been built with no undervoltage lockout (UVLO) hysteresis. Desired UVLO hysteresis is accomplished by an external resistive voltage divider from battery to ground. The resistor divider can consume up to 50 μ A from the battery during power off.

The TPS54160 device families are step-down regulators with an integrated high-side MOSFET. Current-mode control provides simple external compensation and flexible component selection. A low-ripple, pulse-skip mode reduces the no-load, regulated-output supply current to 116 μ A. Using the enable pin (pin 3), the shutdown supply current is reduced to 1.3 μ A. The UVLO is internally set to 2.5 V but can be changed to a higher level using the enable pin.

The output voltage start-up ramp is controlled by the slow start pin (pin 4) that can also be configured for sequencing/tracking. The SS/TR (slow start/tracking) pin is used to minimize inrush currents and provides power supply sequencing during power up. A small value capacitor must be coupled to the SS/TR pin to adjust the slow-start time. A resistor divider can be tied to this pin for critical power supply sequencing requirements. The SS/TR pin is discharged before the output powers up. This discharging ensures a repeatable restart after an overtemperature fault, UVLO fault, or a disabled condition.

Furthermore, the slow-start capacitor discharges during overload conditions with an overload recovery circuit. The overload recovery circuit slow-starts the output from the fault voltage to the nominal regulation voltage once a fault condition is removed.

A frequency foldback internal circuit reduces the switching frequency during start-up and overcurrent fault conditions and helps control the inductor current. The TPS54160 is disabled when the VIN pin (pin 2) voltage falls below 2.5 V.

For applications requiring higher UVLO, the EN pin must be used as shown in Figure 1to adjust the UVLO using the two external resistors.

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Although, it is unnecessary to use the UVLO adjust resistors, for operation it is highly recommended to provide consistent power-up behavior. The EN pin has an internal pullup current source of 0.9 µA. This provides the default condition of the TPS54160 operating when the EN pin is floating. When the EN pin voltage exceeds 1.25 V, an additional 2.9 µA of hysteresis, is added to the pullup current source. This additional current facilitates input voltage hysteresis.

The following two equations (Figure 1) are used to set the external hysteresis for the input voltage (UVLO). For more detail information, see the device data sheet (SLVS922).

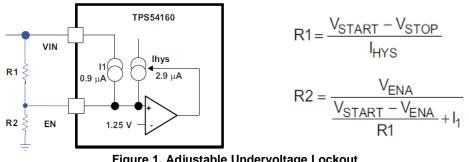


Figure 1. Adjustable Undervoltage Lockout

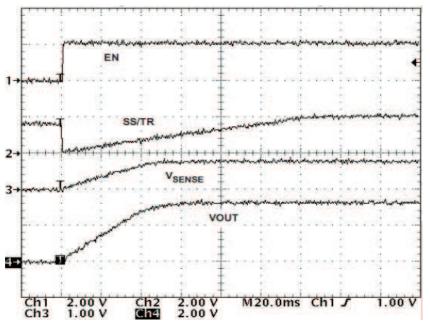


Figure 2. Operation of SS/TR Pin When Starting

The TPS54610 dc/dc converter consumes less than 1 µA at power off. It is desirable to maintain the input voltage at 2.5 V during power recycling. This additional feature is obtained by floating the EN pin and adding the following external components as in Figure 3:

- 1. High-input capacitor value approximately 1000 µF
- 2. Blocking diode varies depending on power requirements

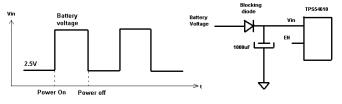


Figure 3. Battery Voltage Recycling

As can be seen, the device UVLO is 2.5 V and has no hysteresis. The device turn ON and turn OFF voltages are equal to 2.5 V. When the device is off and Vin = 2.5 V, the capacitor discharge equation is:

$$I = C \frac{dVin}{dt}$$

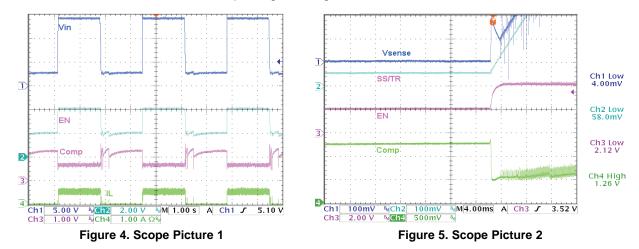
(1)

For example, at a constant discharge current of 1 µA, the Vin drops 50 mV below 2.5 V within 50 seconds.

2 Signals Status Before Power On

When the power is off and the device is enabled (due to 2.5 V present at Vin), the pins signals are as shown in (Figure 5).

- VSENSE = 0 mV error amplifier negative input
- SS/TR = 50 mV error amplifier positive input
- EN = 2 V
- COMP = 1.5 V because the error amplifier gain is high at 10,000 V/V



3 Status at Power On

The 1000- μ F capacitor is exponentially charging with usual RC time constant. R includes the wires, traces, and connector resistance from the battery to input voltage Vin. This resistance value depends on the circuit location.

Vin = Vbatt
$$\left(1 - e^{-\frac{t}{RC}}\right)$$
,

Where:

Vin = device input voltage Vbatt = battery voltage. COMP = 1.5V Error Amplifier output VSENSE = 0 V and the device frequency is reduced to around 15% of the set frequency.

The 1000-µF charging time is big compared to the device switching cycle period. At power ON, the circuit is in open loop because the COMP pin is high and the only controlled parameter is the inductor peak current which is typically around 2.5 A. The output voltage rises with no control and its peak value depends on the following parameters: L, C, ESR, and the load current lo.

(2)

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Status at Power On

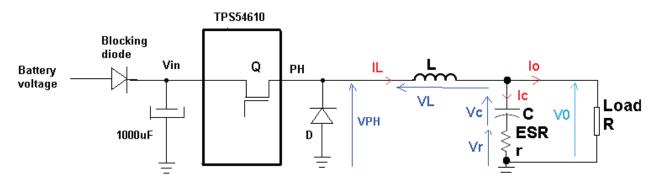


Figure 6. Power Stage Equivalent Circuit Used for In-Depth Calculations

When Q is ON and D is OFF, the resulting equations are:

$$\begin{aligned} & \mathsf{VPH}(t) = \mathsf{VL}(t) + \mathsf{V0}(t) \\ & \mathsf{II}(t) = \mathsf{Ic}(t) + \mathsf{Io}(t) \\ & \mathsf{VO}(t) = \mathsf{Vc}(t) + \frac{\mathsf{rCdVc}}{\mathsf{d}t} \\ & \mathsf{VPH}(t) = \mathsf{VL}(t) + \mathsf{Vc}(t) + \mathsf{Vr}(t) \\ & \frac{\mathsf{d}^2\mathsf{Vc}}{\mathsf{d}t^2} + \frac{\left(\frac{\mathsf{L}}{\mathsf{R}} + \mathsf{rC}\right)}{\mathsf{LC}\left(1 + \frac{\mathsf{r}}{\mathsf{R}}\right)} \frac{\mathsf{d}\mathsf{Vc}}{\mathsf{d}t} + \left(\frac{1}{\mathsf{LC}\left(1 + \frac{\mathsf{r}}{\mathsf{R}}\right)}\right) \mathsf{Vc}(t) = \left(\frac{1}{\mathsf{LC}\left(1 + \frac{\mathsf{r}}{\mathsf{R}}\right)}\right) \times \mathsf{VPH}(t) \\ & \frac{\mathsf{d}^2\mathsf{Vc}}{\mathsf{d}t^2} + \frac{2\zeta \mathsf{wodVc}}{\mathsf{d}t} + \mathsf{wo}^2\mathsf{Vc}(t) = \left(\frac{1}{\mathsf{LC}\left(1 + \frac{\mathsf{r}}{\mathsf{R}}\right)}\right) \times \mathsf{VPH}(t) \\ & \mathsf{V0}(t) = \mathsf{VPH}\left[1 - \left\{\cos \mathsf{wo}\sqrt{(1 - \zeta^2)} t + \frac{(\zeta + \mathsf{rC}\mathsf{wo})}{\sqrt{(1 - \zeta^2)}} \times \sin \mathsf{wo}\sqrt{(1 - \zeta^2)t}\right\} e\left(-\zeta\mathsf{wot}\right)\right] \\ & \mathsf{II}(t) = \frac{\mathsf{V}_{\mathrm{PH}}}{\mathsf{R}}\left[1 - \left\{\cos \mathsf{wo}\sqrt{1 - \zeta^2} t + \frac{(\zeta + \mathsf{rC}\mathsf{wo})}{\sqrt{(1 - \zeta^2)}} \sin \mathsf{wo}\sqrt{(1 - \zeta^2)t}\right\} e^{-\zeta\mathsf{wot}}\right] + \mathsf{V}_{\mathrm{PH}} \frac{\mathsf{C}\mathsf{wo}}{\sqrt{1 - \zeta^2}} \sin \mathsf{wo}\sqrt{1 - \zeta^2} t e^{-\zeta\mathsf{wot}} \end{aligned}$$

Where:

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$$\omega o = \frac{1}{\sqrt{LC\left(1 + \frac{r}{R}\right)}}$$
 is the natural pulsation
$$\zeta = \frac{1}{2} \frac{\left(\frac{L}{R} + rC\right)}{\sqrt{LC\left(1 + \frac{r}{R}\right)}} = \text{is the dumping factor}$$

VPH is the PH node switching voltage V0 is the regulator output voltage Vc is the voltage across the ideal output capacitor Vr is the voltage across the ESR resistance r II is the inductor current Ic is the current into the output capacitor TEXAS INSTRUMENTS

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lo is the load current

Example of detailed calculations using the circuit parameters as shown in Figure 7. VPH average is around 2.2 V $\,$

L = 33 $\mu H,$ C = 44 $\mu F,$ Load R = 8 $\Omega,$ r = 0.05 Ω (estimated), Ton = 38 μs

$$\zeta = \frac{1}{2} \frac{\left(\frac{L}{R} + rC\right)}{\sqrt{LC\left(1 + \frac{r}{R}\right)}} = 0.083$$

$$\omega o = \frac{1}{\sqrt{LC\left(1 + \frac{r}{R}\right)}} = 0.0262 \text{ rd/}\mu \text{s}$$

Calculated output voltage from the equations at 38 μ s, V0 = 1.055 sV Measured output voltage at 38 μ s is approximately 1.1 V (Figure 8 and Figure 9) Calculated inductor current from the equations at 38 μ s II = 2.08 A Measured inductor current at 38 μ s is approximately 2 A, shown in Figure 8 and Figure 9.

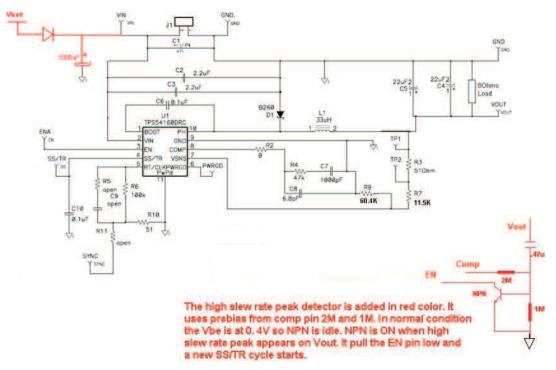


Figure 7. Example of Detailed Calculations

When Q is OFF and D is ON, the resulting equations are:



Status at Power On

$$\begin{aligned} -\mathrm{Vf} &= \mathrm{LC}\left(1+\frac{\mathrm{r}}{\mathrm{R}}\right) \frac{\mathrm{d}^{2}\mathrm{Vc}}{\mathrm{d}t^{2}} + \left(\frac{\mathrm{L}}{\mathrm{R}} + \mathrm{rC}\right) \frac{\mathrm{d}\mathrm{Vc}}{\mathrm{d}t} + \mathrm{Vc}(t), \ \mathrm{Vf} \ \mathrm{is} \ \mathrm{diode} \ \mathrm{forward} \ \mathrm{voltage} \\ &= \frac{\mathrm{d}^{2}\mathrm{Vc}}{\mathrm{d}t^{2}} + \frac{\left(\frac{\mathrm{L}}{\mathrm{R}} + \mathrm{rC}\right)}{\mathrm{LC}\left(1+\frac{\mathrm{r}}{\mathrm{R}}\right)} \frac{\mathrm{d}\mathrm{Vc}}{\mathrm{d}t} + \left(\frac{1}{\mathrm{LC}\left(1+\frac{\mathrm{r}}{\mathrm{R}}\right)}\right) \mathrm{Vc}(t) = -\left(\frac{1}{\mathrm{LC}\left(1+\frac{\mathrm{r}}{\mathrm{R}}\right)}\right) \mathrm{Vf} \\ \mathrm{Vc}(t) &= -\mathrm{Vf} + \left[\mathrm{G} \ \mathrm{cos} \ \mathrm{sos} \sqrt{(1-\zeta^{2})} \ (t-\mathrm{Ton}) + \mathrm{K} \ \mathrm{sin} \ \mathrm{sos} \sqrt{(1-\zeta^{2})}(t-\mathrm{Ton})\right] e^{-\zeta \mathrm{sos}(t-\mathrm{Ton})} \\ \mathrm{Ic}(t) &= \mathrm{Csos} \left[\left(\mathrm{K} \sqrt{(1-\zeta^{2})} - \mathrm{G}\zeta\right) \mathrm{cos} \ \mathrm{sos} \sqrt{(1-\zeta^{2})}(t-\mathrm{Ton}) - \left(\mathrm{K}\zeta + \mathrm{G} \sqrt{(1-\zeta^{2})}\right) \mathrm{sin} \ \mathrm{sos} \sqrt{(1-\zeta^{2})}(t-\mathrm{Ton}) \right] e^{-\zeta \mathrm{sos}(t-\mathrm{Ton})} \\ \mathrm{G} &= \mathrm{Vf} + \mathrm{VPH} \left[1 - \left(\mathrm{cos} \ \mathrm{sos} \sqrt{(1-\zeta^{2})} \ \mathrm{Ton} + \frac{\zeta}{\sqrt{(1-\zeta^{2})}} \mathrm{sin} \ \mathrm{sos} \sqrt{(1-\zeta^{2})} \mathrm{cos} \ \mathrm{sos} \sqrt{(1-\zeta^{2})} \mathrm{cos} \ \mathrm{sos} \sqrt{(1-\zeta^{2})} \mathrm{cos} \ \mathrm{sos} \sqrt{(1-\zeta^{2})} \mathrm{cos} \ \mathrm{sos} \sqrt{(1-\zeta^{2})} \mathrm{Ton} - \frac{\zeta}{\sqrt{(1-\zeta^{2})}} \mathrm{cos} \ \mathrm{sos} \sqrt{(1-\zeta^{2})} \mathrm{Ton} \right] e^{-\zeta \mathrm{sos} \mathrm{Ton}} \\ \mathrm{Vo}(t) &= \mathrm{Vc}(t) + \mathrm{vPH} \right] + \mathrm{vPH} \left(\mathrm{sin} \ \mathrm{sos} \sqrt{(1-\zeta^{2})} \mathrm{Ton} - \frac{\zeta}{\sqrt{(1-\zeta^{2})}} \mathrm{cos} \ \mathrm{sos} \sqrt{(1-\zeta^{2})} \mathrm{Ton} \right) e^{-\zeta \mathrm{sos} \mathrm{Ton}} \\ \mathrm{vo}(t) &= \mathrm{vc}(t) + \mathrm{r} \ \mathrm{Ic}(t) \\ \mathrm{II}(t) &= \mathrm{Ic}(t) + \frac{\mathrm{Vo}(t)}{\mathrm{R}} \end{aligned}$$

(4)

For more detailed calculations, see the appendixes.

Example of Detailed Calculations Using the Circuit Parameters as shown in Figure 7. L = 33 μ H, C = 44 μ F, Load R = 8 Ω , r = 0.05 Ω (estimated), Ton = 38 μ s, T = 50 μ s

$$\zeta = \frac{1}{2} \frac{\left(\frac{L}{R} + rC\right)}{\sqrt{LC\left(1 + \frac{r}{R}\right)}} = 0.083$$

$$\omega o = \frac{1}{\sqrt{LC\left(1 + \frac{r}{R}\right)}} = 0.0262 \text{ rd/}\mu\text{S}$$

 $\sqrt{LC\left(1+\frac{r}{R}\right)}$

Vf = 0.7 V, G = 1.66 V , K = 1.836 V

Calculated voltage across capacitor from the equations at $t = T = 50 \ \mu s \ Vc(T) = 1.388 \ V$ Calculated capacitor current from the equations at $t = T = 50 \ \mu s \ Ic(T) = 1.179 \ A$ Calculated output voltage V0(T) = Vc(T) + r Ic(T) at 50 \ \mu s \ V0 = 1.388 + 0.05x1.179

Measured output voltage at 50 µs is approximately 1.6 V, shown in Figure 8 and Figure 9.

Calculated inductor current $I_L(t) = Ic(t) + \frac{Vo(t)}{R}$ at 50µS

$$I_{L} = 1.179A + \frac{1.447}{8}$$

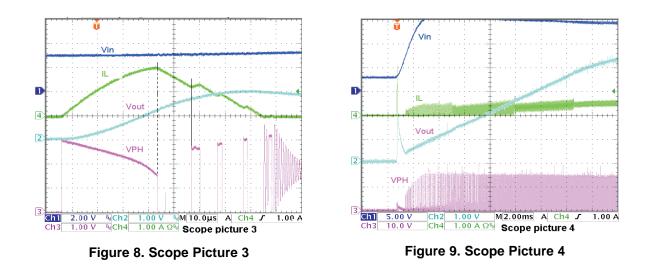
 $II(T) = 1.36 A$

Measured inductor current at 50 µs is approximately 1.2 A, shown in Figure 8 and Figure 9.

The output voltage overshoot depends on the values of these circuit parameters. The overshoot level can be minimized by choosing these parameters: L, C, ESR, and the load current I_0 .



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3.1 Output Voltage Overshoot Remover

The circuit shown in Figure 10 is formed by two resistors, one capacitor, and one NPN small signal transistor and is acting as fast transient detector (Figure 11). The output voltage rises at a low slew rate because of soft start. If the EN pin is left floating, the output voltage slew rate is fast and depends on the battery voltage slew rate. The circuit detects the output voltage slew rate. If the slew rate is fast, the NPN transistor turns ON and pulls the EN pin LO; a new soft start starts the output voltage.

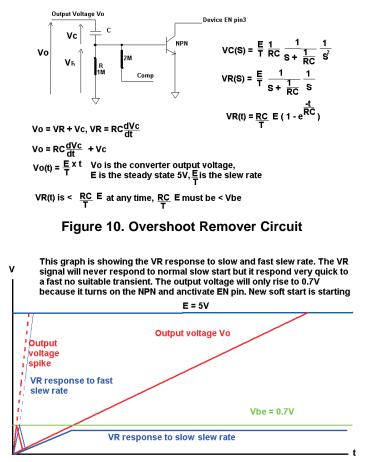


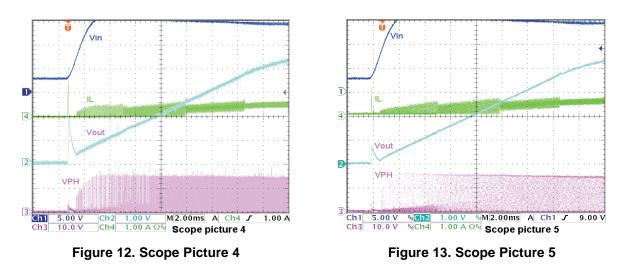
Figure 11. Overshoot Remover Mechanism



Conclusion

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Scope picture 4 (Figure 12) shows output voltage overshoot of 2 V without overshoot remover circuit. Scope picture 5 (Figure 13) shows output voltage overshoot of only 0.7 V with overshoot remover circuit.



4 Conclusion

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In conclusion, the TPS54160 family of switching buck converters as used in this application report saves more than 50 μ A of power drained from the battery during power off. This application report illustrates in-depth information and detailed calculations of the first switching cycle differential equations. It also provides solutions to minimize overshoot and input inrush current during the power-up cycle. It is evident that this device family is optimized for many applications where battery power saving is crucial.

Although, the output voltage overshoot at power up cannot be completely eliminated, it can be minimized to less than 1 V using the appropriate components or by adding the overshoot remover circuit.

Using the example circuit in Figure 7, the following results are achieved:

- Vin stayed above 2.4 V for more than 15 minutes when the battery is disconnected.
- With output capacitor of 220 μF, the output voltage overshoot at power up was less than 1 V.
- With the overshoot remover circuit formed by two resistors, one capacitor, and one NPN transistor, the output voltage overshoot was less than 0.7 V.



Appendix A

A.1 Backup Calculation for Q on and D off

VPH(t) = VL(t) + VO(t)II(t) = Ic(t) + Io(t)

$$V0(t) = Vc(t) + rC \frac{dVc}{dt}$$
(5)

$$VPH(t) = VL(t) + Vc(t) + Vr(t)$$

$$VPH(t) = L\frac{dII}{dt} + Vc(t) + rC\frac{dVc}{dt}$$

$$VPH(t) = L\frac{dIc}{dt} + L\frac{dIo}{dt} + Vc(t) + rC\frac{dVc}{dt}$$

$$VPH(t) = LC\frac{d^{2}Vc}{dt^{2}} + \frac{L}{R} \times \frac{dV0}{dt} + Vc(t) + rC\frac{dVc}{dt}$$

$$VPH(t) = LC\left(1 + \frac{r}{R}\right)\frac{d^{2}Vc}{dt^{2}} + \left(\frac{L}{R} + rC\right)\frac{dVc}{dt} + Vc(t)$$
(7)

$$\frac{d^{2}Vc}{dt^{2}} + \frac{\left(\frac{L}{R} + rC\right)}{LC\left(1 + \frac{r}{R}\right)}\frac{dVc}{dt} + \left(\frac{1}{LC\left(1 + \frac{r}{R}\right)}Vc(t)\right) = \left(\frac{1}{LC\left(1 + \frac{r}{R}\right)}\right)VPH(t)$$
(8)

$$\frac{d^{2}Vc}{dt^{2}} + \frac{\left(\frac{L}{R} + rC\right)}{LC\left(1 + \frac{r}{R}\right)}\frac{dVc}{dt} + \left(\frac{1}{LC(1 + \frac{r}{R})}\right)Vc(t) = 0 \text{ homogeneous equation}$$
(9)

$$\frac{d^2 Vc}{dt^2} + 2\zeta \omega o \frac{dVc}{dt} + \omega_o^2 Vc(t) = 0$$
(10)

$$\omega o = \frac{1}{\sqrt{LC\left(1 + \frac{r}{R}\right)}} \text{ natural pulsation}$$
$$\zeta = \frac{1}{\sqrt{LC\left(1 + \frac{r}{R}\right)}} \text{ dumping factor}$$

(11)

(6)

In this application the dumping factor is $0 < \zeta < 1$ so two distinct complex root for Equation 9

X1,2 = - ζωο $\pm jωo\sqrt{(\zeta^2 - 1)}$

The solution for Equation 8 is obtained by adding a particular solution of it and the general solution of Equation 9.

The homogeneous Equation 9 general solution (A and B are constant) is:

(A cos
$$\omega o \sqrt{(1-\zeta^2)}t$$
 + B sin $\omega o \sqrt{(1-\zeta^2)}t$) e^{- ζωot}



Value of the Constants A and B

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VPH is particular solution of Equation 8

$$Vc(t) = VPH + [A \cos \omega o \sqrt{(1 - \zeta^2)}t + B \sin \omega o \sqrt{(1 - \zeta^2)}t]e^{-\zeta \omega o t}$$

$$Ic(t) = \frac{CdVc}{dt}, VPH \text{ is considered constant during transistor ON}$$
(12)

A.2 Value of the Constants A and B

At t =0, output voltage $V_0=0V$, Inductor current $I_1=0A$, that means $V_0(0)=0$ and $I_0(0)=0$ Vc(0) = VPH + A.A = -VPH $\frac{\mathrm{CdVc}(0)}{\mathrm{ctt}} = 0$ $-\zeta \omega o \left[A \cos \omega o \sqrt{(1-\zeta^2)} 0 + B \sin \omega o \sqrt{(1-\zeta^2)} 0 \right] e^{-\zeta \omega o 0} +$ $\left[\mathsf{B}\omega\mathsf{o}\sqrt{(1-\zeta^2)}\,\cos\,\omega\mathsf{o}\sqrt{(1-\zeta^2)}\mathsf{0}\,-\,\omega\mathsf{o}\sqrt{(1-\zeta^2)}\,\mathsf{A}\sin\,\omega\mathsf{o}\sqrt{(1-\zeta^2)}\mathsf{0}\right]\mathsf{e}^{-\zeta\omega\mathsf{o}\mathsf{0}}\,=\,\mathsf{0}$ $-\zeta \omega \circ A + B \omega \circ \sqrt{(1-\zeta^2)} = 0$, $B = A \frac{\zeta}{\sqrt{(1-\zeta^2)}}$, $B = -VPH \frac{\zeta}{\sqrt{(1-\zeta^2)}}$ $Vc(t) = VPH \left| 1 - \left(\cos \omega o \sqrt{(1 - \zeta^2)} t + \frac{\zeta}{\sqrt{(1 - \zeta^2)}} \sin \omega o \sqrt{(1 - \zeta^2)} t \right) e^{-\zeta \omega o t} \right|$ $Ic(t) = VPH \frac{C\omega o}{\sqrt{1-\zeta^2}} \sin \omega o \sqrt{(1-\zeta^2)} t e^{-\zeta \omega o t}$ $VO(t) = Vc(t) + rC \frac{dVc}{dt}$ (13) $VO(t) = VPH \left| 1 - \left\{ \left(\cos \omega o \sqrt{(1 - \zeta^2)t} + \frac{\zeta}{\sqrt{(1 - \zeta^2)}} \times \sin \omega o \sqrt{1 - \zeta^2} t + \frac{rC\omega o}{\sqrt{1 - \zeta^2}} \sin \omega o \sqrt{1 - \zeta^2} t \right\} \right\} e^{-\zeta \omega o t}$ $VO(t) = VPH \left| 1 - \left\{ \cos \omega o \sqrt{(1-\zeta^2)t} + \frac{\zeta + rC\omega o}{\sqrt{1-\zeta^2}t} \times \sin \omega o \sqrt{1-\zeta^2}t \right\} e^{-\zeta\omega o t} \right|$ II(t) = Ic(t) + Io(t)= Ic(t) + Vo(t)/R $II(t) = \frac{VPH}{R} \left[1 - \left\{ \cos \omega o \sqrt{(1-\zeta^2)}t + \frac{\zeta + rC\omega o}{\sqrt{1-\zeta^2}} \sin \omega o \sqrt{(1-\zeta^2)}t \right\} e^{-\omega\zeta o t} + VPH \frac{C\omega o}{\sqrt{1-\zeta^2}} \sin \omega o \sqrt{(1-\zeta^2)}t e^{-\omega\zeta o t} \right]$ (14)



A.3 Backup Calculation for D on and Q off

$$\begin{aligned} -\mathrm{Vf} &= \mathrm{LC}\left(1+\frac{\mathrm{r}}{\mathrm{R}}\right) \frac{\mathrm{d}^{2}\mathrm{Vc}}{\mathrm{d}t^{2}} + \left(\frac{\mathrm{L}}{\mathrm{R}} + \mathrm{rC}\right) \frac{\mathrm{d}\mathrm{Vc}}{\mathrm{d}t} + \mathrm{Vc}(t), \ \mathrm{Vf} \ \mathrm{is} \ \mathrm{diode} \ \mathrm{forward} \ \mathrm{voltage} \\ \\ &\frac{\mathrm{d}^{2}\mathrm{Vc}}{\mathrm{d}t^{2}} + \frac{\left(\frac{\mathrm{L}}{\mathrm{R}} + \mathrm{rC}\right)}{\left(\mathrm{LC}\left(1+\frac{\mathrm{r}}{\mathrm{R}}\right)\right)} \frac{\mathrm{d}\mathrm{Vc}}{\mathrm{d}t} + \left(\frac{1}{\mathrm{LC}\left(1+\frac{\mathrm{r}}{\mathrm{R}}\right)}\right) \mathrm{Vc}(t) = -\left(\frac{1}{\mathrm{LC}\left(1+\frac{\mathrm{r}}{\mathrm{R}}\right)}\right) \mathrm{Vf} \\ \\ &\frac{\mathrm{d}^{2}\mathrm{Vc}}{\mathrm{d}t^{2}} + \frac{\left(\frac{\mathrm{L}}{\mathrm{R}} + \mathrm{rC}\right)}{\mathrm{LC}\left(1+\frac{\mathrm{r}}{\mathrm{R}}\right)} \frac{\mathrm{d}\mathrm{Vc}}{\mathrm{d}t} + \left(\frac{1}{\mathrm{LC}(1+\frac{\mathrm{r}}{\mathrm{R}})}\right) \mathrm{Vc}(t) = 0 \ \ \mathrm{homogeneous} \ \mathrm{equation} \\ \\ &\omega o = \frac{1}{\sqrt{\mathrm{LC}\left(1+\frac{\mathrm{r}}{\mathrm{R}}\right)}} \ \ \mathrm{natural} \ \mathrm{pulsation} \\ \\ &\zeta = \frac{1}{2} \left(\frac{\frac{\mathrm{L}}{\mathrm{R}} + \mathrm{rC}}{\sqrt{\mathrm{LC}\left(1+\frac{\mathrm{r}}{\mathrm{R}}\right)}}\right) \ \ \mathrm{dumping} \ \mathrm{factor} \end{aligned}$$

In this application the dumping factor is $0 < \zeta < 1$ so two distinct complex root for

X1,2 =
$$-\zeta \omega o \pm j \omega o \sqrt{(1-\zeta^2)}$$

The solution for Equation 3 is obtained by adding a particular solution of Equation 7 and the general solution of Equation 9

The homogeneous Equation 9 general solution (G and K are constant) is:

$$\left(G \cos \omega o \sqrt{(1 - \zeta^2)} t + K \sin \omega o \sqrt{(1 - \zeta^2)} t \right) e^{-\zeta \omega o t}$$

$$-Vf \text{ is particular solution of equation 7}$$

$$Vc(t) = -Vf + \left[G \cos \omega o \sqrt{(1 - \zeta^2)} (t - Ton) + K \sin \omega o \sqrt{(1 - \zeta^2)} \right] (t - Ton) e^{-\zeta \omega o (t - Ton)}$$

$$Ic(t) = \frac{C dVc}{dt}, \text{ Vf is constant during diode D is ON}$$

$$Ic(t) = -\zeta C \omega o \left[G \cos \omega o \sqrt{(1 - \zeta^2)} (t - Ton) + K \sin \omega o \sqrt{(1 - \zeta^2)} (t - Ton) \right] e^{-\zeta \omega o (t - Ton)}$$

$$+ \left[KC \omega o \sqrt{(1 - \zeta^2)} \cos \omega o \sqrt{(1 - \zeta^2)} (t - Ton) - GC \omega o \sqrt{(1 - \zeta^2)} \sin \omega o \sqrt{(1 - \zeta^2)} (t - Ton) \right] e^{-\zeta \omega o (t - Ton)}$$

$$Ic(t) = C \omega o \left[(K \sqrt{(1 - \zeta^2)} - G \zeta \cos \omega o \sqrt{(1 - \zeta^2)} (t - Ton) - (K \zeta + G \sqrt{(1 - \zeta^2)}) \sin \omega o \sqrt{(1 - \zeta^2)} (t - Ton) \right]$$

$$e^{-\zeta \omega o (t - Ton)} \left(K \sqrt{(1 - \zeta^2)} - G \zeta \right)$$

$$(16)$$

A.4 Value of the Constants G and K

The inductor current II(t) and the capacitor voltage Vc(t) are continuous functions

At t = Ton switching transition time, the capacitor voltage is continuous function:

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(15)



Value of the Constants G and K

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$$Vc(t) = VPH \left[1 - \left(\cos \omega o \sqrt{(1 - \zeta^2)} t + \frac{\zeta}{\sqrt{(1 - \zeta^2)}} \sin \omega o \sqrt{(1 - \zeta^2)} t \right) e(-\zeta \omega o t) \right]$$
$$Vc(Ton) = VPH \left[1 - \left(\cos \omega o \sqrt{(1 - \zeta^2)} Ton + \frac{\zeta}{\sqrt{(1 - \zeta^2)}} \sin \omega o \sqrt{(1 - \zeta^2)} Ton \right) e(-\zeta \omega o Ton) \right]$$
$$= -Vf + \left[G \cos \omega o \sqrt{(1 - \zeta^2)} (Ton - Ton) + K \sin \omega o \sqrt{(1 - \zeta^2)} (Ton - Ton) \right] e^{-\zeta \omega o (Ton - Ton)} = -Vf + G$$

$$G = Vf + VPH \left[1 - \left(\cos \omega o \sqrt{(1 - \zeta^2)} Ton + \frac{\zeta}{\sqrt{(1 - \zeta^2)}} \sin \omega o \sqrt{(1 - \zeta^2)} Ton \right] e(-\zeta \omega o Ton) \right]$$
(17)

At t = Ton switching transition time, the capacitor current is continuous function:

$$\begin{split} \mathsf{lc}(\mathsf{t}) &= \left(\frac{\mathsf{VPHxC}_{\Theta \Theta}}{\sqrt{(1-\zeta^2)}} \operatorname{sin}_{\Theta \circ \sqrt{(1-\zeta^2)}} \mathsf{Ton}\right) e^{-\zeta_{\Theta \circ} \mathsf{Ton}} \\ &= \mathsf{C}_{\Theta \circ} \Big[\left(\mathsf{K} \sqrt{(1-\zeta^2)} - \mathsf{G} \zeta \right) \operatorname{cos}_{\Theta \circ \sqrt{(1-\zeta^2)}} (\mathsf{Ton-Ton}) - \left(\mathsf{G} \sqrt{(1-\zeta^2)} + \mathsf{K} \zeta \right) \operatorname{sin}_{\Theta \circ \sqrt{(1-\zeta^2)}} (\mathsf{Ton-Ton}) \Big] \\ &= \left(\mathsf{K} \sqrt{(1-\zeta^2)} - \mathsf{G} \zeta \right) \mathsf{C}_{\Theta \circ} \circ \Big[\left(\frac{\mathsf{VPHxC}_{\Theta \circ}}{\sqrt{(1-\zeta^2)}} \operatorname{sin}_{\Theta \circ \sqrt{(1-\zeta^2)}} \mathsf{Ton} \right) e^{-\zeta_{\Theta \circ} \mathsf{Ton}} \Big] \\ &= \left(\mathsf{K} \sqrt{(1-\zeta^2)} - \mathsf{G} \zeta \right) \mathsf{C}_{\Theta \circ} \circ \Big[\left(\frac{\mathsf{VPHxC}_{\Theta \circ}}{\sqrt{(1-\zeta^2)}} \operatorname{sin}_{\Theta \circ \sqrt{(1-\zeta^2)}} \mathsf{Ton} \right) e^{-\zeta_{\Theta \circ} \mathsf{Ton}} + \frac{\mathsf{G} \zeta}{\sqrt{(1-\zeta^2)}} \Big] \\ &= \left(\frac{\mathsf{VPH}}{(1-\zeta^2)} \operatorname{sin}_{\Theta \circ \sqrt{(1-\zeta^2)}} \mathsf{Ton} \right) e^{-\zeta_{\Theta \circ} \mathsf{Ton}} + \frac{\zeta}{\sqrt{(1-\zeta^2)}} \mathsf{X} \\ &\left\{ \mathsf{Vf} + \mathsf{VPH} \left[1 - \left(\operatorname{cos}_{\Theta \circ \sqrt{(1-\zeta^2)}} \mathsf{Ton} + \frac{\zeta}{\sqrt{(1-\zeta^2)}} \operatorname{sin}_{\Theta \circ \sqrt{(1-\zeta^2)}} \mathsf{Ton} \right) e^{-\zeta_{\Theta \circ} \mathsf{Ton}} \right] \right\} \\ &= \left(\frac{\zeta}{(1-\zeta^2)} (\mathsf{Vf} + \mathsf{VPH}) + \mathsf{VPH} \left(\operatorname{sin}_{\Theta \circ \sqrt{(1-\zeta^2)}} \mathsf{Ton} - \frac{\zeta}{(1-\zeta^2)} \operatorname{cos}_{\Theta \circ \sqrt{(1-\zeta^2)}} \mathsf{x} \operatorname{Ton} \right) e^{-\zeta_{\Theta \circ} \mathsf{Ton}} \\ &\mathsf{V0}(\mathsf{t}) = \mathsf{Vc}(\mathsf{t}) + \mathsf{r} \operatorname{Ic}(\mathsf{t}) \\ & \mathsf{I}(\mathsf{t}) = \mathsf{Ic}(\mathsf{t}) + \mathsf{Vo}(\mathsf{t})/\mathsf{R} \end{split}$$

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