Adaptive Control of Induction Motors with Unknown Load and Rotor Resistance

Abstract
This paper considers the design of an observer-based adaptive controller for variable speed three-phase AC induction motors. The proposed controller allows for the simultaneous and independent control of the speed (torque) and the flux of the motor without requiring the measurement of the flux, and without the knowledge of the rotor resistance and the motor load. The control adaptively estimates the flux variables and the unknown parameters, using only the measured signals. The control strategy is designed for the equivalent two-phase field-oriented (d-q) model of the motor. Therefore, it does not have stiff nonlinearities and, hence, it is suitable for the discretization and digital implementation with DSPs.

Keywords: Induction motor, rotor resistance, adaptive control, flux observer, DSP.

Introduction
AC induction motors are very popular in the industry since they are rugged, inexpensive, and are maintenance free. Unfortunately, unlike DC motors, they have nonlinear dynamics and, for variable speed servo applications, they require advanced control schemes [1-4]. Fortunately, however, using power electronics and fast digital signal processors (DSP), the implementation of such advanced controllers is now becoming practical [5-6].

In the past recent years, many techniques have been developed for the control of variable speed induction motors [1-6]. Although DSPs have provided the computational power for the implementation of such advanced control schemes, in real applications, these techniques are either very difficult to implement or do not perform as desired. The problems are due to the fact that induction motors have nonlinear dynamics; their rotor flux variables cannot be measured for control application; and that rotor resistance varies up to 200% due to heating of the motor. These have been the topic of research in the recent past years and some solutions have been proposed [7-11]. However, no simple solution has yet been provided.

In this paper the design of a field-oriented adaptive controller is considered for variable speed three-phase AC induction motors with unknown load and rotor resistance. The control is designed for an equivalent two-phase field-oriented (d-q) model of the motor. It allows for the simultaneous and independent control of the speed (torque) and the flux of the motor, without requiring the measurement of the flux. The control does not require the knowledge of the rotor resistance or the load. It adaptively estimates these parameters, using only measurable signals and guarantees the stability of the closed-loop system. The proposed controller also eliminates
the stiff nonlinearities in the model, which are due to the simultaneous existence of slow and fast modes, and hence is suitable for the discretization and digital implementation with DSPs.

**Dynamic model of induction motors**

The dynamic model of three-phase induction motors, Figure 1, is very nonlinear with strong cross couplings. It also contains both slow and fast modes, which makes digital implementation of most control techniques difficult.

![Three-Phase Induction Motor](image1)

*Figure 1*

An equivalent two-phase dynamic model of a three-phase induction motor [1-4], as shown in Figure 2, is given by

![Two-Phase Induction Motor](image2)

*Figure 2*

\[
\begin{align*}
\dot{\omega} &= a_1 (\phi_\alpha i_\beta - \phi_\beta i_\alpha) - a_0 T_L \\
\dot{i}_\alpha &= \frac{1}{L_\alpha} (v_\alpha - R_\alpha i_\alpha) - \frac{R}{L_\alpha L_p} (L_{m_\alpha} i_\alpha - \phi_\alpha) + \frac{1}{L_p} n \omega \phi_\beta \\
\dot{i}_\beta &= \frac{1}{L_\beta} (v_\beta - R_\beta i_\beta) - \frac{R}{L_\beta L_p} (L_{m_\beta} i_\beta - \phi_\beta) - \frac{1}{L_p} n \omega \phi_\alpha \\
\dot{\phi}_\alpha &= \frac{R}{L_r} (L_{m_\alpha} i_\alpha - \phi_\alpha) - n \omega \phi_\beta \\
\dot{\phi}_\beta &= \frac{R}{L_r} (L_{m_\beta} i_\beta - \phi_\beta) + n \omega \phi_\alpha
\end{align*}
\]

(1)

where \(\omega, \phi_{\alpha,\beta}, i_{\alpha,\beta}, v_{\alpha,\beta}\) are rotor speed, induced fluxes, stator currents and stator voltages in the stator’s fixed \(\alpha-\beta\) coordinate system (frame). \(T_L\) is the load torque, which is usually a function...
of rotor’s speed. Moreover, \( a_0 = \frac{1}{r} \), \( a_1 = \frac{n L_{in}}{2} \), \( L_\mu = \frac{\sigma}{r} L_m \), \( L_\sigma = \sigma L_s \) (leakage inductance), \( \sigma = 1 - \frac{L_{in}^2}{L_m L_s} \) (leakage factor), and that \( R_r, R_s, L_r \) and \( L_s \) are the resistances and inductances of the rotor and the stator, and that \( L_m \) is the mutual inductance of the motor. Some of these parameters, such as rotor’s effective resistance \( R_r \), change drastically with temperature [7-11]. Also, the flux variables \( \phi_\alpha \) and \( \phi_\beta \), generally, cannot be measured directly. Due to these facts, the controller design for the induction motors is quite challenging. Nevertheless, due to their simple structure, low maintenance, and high torque generation, these motors are very popular in the industry and the development of high performance controllers for these machines is of great importance.

Consider the generalized d-q field-oriented coordinate transformation, as shown in Figure 3, given by

\[
X_{\alpha q} = e^{-j\rho} X_{\alpha\beta}
\]

\[
\rho = \omega_s
\]

where \( X_{\alpha\beta} = x_\alpha + jx_\beta \) is an arbitrary vector in the stator’s fixed \( \alpha-\beta \) frame and that \( X_{\alpha q} = x_d + jx_q \) is the same vector expressed in the d-q frame. The variable \( \rho \) is the angle between the \( \alpha-\beta \) coordinate system and the d-q coordinate system. The above coordinate transformation can also be expressed as the following matrix multiplication

\[
\begin{bmatrix}
  x_d \\
  x_q
\end{bmatrix} =
\begin{bmatrix}
  \cos \rho & \sin \rho \\
  -\sin \rho & \cos \rho
\end{bmatrix}
\begin{bmatrix}
  x_\alpha \\
  x_\beta
\end{bmatrix}
\]

(3)

Applying the above coordinate transformation, the motor’s dynamics may be written as

\[
\dot{x}_d = a_1 (\phi_d i_q - \phi_q i_d) - a_0 T_L
\]

\[
\dot{i}_d = \frac{1}{L_s} (v_d - R_s i_d) - \frac{R}{L_s L_\mu} (L_m i_d - \phi_d) + \frac{j}{L_s} n \omega \phi_q + \omega_s i_d
\]

\[
\dot{i}_q = \frac{1}{L_s} (v_q - R_s i_q) - \frac{R}{L_s L_\mu} (L_m i_d - \phi_q) - \frac{j}{L_s} n \omega \phi_d - \omega_s i_d
\]

\[
\dot{\phi}_d = \frac{R}{L_s} (L_m i_d - \phi_d) - (n \omega - \omega_s) \phi_q
\]

\[
\dot{\phi}_q = \frac{R}{L_s} (L_m i_d - \phi_q) + (n \omega - \omega_s) \phi_d
\]

where \( \omega_s \) is the angular speed of \( \alpha-\beta \) frame with respect to d-q frame. Here, the variable \( \omega_s \), known as the slip frequency, is considered as an input variable that is to be selected.
Controller Design

There are two common techniques for implementing a controller for an induction motor, current source inverter (CSI)-fed control and voltage source inverter (VSI)-fed control. For CSI-fed control, the stator currents, together with $\omega_s$, are considered as the control variables and their values are determined. High-gain inner-loop current controllers are then used to implement these desired currents. For VSI-fed control, the stator voltages, together with $\omega_s$, are considered as the control variables. These voltages are then determined and implemented directly. In both cases the applied currents and voltages must be kept bounded. In this paper, the design of a CSI-fed control strategy for induction motors is considered. Generally, for the CSI control design, only the first and the last two equations in (4) are required for control derivations.

Case 1 – Known Parameters

Let $\omega_R$ and $\phi_R$ denote smooth references for the rotor’s speed $\omega$ and flux magnitude $\phi_d$, where both, as well as their derivatives, are assumed to be known. Moreover, let the reference value for $\phi_q$ be zero. The corresponding tracking errors for these signals are defined as $e_\omega=\omega-\omega_R$, $e_\phi=\phi-\phi_R$ and $\phi_q$, and their dynamic equations are given by

\begin{align}
\dot{e}_\omega &= a_1 (e_\phi i_q - \phi_q i_d) + \left[ a_1 \phi_R i_q - a_0 T_L - \dot{\omega}_R \right] \\
\dot{e}_\phi &= -\frac{R}{L_m} e_\phi - (n\omega - \omega) \phi_q + \left[ \frac{R}{L_m} (L_m i_d - \phi_R) - \dot{\phi}_R \right] \\
\dot{\phi}_q &= -\frac{R}{L_m} \phi_q + (n\omega - \omega) e_\phi + \left[ \frac{R}{L_m} L_m i_q + (n\omega - \omega) \phi_R \right]
\end{align}

(5)

Now consider the control strategy, given as

\begin{align}
i_d &= m_2 - \frac{1}{L_m} \frac{K_p}{R_s} \frac{L_m}{L_s} \gamma T a_i m_i e_\omega \\
i_q &= m_i \\
\omega_s &= n\omega + \frac{1}{Q_e} \left[ \frac{K_p}{R_s} L_m m_i - \gamma T a_i \left( m_2 - \frac{1}{L_m} \frac{K_p}{R_s} \gamma T a_i m_i e_\omega \right) \right]
\end{align}

(6)

with

\begin{align}
m_i &= \frac{1}{L_m} \left[ \omega_R + a_0 T_L - \left( c_1 + c_2 e_\omega^2 \right) e_\omega \right] \\
m_2 &= \frac{1}{L_m} \left( \phi_R + \frac{1}{L_m} \dot{\phi}_R \right)
\end{align}
where $\gamma = \frac{\gamma_0}{1 + f c}$, $\gamma = 1 - 0.5 \frac{f c}{1 + f c}$, $f c = \gamma_0 a i_{\text{max}} | e_o |$, $i_{\text{max}}$ is the upper-bound for the magnitude of the currents, and that $c_1$, $c_{11}$, $\gamma_0$ and $\gamma_0$ are positive constants. Note that $0.5 < \gamma \leq 1$. Also note that the proposed control strategy is just a nonlinear static feedback control in the d-q frame. The control does not require the measurement of the rotor flux. Moreover, it is well defined everywhere, provided that the reference command for the rotor’s flux magnitude, $\phi_R$, is kept non-zero and positive at all times, which is always possible. With the above controller, the dynamics of the closed-loop error system can be written as

$$
\dot{e}_o = -(c_1 + c_{11} e_o^2) e_o + a_1 (e_q i_q - \phi_q i_d)
$$

$$
\dot{e}_q = -\frac{R_c}{L_c} e_o - (n\omega - \omega_e)\phi_q - \gamma a_i e_o
$$

$$
\dot{\phi}_q = -\frac{R_c}{L_c} \phi_q + (n\omega - \omega_e) e_\phi + \gamma a_i i_d e_o
$$

Using a candidate Lyapunov function, such as $V = \frac{1}{2} [\gamma e_o^2 + e_\phi^2 + \phi_q^2]$, it can be shown that all the closed-loop signals are globally bounded and that the errors converge to zero exponentially. Therefore, when all the motor parameters are constant and known, the proposed static feedback control (6) can achieve the motor speed servo control objectives. For implementation, of course, the equivalent control currents $i_\alpha$ and $i_\beta$ in the $\alpha-\beta$ frame must be applied, using equations (2) and (3). Obviously, since equation (2) must be used for the control implementation, the resulting controller in the $\alpha-\beta$ frame would be of first order.

**Case 2 – Unknown Parameters**

Now let us assume that both load torque $T_L$ and rotor’s electrical resistance $R_r$ change during the operation. In that case, the control strategy (6) must be modified to account for the variations in these parameters. Here we will consider an adaptive control strategy.

Assume that both $T_L$ and $R_r > 0$ are unknown. Denote the inverse of rotor resistance as $\rho = \frac{1}{R_r}$, which is also positive and unknown. Then the tracking errors (5) are written as

$$
\dot{e}_o = a_1 (e_\phi i_q - \phi_q i_d) - a_0 T_L + \left[ a_1 \phi_R i_q - a_0 \tilde{T}_L - \tilde{\omega}_R \right]
$$

$$
\dot{e}_q = -\frac{R_c}{L_c} e_\phi - (n\omega - \omega_e)\phi_q - R_r \tilde{\rho} \frac{L_m i_q}{L_r} + \left[ L_m i_q \right]
$$

$$
\dot{\phi}_q = -\frac{R_c}{L_c} \phi_q + (n\omega - \omega_e) e_\phi + \frac{R_r}{L_r} L_m i_q + \left[ \frac{R_r}{L_r} L_m i_q + (n\omega - \omega_e) \phi_R \right]
$$

where the parameter errors are defined as $\tilde{T}_L = T_L - \hat{T}_L$, $\tilde{R}_r = R_r - \hat{R}_r$, and $\tilde{\rho} = \rho - \hat{\rho}$, and that $\hat{T}_L$, $\hat{R}_r$, and $\hat{\rho}$ are the estimates of $T_L$, $R_r$, and $\rho = \frac{1}{R_r}$. The unknown parameter $T_L$ appears only in the dynamic equation of $e_o$. But since $e_o$ is known, then $T_L$ can be estimated. The unknown parameters $R_r$ and $\rho = \frac{1}{R_r}$, on the other hand, appear in the flux equations $\phi_q$ and $e_\phi$. Obviously, if $\phi_d$ and $\phi_q$ were known, $R_r$ and $\rho$ also could be estimated. However, since the flux variables are not known, they must first be estimated or calculated, using only the measured variable.
Let us first assume that the rotor flux $\phi_d$ and $\phi_q$ are known. Now consider the adaptive control strategy, given by

$$i_d = m_2 - \frac{1}{l_m} \hat{\rho} L_r \gamma_1 a_1 m_1 e_\omega$$
$$i_q = m_1$$
$$\omega = n_0 + \frac{1}{\phi_q} \left[ \frac{R}{l_m} L_m m_1 - \gamma_1 a_1 \left( m_2 - \frac{1}{l_m} \hat{\rho} L_r \gamma_1 a_1 m_1 e_\omega + c_2 \phi_q \right) \right]$$
$$\hat{T}_L = -\gamma_1 \gamma_1 \hat{a}_1 m_1 e_\omega$$
$$\hat{R}_r = \gamma_2 \frac{1}{l_m} i_q \phi_q$$
$$\hat{\phi}_q = -\gamma_3 \left( \hat{\phi}_R - \gamma_2 \gamma_1 a_1 i_q e_\omega - c_2 e_\phi \right) e_\phi$$

with

$$m_1 = \frac{1}{\phi_q} \left[ \omega_R + a_0 \hat{T}_L - (c_1 + c_{11} e_\omega^2) e_\omega \right]$$
$$m_2 = \frac{1}{l_m} \left[ \phi_R + \hat{\rho} L_r (\hat{\phi}_R - c_2 e_\phi) \right]$$

where $\gamma_1$, $\gamma_2$, $c_1$ and $c_{11}$ are defined as before, and that $c_2$, $\gamma_1$, $\gamma_2$ and $\gamma_3$ are positive constants. Note again that the proposed controller is defined everywhere, provided that $\phi_R > 0$ at all times, which is always possible. With the above adaptive control, the dynamics of the closed-loop error system is written as

$$\dot{e}_\omega = -(c_1 + c_{11} e_\omega^2) e_\omega + a_1 (e_\phi i_q - \phi_q i_d) - a_0 \hat{T}_L$$
$$\dot{e}_\phi = -\frac{R}{l_m} e_\phi - (n_0 - \omega) \phi_q - R_i \hat{\rho} \left( \hat{\phi}_R - \gamma_1 a_1 i_q e_\omega - c_2 e_\phi \right) - \gamma_1 a_1 i_q e_\omega - c_2 e_\phi$$
$$\dot{\phi}_q = -\frac{R}{l_m} \phi_q + (n_0 - \omega) e_\phi + \frac{R}{l_m} L_m i_q + \gamma_2 \gamma_1 a_1 i_d e_\omega - c_2 \phi_q$$

Again using a candidate Lyapunov function, such as $V = \frac{1}{2} \left[ \gamma e_\omega^2 + e_\phi^2 + \phi_q^2 + \frac{1}{l_m} \hat{T}_L^2 + \frac{1}{l_m} \hat{R}_r^2 + \frac{R}{\gamma_3} \hat{\rho}^2 \right]$, it can be shown that, with the control strategy (9), all the closed-loop signals are globally bounded and that the tracking errors converge to zero asymptotically. The parameter errors, on the other hand, will always stay bounded, but they may or may not converge to zero. However, as in any direct adaptive control strategy, the convergence of parameter estimates is not of primary importance. Therefore, the proposed dynamic control (9) can achieve the motor speed servo control objectives.

The above algorithm, however, requires the knowledge of $e_\phi$ and $\phi_q$. In order to find these variables, note from equation (4) that, one may find the dynamic model of the stator flux variables, which do not depend on the unknown rotor resistance [9-11]. Then, using the algebraic relationships between the stator and the rotor flux variables, which also are independent of the rotor resistance, one would be able to calculate the rotor flux variables [11]. The corresponding observer for the rotor flux error variables $e_\phi$ and $\phi_q$ are then given as
\[ \begin{align*}
\dot{\psi}_d &= \omega_e \psi_q + \frac{1}{L_d} (v_d - R_s i_d) \\
\dot{\psi}_q &= -\omega_e \psi_d + \frac{1}{L_q} (v_q - R_s i_q) \\
e_\phi &= \psi_d - L_{\mu} i_d - \phi_R \\
\hat{\phi}_q &= \psi_q - L_{\mu} i_q
\end{align*} \] (11)

where \( \psi_d = \phi_d + L_{\mu} i_d \) and \( \psi_q = \phi_q + L_{\mu} i_q \) are the stator flux variables. As it can be seen, the above equations are independent of the unknown parameters and all the variables in the right side of the equations are measurable. Hence, the above flux observer can be used for the on line calculation of the rotor flux variables. The calculated rotor flux can then be used for the estimation of \( R_r \) and \( \rho \).

However, these equations lack leakage terms and hence initial errors, if they exist, may not die out [11]. One possible remedy is to include, as the correcting term, a bounded function of the tracking errors in the dynamics of the above rotor flux observer, as

\[ \begin{align*}
\dot{\hat{\psi}}_d &= \omega_e \hat{\psi}_q + \frac{1}{L_d} (v_d - R_s i_d) - z_d \\
\dot{\hat{\psi}}_q &= -\omega_e \hat{\psi}_d + \frac{1}{L_q} (v_q - R_s i_q) - z_q \\
\hat{e}_\phi &= \hat{\psi}_d - L_{\mu} i_d - \phi_R \\
\hat{\phi}_q &= \hat{\psi}_q - L_{\mu} i_q
\end{align*} \] (12)

where the correcting terms \( z_d \) and \( z_q \) are bounded functions of the tracking errors \( e_{\omega}, \hat{e}_\phi \) and \( \hat{\phi}_q \). A variety of functions can be selected for the above correcting terms that guarantee the convergence of the observer dynamics. Such choices and their corresponding proof of convergence will be presented in future works.

**Simulation/ Emulation Example**

In this section the effectiveness of the proposed algorithm for speed control of an AC induction motor is verified by computer simulation/ emulation, according to Figure 5.

![Figure 5: Induction Motor Control Set-up](image-url)
The specifications for the AC induction motor are: motor torque=15 KW (rated), load torque=70 Nm (rated), rotor flux linkage=1.3 Wb (rated), angular speed=220 rad/s (rated), \( n=1 \), \( J=0.0586 \) Kgm\(^2\), \( R_s=0.18 \) \( \Omega \), \( R_r=0.15 \) \( \Omega \), \( L_s=0.0699 \) H, \( L_r=0.0699 \) H, \( L_m=0.068 \) H. A DSP application development board, with TI’s TMS320C31 floating-point DSP chip, is considered for implementation. For the simulation, the DSP’s A/D sampling time is chosen as \( T_s=1 \) msec, and the actual load torque is taken to be \( T_L=70 \) Nm. Also, the PWM inverter is assumed to perform in an ideal manner. Moreover, the reference commands for the motor speed and rotor flux magnitude are given by

\[
\omega_R + 2\omega_R = 2\omega_c \\
\dot{\phi}_R + 2\phi_R = 2\phi_c
\]  

(13)

where \( \omega_c \) is a square-wave with amplitude of \( \pm 10 \) and frequency of \( f=0.2 \) Hertz, for \( 0 \leq t \leq 10 \) seconds, while \( \phi_c=1 \) over the same time interval. The corresponding initial conditions are \( \omega_R(0)=0 \) and \( \phi_R(0)=1 \).

The proposed field-oriented observer-based adaptive controller was applied to the computer simulated motor, for the time interval of \( t \in [0,10] \) seconds. Figure 6 shows the desired and the actual values for the speed and the flux magnitude, respectively. Figure 7 shows the control currents, slip frequency, and the applied voltages, respectively. Figure 8 shows the estimates of the load torque, rotor resistance and the inverse of the rotor resistance.

Figure 6: Motor’s speed and flux variables
As it can be seen from the figures, the proposed controller achieves the speed servo control objective quite satisfactorily, without rotor flux measurements and without the knowledge of the load torque and rotor resistance. The speed and flux, as can be seen, are controlled independently and simultaneously, as desired.
Conclusions

In this paper, an adaptive nonlinear control technique for speed servo control of AC induction motors is presented. The proposed controller is designed for the field-oriented (d-q) model of the motor. The d-q model eliminates the stiff nonlinearities (simultaneous slow and fast modes) inherent in the induction motor’s dynamics and, hence, it is more suitable for discretization and digital control implementation. The controller results in the decoupling and I/O linearization of the motor’s dynamics, which allows for better transient response. The control includes a flux estimator, which does not require the knowledge of the rotor resistance. The proposed controller is adaptive in the sense that it includes estimators for the unknown torque load, rotor resistance and the inverse of the rotor resistance (to avoid division by zero). The adaptive property of the controller makes it more practical, since in real applications the load is not exactly known in advance and that the rotor resistance may vary quite a bit, due to heating. In addition, the controller is asymptotically stable and does not have singularities.

References


