REAL-TIME PERIOD ESTIMATION FOR PREDICTIVE MAINTENANCE

D. Darian Muresan

Department of Electrical Engineering
Cornell University
Ithaca, NY 14853

Thomas W. Parks

Department of Electrical Engineering
Cornell University
Ithaca, NY 14853

ABSTRACT

The detection and estimation of machine vibration multi-periodic signals of unknown frequencies in white Gaussian noise is investigated. New estimates for the sub-signals (signals making up the received signal) and their periods are derived using an orthogonal subspace decomposition approach. The introduction of synchronous sampling is shown to dramatically improve the performance of our estimators.

1. INTRODUCTION

The analysis of machine vibrations has proven to be a very valuable application of signal processing. Such study facilitates the judicious use of predictive maintenance to lessen or prevent catastrophic failures of machinery which can lead to expensive down-times. There are a variety of well-known techniques used in this area, [1], and there is also serious inquiry into the development of more powerful methods. One such method involves periodic time-frequency analysis, using distributions designed to analyze the waveform of periodic sub-signals of a machine vibration signal. Implicit in such studies is the requirement that good period and periodic sub-signal estimates can be made.

Our aim is to develop a method for period and periodic sub-signal estimation. Using a vibration signal, recorded from a General Motors gear box (see Fig. 1), we present a method for estimating the periods and the corresponding periodic sub-signals, hidden in the original signal.

In [2] a period and signal estimation scheme, based on a maximum likelihood formulation was presented. The results of [2] were extended [3] to the estimation of two periods and the respective periodic sub-signals, with the restriction that the periods were relatively prime.

Other related work includes the *matrix algebraic separation* (MAS) algorithm, for estimating two periodic sub-signals [4], [5] and [6]. The assumption behind the (MAS) algorithm is that we have exact, prior knowledge of the sub-signal periods. Another approach to estimating two combined periods is made in [7]. The authors suggest a *double difference function* (DDF) algorithm, where the composite signal is fed through two cascaded comb filters of the form $h_i(t) = \delta(t) - \delta(t - \tau_i)$. When the lag parameters, τ_i, correspond to the two periods, the output is everywhere zero.

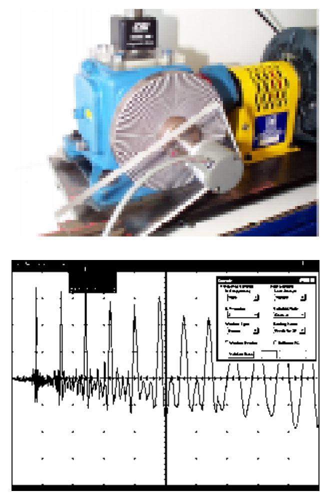

Figure 1: General Motors gear box used for our real time synchronous sampling. Sampling is synchronized with the position of the large wheel. Also shown, is a screen shot of our real time software display.

Using a maximum likelihood approach reformulated as a mean square error approximation we extend the results from [2] and [3] to the case where there are several periodic sub-signals, not necessarily of relatively prime periods, additively combined in the vibration signal. Our method estimates both the periods and the respective periodic sub-signals.

The idea behind our approach is to assume that each periodic sub-signal lives in some unique subspace. When we are assuming only one periodic sub-signal,

Supported by ONR N00014-94-1-0102

this subspace is clearly defined and it's spanned by an orthonormal basis set. The problem becomes that of finding the basis set that minimizes the two norm of the error between the subspace and the received signal.

If we assume two periodic sub-signals, the subspace is spanned by the union of the two basis sets corresponding to each period. The union set does not generally form a linearly independent set, making it harder to find the two norm error between the subspace (spanned by the union set) and the signal.

2. SYNCHRONOUS SAMPLING FOR ESTIMATION OF PERIODIC SIGNALS

In [2], Caprio presented a maximum likelihood pitch estimation method. One difficulty with almost any period estimation method is that the period of the analog signal does not contain an integer number of samples. In other words, the sampled data is never really periodic. Our approach is to force an integer number of samples per period by doing synchronous sampling. Instead of sampling our vibration signal using a fixed frequency clock, our sampling is synchronized with the position of the gears (Fig. 1). Assuming that the periodic sub-signals are generated by different gears in the box, a synchronous sampling will usually provide an integer number of samples per period. One starts with the assumption that all the periodic sub-signals are generated by the rotation of different gears. In our case, all the gears are connected with each other via metal teeth. A valid assumption is therefore that synchronizing with one gear, is enough to provide synchronous sampling for all the other gears. So we have an integer number of samples for each period corresponding to different sub-signals.

Based on the work done in [2], let $S_1, \ldots, S_m$ be periodic repetitions of the length $P_1, \ldots, P_m$ sequences $Q_1, \ldots, Q_m$ respectively (as in Fig. 2). The received signal R, of length K_0 (K_0 is a multiple of the least common multiplier of $P_1, \ldots, P_m$)[1], then consists of $S_1, \ldots, S_m$ plus noise N

$$R = S_1 + \ldots + S_m + N$$

If the signal has zero mean, the log ML estimator requires minimization of

$$\hat{\sigma}^2 = \frac{1}{K_0} \|R - \hat{S}_1 - \ldots - \hat{S}_m\|^2 \qquad (1)$$

where $\hat{S}_i$ are the estimated periodic sub-signals.

[1] in [2], Caprio explains that if K_0 is sufficiently large, we can relax the restriction that it be a multiple of the least common multiplier

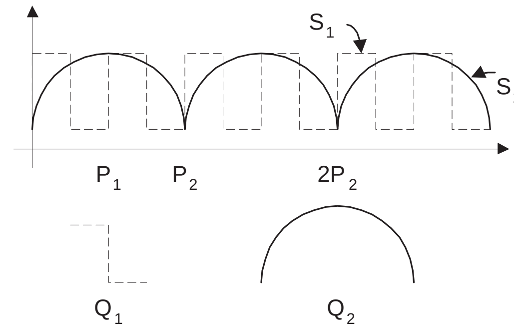

Figure 2: Sub-signals S_1 and S_2 of respective periods P_1 and P_2

We are seeking to minimize (1) by the choice of periodic sub-signals and their periods. In other words we are seeking to minimize the distance between R and $\left(\hat{S}_1 + \ldots + \hat{S}_m\right)$. But, $\hat{S}_i$ can be any periodic signal with period P_i. If we can find a basis set spanning a subspace in which periodic signals of period P_i live (call this basis set Ψ_i), then minimizing (1) is the same as minimizing the distance between R and the subspace spanned by the union of all the basis sets Ψ_i. This is the MSE period estimation approach.

For the one period estimation case, we have to minimize the distance between R and the subspace spanned by the orthonormal basis set:

$$\begin{aligned}
\psi_0 &= \sqrt{\tfrac{1}{M}}[100\ldots0100\ldots0100\ldots]^T \\
\psi_1 &= \sqrt{\tfrac{1}{M}}[010\ldots0010\ldots0010\ldots]^T \\
&\vdots \\
\psi_{P-1} &= \sqrt{\tfrac{1}{M}}[000\ldots1000\ldots1000\ldots]^T
\end{aligned}$$

where $K_0 = M \times P$. Let Ψ be the orthonormal matrix having $\psi_0 \ldots \psi_{P-1}$ as column vectors.

With $\phi_R(k)$, the autocorrelation function of R, defined as

$$\phi_R(k) = \sum_{j=0}^{K_0-1-k} r_j r_{j+k}$$

minimizing the distance between R and the column space of (Ψ) is equivalent to maximizing $\|\hat{S}\|^2$ ([2] and [3])

$$\|\hat{S}\|^2 = \frac{P}{K_0}\left[\phi_R(0) + 2\sum_{l=1}^{N-1} \phi_R(lP)\right] \qquad (2)$$

From (1) we have that

$$\hat{\sigma}^2 = \frac{1}{K_0}\sum (r_i - \hat{s}_i)^2 = \frac{1}{K_0}(\|R\|^2 - \|\hat{S}\|^2)$$

and

$$E(\hat{\sigma}^2) = \frac{M-1}{M}\sigma^2 \tag{3}$$

In other words, our variance estimator is biased (although, it is asymptotically unbiased). To get an unbiased estimator, let

$$\hat{\sigma}^2_{UB} = \frac{M}{M-1}\hat{\sigma}^2$$

This implies that instead of maximizing (2) we now maximize

$$\frac{2P}{K_0 - P}\sum_{l=1}^{M-1}\phi_R(lP) \tag{4}$$

To further enhance peaks at smaller periods, we eliminate P from the denominator, to obtain

$$g_{(P,R)} = \frac{2P}{K_0}\sum_{l=1}^{M-1}\phi_R(lP) \tag{5}$$

Maximizing the function $g_{(P,R)}$ will give the best signal period estimation when we are assuming the signal contains one periodic sub-signal.

In figures 4 and 5, we have plotted the function $g_{(P,R)}$ calculated from vibration data obtained with synchronous sampling and from vibration data obtained with a fixed 25KHz sampling clock. We can clearly detect the difference in the performance of the two approaches. Synchronous sampling gives almost ideal results on real vibration data, while using conventional sampling is clearly less satisfactory. The peaks, corresponding to $g(P,R)$ are not as well defined and clean as in the synchronous case.

3. SOLVING THE TWO PERIOD ESTIMATION PROBLEM USING MSE

We first solve the problem assuming that the signal consists of only two periods and then extend our result to multiple periods. When we have two periodic subsignals hidden in our received signal R, we have to maximize the projection of R onto the subspace spanned by the union of the two basis sets: Ψ and Ω (where the two basis sets correspond to periods P_1 and P_2 respectively). We also require that the signal length K_0 be a multiple of the least common multiplier of P_1 and P_2; $K_0 = P_1 \times M_1 = P_2 \times M_2$). For the sake of clarity let $P_1 = 4$ and $P_2 = 6$. By definition Ψ is the collection of

$$\begin{aligned}
\psi_0 &= \sqrt{\tfrac{1}{M_1}}[100010001000...]\\
\psi_1 &= \sqrt{\tfrac{1}{M_1}}[010001000100...]\\
\psi_2 &= \sqrt{\tfrac{1}{M_1}}[001000100010...]\\
\psi_3 &= \sqrt{\tfrac{1}{M_1}}[000100010001...]
\end{aligned}$$

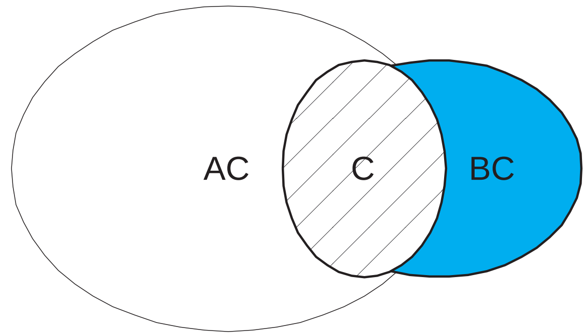

Figure 3: Subspace spanned by the columns of $[\Psi\Omega]$. Subspaces AC, C and BC are mutually orthogonal. The column space of $\Psi = AC + C$ and the column space off $\Omega = BC + C$.

and Ω is the collection of

$$\begin{aligned}
\omega_0 &= \sqrt{\tfrac{1}{M_2}}[100000100000...]\\
\omega_1 &= \sqrt{\tfrac{1}{M_2}}[010000010000...]\\
\omega_2 &= \sqrt{\tfrac{1}{M_2}}[001000001000...]\\
\omega_3 &= \sqrt{\tfrac{1}{M_2}}[000100000100...]\\
\omega_4 &= \sqrt{\tfrac{1}{M_2}}[000010000010...]\\
\omega_5 &= \sqrt{\tfrac{1}{M_2}}[000001000001...]
\end{aligned}$$

The space spanned by Ψ and Ω is the column space of the matrix $[\Psi\ \Omega]$.

Since $[\Psi\ \Omega]$ is not an orthonormal matrix, the maximum projection of R on the space spanned by Ψ and Ω is not equal to the projection of R onto Ψ plus the projection of R onto Ω. In fact, although Ψ and Ω are each orthonormal, $[\Psi\ \Omega]$ is not even full rank. In other words, there is a common subspace which is the intersection of the range of Ψ and the range of Ω, as shown in Fig. 3.

Although the matrix $[\Psi\ \Omega]$ is not full rank, it's range can be decomposed into three mutually orthogonal subspaces [6] (Fig. 3):

$$\begin{aligned}
AC &= \text{span of } (\Psi - \text{Projection of } \Psi \text{ onto } \Omega)\\
C &= \text{span of } (\text{Projection of } \Omega \text{ onto } \Psi)\\
&= \text{span of } (\text{Projection of } \Psi \text{ onto } \Omega)\\
BC &= \text{span of } (\Omega - \text{Projection of } \Omega \text{ onto } \Psi)
\end{aligned}$$

which gives

$$\begin{aligned}
AC &= \text{span of } (\Psi - \Omega\Omega^T\Psi)\\
C &= \text{span of } (\Psi\Psi^T\Omega)\\
&= \text{span of } (\Omega\Omega^T\Psi)\\
BC &= \text{span of } (\Omega - \Psi\Psi^T\Omega)
\end{aligned}$$

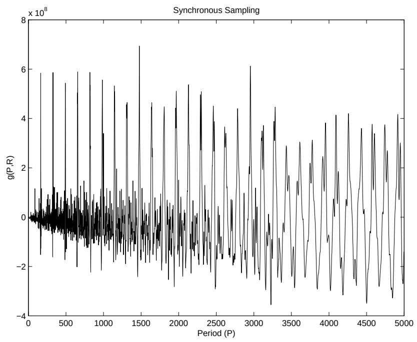

Figure 4: Plot of $g(P,R)$ for synchronous sampling

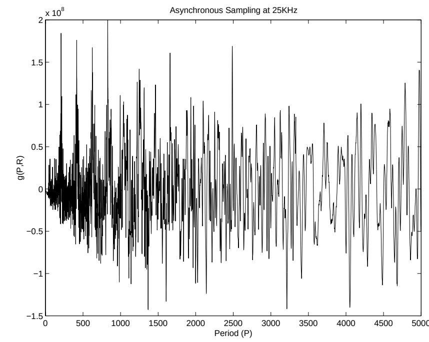

Figure 5: Plot of $g(P,R)$ for 25KHz, asynchronous sampling

From here we are now ready to give the theorem for finding the best period estimate when our received signal contains two periods: P_1 and P_2.

Theorem 1 *The projection of the received signal R onto the subspace spanned by Ψ and Ω is equivalent to adding the projections of R onto Ψ and the projection of R onto Ω and then subtracting the projection of R onto C.*

$$\hat{S}_1 + \hat{S}_2 = P_{[\Psi\Omega]}R = P_{[\Psi]}R + P_{[\Omega]} - P_{[C]}R \qquad (6)$$

From theorem 1, if the signal projected onto $[\Psi\Omega]$ is $\hat{R} = R - P_{[C]}R$ then we obtain

$$\begin{aligned} P_{[\Psi\Omega]}\hat{R} &= P_{[\Psi]}\hat{R} + P_{[\Omega]}\hat{R} \\ &= g_{(P_1,\hat{R})} + g_{(P_2,\hat{R})} \end{aligned}$$

In other words, theorem 1 says that to find P_1 and P_2 we can:

1. Maximize $G_{(P_1,P_2,R)} = g_{(P_1,R)} + g_{(P_2,R)} - g_{(gcd(P_1,P_2),R)}$ or
2. Subtract from R the projection of R onto the subspace common to $Range(\Psi)$ and $Range(\Omega)$, and then maximize $g_{(P_1,\hat{R})} + g_{(P_2,\hat{R})}$.

These results can easily be extended to multiple periodic sub-signals [3].

4. CONCLUSION

We have discussed the application of period and periodic sub-signal estimation as it pertains to machine vibration signals. Using a mean square error approach, we have shown that the performance of our estimators can be significantly improved through the use of synchronous sampling.

5. REFERENCES

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