Spectral Optimization for Symmetric Bit-rate Communication in the Presence of Crosstalk

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Abstract

We present a framework for maximizing capacity (or bit rates) of symmetric bit-rate communication services dominated by crosstalk, in particular *Digital Subscriber Line* (DSL) services. We assume the DSL channel is a Gaussian channel, so transmit power spectral density (PSD) design is sufficient to maximize bit rates. Using the channel, noise, and crosstalk transfer functions, we set up an optimization problem and solve for transmit PSDs to maximize the joint capacity of same-service users. Joint signaling techniques and optimal power distribution yield significant gains in bit rates (or performance margins) over current schemes. Furthermore, by design, the transmit PSDs are spectrally compatible with existing services on neighboring lines. The framework is quite general; it does not depend on the exact choice of modulation scheme, for example. It is also extremely simple and of low computational complexity. The framework can also apply to other channels besides DSLs, for example, wireless channels, coaxial cables, power lines, and telemetry cables used in geophysical well-logging tools.

Keywords

Digital Subscriber Line (DSL) systems, crosstalk, information rates, multiuser communication, spectral compatibility.

I. Introduction

Many communication systems rely on significant resource-sharing among multiple users, in particular transmission bandwidth. Proximity of different paths or channels between users can lead to *multiuser interference* or *crosstalk*, for example, crosstalk between digital subscriber lines (DSLs) in a telephone cable or users in a wireless channel. Crosstalk lowers channel capacities and can be a limiting factor for achievable bit rates. We wish to optimally allocate bandwidth among users in the presence of crosstalk to maximize bit rates.

For concreteness we frame our discussion in terms of the DSL technology, though our results are more general as we indicate later. We define a *service* as a transmission protocol

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(based on bit-rate and transmit power characteristics) over a standard telephone line. A service can be a DSL service, (for example, ADSL, VDSL, or HDSL) [6] or non-DSL service, (for example, T1, ISDN, or POTS) [6]. A telephone cable contains several lines, typically 50 in USA, carrying different services (DSL and non-DSL) packed closely together. Every transmitter intends to communicate with the corresponding receiver at the other end. However, transmitted energy from a given line leaks (via electro-magnetic coupling) into other receivers of neighboring lines as a result of proximity of lines and absence of proper shielding. This leads to significant crosstalk between lines in a telephone cable.

We wish to introduce a new DSL service (for example HDSL2) over some lines of the telephone cable. A new-service line faces crosstalk from existing services (different-service interference) like ADSL, HDSL, T1, and ISDN as well as crosstalk from similar new-service lines (self-interference). We seek to jointly maximize the bit rates of all lines carrying this new service in the presence of different-service interference and self-interference. We assume that the DSL channel is a Gaussian channel. Hence, second order statistics are sufficient to characterize the channel [1] and this leads to the design of optimal transmit *power spectral densities* (PSDs), also referred to as *transmit spectra*. Further, when we optimize the bit rate of any new service we need to make sure that the new service does not significantly inject crosstalk power into the existing DSL services. In other words, we need to check if the service being optimized is *spectrally compatible* with existing services. A *frequency plan* defines the set of rules that limit the transmit spectra of each service to ensure spectral compatibility.

Consider a telephone cable with I new-service lines with index $i \in \{1, \ldots, I\}$, and denote the direction of transmission with index $o \in \{u, d\}$, with $u =$ upstream (to the central office (a building that connects telephone subscribers and houses network equipment like switches)) and $d =$ downstream (from the central office) [1]. Let $\overline{o}$ be the complement direction of o: $\overline{u} = d$, $\overline{d} = u$. We denote the transmit and received PSDs on line i and direction o as $S_i^o(f)$ and $Z_i^o(f)$ respectively. Similarly, $N_i^o(f)$ denotes the total additive Gaussian noise (AGN) PSD (this consists of channel noise plus different-service interference [2]). Let $H_C(f)$ denote the channel transfer function (we assume that all the I channels are the same). In the absence of self-interference we can write the received PSD as $Z_i^o(f) = H_C(f) S_i^o(f) + N_i^o(f)$. However, self-interference is significant and for each receiver on line i power couples from neighboring transmitters in both directions (upstream and downstream). $H_X(f)$ and $H_F(f)$ denote the self *far-end crosstalk* (self-FEXT) and self *near-end crosstalk* (self-NEXT) transfer functions respectively from channel i to channel j $(i \neq j)$ (see Figure 1). (We assume that all the self-NEXT and self-FEXT transfer functions are the same between any two channels i and j.)

We can characterize such a communication system as a multiple-input-multiple-output (MIMO) system and express the upstream received PSDs as

$$\overbrace{\begin{bmatrix} Z_1^u \\ Z_2^u \\ \vdots \\ Z_I^u \end{bmatrix}}^{\underline{Z}^u(f)} = \overbrace{\begin{bmatrix} H_C & H_F & \cdots & H_F \\ H_F & H_C & \cdots & H_F \\ \vdots & \vdots & \ddots & \vdots \\ H_F & H_F & \cdots & H_C \end{bmatrix}}^{\mathbf{U}(f)} \overbrace{\begin{bmatrix} S_1^u \\ S_2^u \\ \vdots \\ S_I^u \end{bmatrix}}^{\underline{S}^u(f)} + \overbrace{\begin{bmatrix} 0 & H_X & \cdots & H_X \\ H_X & 0 & \cdots & H_X \\ \vdots & \vdots & \ddots & \vdots \\ H_X & H_X & \cdots & 0 \end{bmatrix}}^{\mathbf{V}(f)} \overbrace{\begin{bmatrix} S_1^d \\ S_2^d \\ \vdots \\ S_I^d \end{bmatrix}}^{\underline{S}^d(f)} + \overbrace{\begin{bmatrix} N_1^u \\ N_2^u \\ \vdots \\ N_I^u \end{bmatrix}}^{\underline{N}^u(f)} \tag{1}$$

Similarly, we can write an expression for the downstream received PSDs as

$$\underline{Z}^d(f) \;=\; \mathbf{U}(f)\underline{S}^d(f) + \mathbf{V}(f)\underline{S}^d(f) + \underline{N}^d(f). \tag{2}$$

Equations (1) and (2) hold for all frequencies. Note that all the vector and matrix elements are a function of frequency.

We seek to maximize capacity of each individual new-service line i by choosing $S_i^u(f)$ and $S_i^d(f)$. Simultaneously we need to ensure that the new-service transmit spectra are spectrally compatible with existing services.

Complete cancellation or suppression of crosstalk is not always possible, since it is difficult and/or expensive [3]. Rather, we employ *crosstalk avoidance* techniques to design optimal transmit spectra. We avoid crosstalk (self-interference only, since we have no control over transmit spectra of existing services) by separating $S_i^u(f)$ and $S_i^d(f)$ in appropriate frequency regions. A similar problem was first solved in [4] for symmetric-bit-rate services facing self-NEXT and white AGN. Here, we solve the problem for symmetric-bit-rate [5] services in the presence of self-NEXT, self-FEXT, AGN, and interference from other services. Given the channel, noise, and crosstalk characteristics we solve an optimization problem to maximize the joint channel capacity of the new-service neighboring lines under an average power constraint. However, the resulting spectra from [4] and our solution are *discrete* (noncontiguous allocation of power over frequency). We present algorithms for optimal grouping of frequency bins to obtain contiguous transmit spectra (see Figure 7). Contiguous spectra are desirable to implement various broadband modulation schemes, for example QAM or CAP [6]. We also discuss a general algorithm to design optimal spectra over non-monotonic channels as in the case of bridged taps. Finally, we present extensions of our optimization ideas in the case of an additional peak frequency-domain power constraint [7]. Power distribution might be constrained in frequency by standardized fixed PSDs, or due to spectral compatibility considerations.

Our results indicate that optimal transmit spectra vary significantly with neighboring interference. They yield large performance margin gains up to 16 dB over current fixed-spectra schemes (see table I). Further, they are inherently spectrally compatible with existing services. Optimal spectra can be implemented using any modulation scheme and can be easily adapted to varying noise and interference environments. Finally, near-optimal spectra can be trivially computed.

Section II outlines our definitions, notation, and assumptions. Details on obtaining optimal transmit spectra are presented in Section III. Significant extensions to optimal transmit spectra ideas are presented in Section IV. We discuss simulation results in Section V and present conclusions in Section VI.

II. Definitions, Notation, and Assumptions

We use the standard DSL environment setup [8].

A. Channel and crosstalk

There are two types of crosstalk in a telephone cable (see Figure 1):

NEXT: Interference between neighboring lines that arises when signals are transmitted in opposite directions. If the neighboring lines carry the same type of service, then the interference is called self-NEXT; otherwise, it is called different-service (DS) NEXT.

FEXT: Interference between neighboring lines that arises when signals are transmitted in the same direction. If the neighboring lines carry the same type of service, then the interference is called self-FEXT; otherwise, it is called different-service (DS) FEXT.

The term *self-interference* refers to the total combined self-NEXT and self-FEXT.

We assume that the channel can be characterized as a linear time-invariant system. We divide the *transmission bandwidth* B of the channel into K narrow frequency bins; each of width W Hz and center frequencies f_k with index $k \in \{1, \ldots, K\}$ (see Figure 2). We assume that the channel, noise, and the crosstalk characteristics vary slowly enough with frequency that they can be approximated as constant over each bin [8]. Figure 2 illustrates the channel, self-NEXT, and self-FEXT transfer functions. We use the following notation for the magnitude squared channel transfer function [9],

$$H_k = |H_C(f_k)|^2, \quad \text{for } |f - f_k| \leq \frac{W}{2}, \quad 0 \leq f \leq B, \tag{3}$$

self-NEXT transfer function [10],

$$X_k = |H_N(f_k)|^2, \quad \text{for } |f - f_k| \leq \frac{W}{2}, \quad 0 \leq f \leq B, \tag{4}$$

and self-FEXT transfer function [10]

$$F_k = |H_F(f_k)|^2, \quad \text{for } |f - f_k| \leq \frac{W}{2}, \quad 0 \leq f \leq B. \tag{5}$$

We consider real signals with symmetric frequency responses. Thus, we deal only with quantities over the non-negative frequency region.

In addition we assume that:

1. The channel transfer function is monotone decreasing. In case the channel transfer function is non-monotonic (e.g., in the case of bridged taps on the line [6]), the techniques that follow can be applied in each individual bin independently. This scenario makes the power distribution problem more difficult, however (see Section IV-C).
2. The level of self-FEXT is low enough in all the bins that it is not necessary to separate in frequency same-direction transmit spectra $S_i^o(f)$ and $S_j^o(f)$ of new-service lines.
3. Channel noise ($N_o(f)$) can be modeled as AGN. We assume that we have no control over the transmit spectra of existing services and treat this different-service interference as AGN [2] for capacity purposes. We combine the DS-NEXT ($DS_N(f)$), DS-FEXT ($DS_N(f)$), and channel noise to obtain the total Gaussian noise [2] as

$$N(f) := N_o(f) + DS_N(f) + DS_F(f). \tag{6}$$

4. We assume symmetric bit-rates in both directions of transmission for all the new-service lines. Further, we assume that the channel, self-NEXT, self-FEXT, different-service interference, and channel noise are the same for all new-service lines as in (1). Symmetric bit-rates and (1) implies that all the upstream PSDs are the same ($S^u(f)$) and all the downstream PSDs ($S^d(f)$) are the same of the new-service lines.
Finally, let $s^u(f)$ and $s^d(f)$ denote the PSDs in a single frequency bin k.

B. Signaling schemes

DSL modems transmit in two directions on the same line via a 4–2 line hybrid circuit. A *No Division Signaling* (NDS)[1] scheme in frequency bin k is one for which $s^u(f) = s^d(f) \neq 0$ for all f in the bin. (that is, both upstream and downstream transmissions occupy the band $|f - f_k| \leq \frac{W}{2}$ in the same way). A *Frequency Division Signaling* (FDS) scheme in frequency bin k is one for which $s^u(f) = 0$ when $s^d(f) \neq 0$ for all f in the bin and vice versa (that is, both transmissions occupy orthogonal frequency bands within $|f - f_k| \leq \frac{W}{2}$). FDS is an example of the general concept of orthogonal signaling that includes other schemes like time-division signaling (TDS) and code-division signaling (CDS). Figure 3 illustrates NDS and FDS schemes in a single frequency bin k. FDS eliminates self-NEXT and therefore increases capacity; however, FDS also cuts the transmission bandwidth into half the total bandwidth, thus reducing capacity.

III. Optimized Signaling Techniques

We focus on jointly maximizing bit rates of a new DSL service by designing optimal transmit spectra. The joint maximization allows us a control only of the self-interference but not of the different-service interference from existing services. Hence, we look at two possible scenarios: optimal signaling in the absence and presence of self-interference.

A. Absence of self-interference

In the absence of self-NEXT and self-FEXT, the interference combination consists exclusively of different service interferers and AGN. This total interference can be lumped together into one AGN source as in (6) [2]. Under the Gaussian channel assumption the

[1]NDS was referred to as EQPSD signaling in [4] and our standards contributions [5], [7], and [11].

optimal power distribution in each direction of transmission is obtained by the classical water-filling solution [12].

B. Presence of self-interference

When present, self-NEXT and self-FEXT severely limit the achievable bit rates in symmetric-bit-rate DSL services. In this paper (and for long lines with small bandwidths in general) self-NEXT can be assumed to dominate self-FEXT and self-FEXT can be assumed small (see Figure 2). This is the case of interest for high-bit-rate DSL2 (HDSL2) [10]. However, self-FEXT still factors into our design in a significant way. This is a new, non-trivial extension of the work of [4].

Self-FEXT is significant in the case of small number of short lines with large bandwidths. To eliminate self-FEXT using orthogonal signaling, we would force each transmission $S_i^o(f)$ to be orthogonal to all other transmissions S_j^o, $j \neq i$ (known as multi-line FDS) [13].

Problem statement: We wish to maximize the capacity of a new DSL service in the presence of channel noise, interference (DS-NEXT and DS-FEXT) from other services, and self-NEXT and self-FEXT under two constraints: (1) the average DSL input power in each direction of transmission be limited to $P_{\max}$, and (2) equal capacity in both directions (upstream and downstream) of transmission.

B.1 Joint signaling scheme

The level of self-NEXT varies with frequency (recall Figure 2). In high self-NEXT regions FDS might be useful to reject the self-NEXT. However, in low self-NEXT regions, the loss of transmission bandwidth of FDS could outweigh any capacity gain due to self-NEXT rejection. Therefore, we would like our signaling scheme to be general enough to encompass both FDS, NDS, and the array of choices in between. Our approach is related to that of [4].

Consider the case of two neighboring lines carrying the same new service. Line 1 upstream capacity is C^u and line 2 downstream capacity is C^d. Under the Gaussian channel assumption, we can write these capacities in bits per second (bps) as

$$C^u = \sup_{S^u(f), S^d(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 S^u(f)}{N(f) + |H_N(f)|^2 S^d(f) + |H_F(f)|^2 S^u(f)} \right] df \qquad (7)$$

and

$$C^d = \sup_{S^u(f),S^d(f)} \int_0^{\infty} \log_2 \left[1 + \frac{|H_C(f)|^2 S^d(f)}{N(f) + |H_N(f)|^2 S^u(f) + |H_F(f)|^2 S^d(f)} \right] df. \tag{8}$$

The supremum is taken over all possible $S^u(f)$ and $S^d(f)$ satisfying

$$S^u(f) \geq 0, \quad S^d(f) \geq 0 \quad \forall f, \tag{9}$$

and the average power constraints for the two directions

$$2\int_0^{\infty} S^u(f)df \leq P_{\max}, \quad 2\int_0^{\infty} S^d(f)df \leq P_{\max}. \tag{10}$$

We can solve for the capacities C^u and C^d using classical water–filling [12] if we impose the restriction of NDS, that is $S^u(f) = S^d(f)$ $\forall f$. However, this gives low capacities. Therefore, we employ FDS ($S^u(f)$ orthogonal to $S^d(f)$) in spectral regions where self-NEXT is large enough to limit our capacity and NDS in the remaining spectrum. This gives much improved performance.

B.2 Optimal spectrum: One frequency bin

We continue our analysis on a single frequency bin k assuming the frequency bin responses (3)–(5). For ease of notation, in this section set

$$H := H_k, \quad X := X_k, \quad F := F_k \quad \text{in (3)–(5)}. \tag{11}$$

and let $N := N(f_k)$ in (6) denote the total noise PSD in bin k. Let $s^u(f)$ denote the PSD in bin k of line 1 upstream direction and $s^d(f)$ denote the PSD in bin k of line 2 downstream direction. For simplicity purposes we will consider the bin k demodulated to baseband, $f \in [0, W]$. Denote the corresponding capacities of bin k by c^u and c^d.

We divide bin k in *half* and set

$$s^u(f) = \begin{cases} \alpha\frac{2P_m}{W} & \text{if } 0 \leq f \leq \frac{W}{2}, \\ (1-\alpha)\frac{2P_m}{W} & \text{if } \frac{W}{2} < f \leq W, \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

$$s^d(f) = \begin{cases} (1-\alpha)\frac{2P_m}{W} & \text{if } 0 \leq f \leq \frac{W}{2}, \\ \alpha\frac{2P_m}{W} & \text{if } \frac{W}{2} < f \leq W, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Here P_m is the average power over the bandwidth W in bin k and $0.5 \leq \alpha \leq 1$. The factor α controls the power distribution in the bin. When $\alpha = 0.5$, $s^u(f) = s^d(f)\ \forall f \in [0, W]$ (NDS); when $\alpha = 1$, $s^u(f)$ and $s^d(f)$ are disjoint (FDS). These two extreme transmit spectra along with other possible spectra (for different values of α) are illustrated in Figure 4. The PSDs $s^u(f)$ and $s^d(f)$ are "symmetrical" or power complementary to each other.

<u>Claim:</u> The optimal signaling strategy uses only FDS or NDS in each frequency bin.

<u>Proof:</u> Define the achievable rate as

$$R_A(s^u(f), s^d(f)) \quad = \quad \int_0^W \log_2 \left[1 + \frac{s^u(f)H}{N + s^d(f)X + s^u(f)F}\right] df. \tag{14}$$

Then, we have

$$c^u = \max_{0.5 \leq \alpha \leq 1} R_A(s^u(f), s^d(f)) \quad \text{and} \quad c^d = \max_{0.5 \leq \alpha \leq 1} R_A(s^d(f), s^u(f)). \tag{15}$$

The power complementarity of $s^u(f)$ and $s^d(f)$ ensures that the upstream and downstream capacities are equal ($c^u = c^d$). Hence, we will only consider the c^u expression. Further, we will use the shorthand R_A for $R_A(s^u(f), s^d(f))$ in the remainder of this section.

Substituting the PSDs (12) and (13) into (14) and using (15), we obtain

$$\begin{aligned} c^u = \frac{W}{2} \max_{0.5 \leq \alpha \leq 1} \Bigg\{ & \log_2 \left[1 + \frac{\frac{\alpha 2 P_m H}{W}}{N + \frac{(1-\alpha)2P_m X}{W} + \frac{\alpha 2 P_m F}{W}}\right] \\ & + \log_2 \left[1 + \frac{\frac{(1-\alpha)2P_m H}{W}}{N + \frac{\alpha 2 P_m X}{W} + \frac{(1-\alpha)2P_m F}{W}}\right] \Bigg\}. \end{aligned} \tag{16}$$

Let $G = \frac{2P_m}{WN}$ denote the signal to noise ratio (SNR). Then, we can rewrite (16) as

$$\begin{aligned} c^u = \max_{0.5 \leq \alpha \leq 1} \frac{W}{2} \Bigg\{ & \log_2 \left[1 + \frac{\alpha G H}{1 + (1-\alpha)GX + \alpha G F}\right] \\ & + \log_2 \left[1 + \frac{(1-\alpha)GH}{1 + \alpha G X + (1-\alpha)GF}\right] \Bigg\}. \end{aligned} \tag{17}$$

Note from (15) and (17) that the expression after the max in (17) is the achievable rate R_A. Differentiating the R_A expression in (17) with respect to α yields

$$\frac{\partial R_A}{\partial \alpha} \quad = \quad \frac{W}{2 \ln 2} \Bigg\{ \Bigg[\frac{1 + (1-\alpha)GX + \alpha G F}{1 + (1-\alpha)GX + \alpha G F + \alpha G H}$$

$$\times \frac{GH(1+(1-\alpha)GX+\alpha GF)-\alpha GH(-GX+GF)}{(1+(1-\alpha)GX+\alpha GF)^2}\Bigg]$$

$$+\Bigg[\frac{1+\alpha GX+(1-\alpha)GF}{1+\alpha GX+(1-\alpha)GF+(1-\alpha)GH}$$

$$\times \frac{-GH(1+\alpha GX+(1-\alpha)GF)-(1-\alpha)GH(GX-GF)}{(1+\alpha GX+(1-\alpha)GF)^2}\Bigg]\Bigg\}$$

$$= G(2\alpha-1)\left[2(X-F)+G(X^2-F^2)-H(1+GF)\right]L, \quad (18)$$

with constant $L > 0\ \forall \alpha \in (0,1]$.

Setting $\frac{\partial R_A}{\partial \alpha} = 0$ it can be shown [14] that we obtain the single stationary point $\alpha = 0.5$. The achievable rate R_A is monotonic in the interval $\alpha \in (0.5, 1]$. If the value $\alpha = 0.5$ corresponds to a maximum, then it is optimal to perform NDS in this bin. If the value $\alpha = 0.5$ corresponds to a minimum, then the maximum is achieved by the value $\alpha = 1$, meaning it is optimal to perform FDS signaling in this bin. *No other values of α are an optimal option.* □

We can write test conditions to determine the signaling nature (FDS or NDS) in a given bin by solving (18):

$$\text{if } X^2 - F^2 - HF < 0, \text{ then}$$

$$G = \frac{2P_m}{NW} \underset{\text{FDS}}{\overset{\text{NDS}}{\gtrless}} \frac{H-2(X-F)}{X^2-F^2-HF}, \quad (19)$$

$$\text{else if } X^2 - F^2 - HF > 0, \text{ then}$$

$$G = \frac{2P_m}{NW} \underset{\text{FDS}}{\overset{\text{NDS}}{\lessgtr}} \frac{H-2(X-F)}{X^2-F^2-HF}. \quad (20)$$

Thus, we can write the upstream capacity c^u in a frequency bin k as

$$c^u = \begin{cases} W \log_2\left[1+\frac{P_m H}{NW+P_m(X+F)}\right], & \text{if } \alpha = 0.5, \\ \frac{W}{2}\log_2\left[1+\frac{P_m H}{N\frac{W}{2}+P_m F}\right], & \text{if } \alpha = 1. \end{cases} \quad (21)$$

Note: FDS is a special case of the more general orthogonal signaling concept, for example, TDS and CDS. However, of all orthogonal signaling schemes, FDS signaling gives the best results in terms of spectral compatibility under an average power constraint and hence is used here (see proof in [14]).

B.3 Optimal spectra: All frequency bins

The above analysis dealt with only a single frequency bin centered around frequency f_k (see Figure 2). To obtain the complete optimal spectra, we apply the test conditions (19) and (20) to each frequency bin in $[0, B]$. In the low self-FEXT case we obtain an NDS frequency region to the left of a switch-over bin M_{N2F} and FDS frequency region to the right of it [14] (see Figure 5). M_N is the largest bin for which the right side of (19) < 0, i.e., the bins $[1, M_N]$ employ NDS (since power, and hence SNR in any bin is nonnegative). Similarly, M_F is the smallest bin for which the right side of (20) < 0, i.e., the bins $[M_F, K]$ employ FDS signaling. Note that the NDS to FDS switch-over bin M_{N2F} lies between M_N and M_F. We use a variation of water-filling for the power distribution. A simple iterative algorithm yields the complete optimal transmit spectra [14].

Algorithm 1: Discrete optimal transmit spectra

1. Initialize $M_{\text{N2F}} = M_N$.
2. Choose some power P_1 over the NDS region and the remaining power $P_{\max} - P_1$ over the FDS region. Distribute these powers within the NDS and FDS bins using the optimization in the presence of self-interference technique [8]. Compute channel capacity. Optimize over different choices of power P_1 to yield the maximum channel capacity.
3. Update $M_{\text{N2F}} = M_{\text{N2F}} + 1$.
4. Loop between 2 and 3 until we find the optimal switch-over bin M_{N2F} that maximizes the channel capacity.

The search algorithm is guaranteed to converge since we assumed that the channel and self-interference transfer functions are monotonic (this ensures that there is a single NDS to FDS switch-over bin). The algorithm can be computationally expensive since we simultaneously optimize over both the switch-over bin and the power distribution over all bins. We can use a simple fast algorithm like the Golden Section Search [15] to find the optimal switch-over bin M_{N2F} in step 3 and thus speed up the convergence of the algorithm. □

IV. Extensions

The optimization techniques developed in Section III yield discrete transmit spectra with a NDS region to the left of a switch-over bin M_{N2F} and a FDS region to the right.

These techniques can be easily customized. We present a few significant extensions including contiguous transmit spectra, optimal spectra for non-monotonic channels, and optimization under a frequency-domain power constraint.

A. Near-optimal spectra: All frequency bins

The iterative algorithm for optimal power distribution discussed in Section III-B.3 can be computationally expensive. Fortunately, there exists a fast near-optimal solution with greatly reduced computational complexity. Set the switch-over bin $M_{\mathrm{N2F}} = M_N$ (since, in several different simulations we found that the bin M_{N2F} is closer to M_N than M_F). This approximation reduces the algorithm of Section III-B.3 to a single, computationally simple step of power distribution.

B. Optimal grouping of bins

FDS signaling divides each bin into even width halves — one half is used by upstream transmission and the other by the downstream. The resulting spectra are discrete and do not have contiguous power allocation over frequency (see Figure 5). Large number of transitions in transmit spectra requires design of many sharp cut-off filters, for example, using discrete multi-tone modulation (DMT). This is an undesirable effect. Contiguous spectra yield reduced number of transitions and are hence desirable for implementing broadband modulation schemes such as carrier-less amplitude phase modulation (CAP) or quadrature amplitude modulation (QAM). In this section, we present optimal ways of grouping bins to yield contiguous spectra.

Only the bins employing FDS are grouped together and the leftmost bins employing NDS are retained as they are. The upstream transmit spectrum is completely contiguous while the downstream spectrum is contiguous except for one gap as shown in Figure 7.

We present a way of grouping bins that achieve *equal performance margins* and *equal upstream and downstream average powers*. This is not the only way that the bins can be grouped. The bins can be grouped in a variety of different ways giving rise to many different equally optimal transmit spectra. Particular modulation schemes and spectral compatibility with neighboring services may influence the way bins are grouped.

We denote the spectral region employing FDS signaling by E_{FDS} and the spectral region employing NDS by E_{NDS}.

Algorithm 2: Contiguous optimal transmit spectra

1. Solve for the optimal transmit spectrum $S^u(f)$ according to the algorithms in Sections III-B.3 or IV-A. This gives a discrete transmit spectrum $S^u(f)$.
2. Obtain $S^d(f)$ from $S^u(f)$ by symmetry, i.e., $S^d(f) = S^u(f) \quad \forall f$ in E_{NDS} and $S^d(f)$ orthogonal to $S^u(f) \quad \forall f$ in E_{FDS}. Merge $S^d(f)$ and $S^u(f)$ to form $S(f)$ as

$$\begin{aligned} S(f) &= S^u(f) = S^d(f) \quad \forall f \in E_{\text{NDS}}, \\ S(f) &= S^u(f) + S^d(f) \quad \forall f \in E_{\text{FDS}}. \end{aligned} \tag{22}$$

3. Choose a bin $M_C \in (M_{\text{N2F}}, K]$. Estimate bin $M_G \in (M_C, K]$ such that we have equal average powers in both directions. Group the bins of $S(f)$ to obtain upstream and downstream transmit spectra as

$$S^u_{\text{opt}}(f) = \begin{cases} S(f) & \forall f \text{ in } E_{\text{NDS}}, \text{ and} \\ & \forall f \text{ in bins } (M_C, M_G], \\ 0 & \text{otherwise,} \end{cases} \tag{23}$$

$$S^d_{\text{opt}}(f) = \begin{cases} S(f) & \forall f \text{ in } E_{\text{NDS}}, \text{ and} \\ & \forall f \text{ in bins } (M_{\text{N2F}}, M_C], \text{ and} \\ & \forall f \text{ in bins } (M_G, K], \\ 0 & \text{otherwise.} \end{cases} \tag{24}$$

4. Iterate step 3 for different choices of M_C and M_G. The final values for bins M_C and M_G are chosen such that we achieve equal performance margins and equal average powers in both directions of transmission. □

Since we have two degrees of freedom (bin choices M_C and M_G) to obtain equal margins and equal average powers the algorithm is guaranteed to converge.

C. Optimal transmit spectra for non-monotonic channels

Optimal signaling is more computationally expensive to implement in the presence of non-monotonic channels [4] since now we may have more than one NDS to FDS (and vice-versa) switch-over bins. In this scenario, even the self-FEXT transfer function has nulls. Bridged taps lead to non-monotonic channel transfer functions. *Bridged taps* (BTs) are short segments of twisted pairs that attach to another twisted pair that carries data between the subscriber and the CO. BTs are terminated at the other end with some characteristic impedance. BTs reflect back the signals from the data-carrying line. These reflections destructively interfere with the transmitted signal over certain frequencies. This leads to nulls in the channel transfer function and the self-FEXT transfer function at these frequencies. These nulls in the channel transfer function significantly reduce the data transmission rate. Thus, bridged taps pose an important problem in achieving high bit rates over DSLs.[2] Nevertheless, the optimal transmit spectra can be obtained by a bin by bin analysis.

Algorithm 3: Optimal transmit spectra for non-monotonic channels

1. Choose an initial power distribution of $P_{\max}$ over all bins.
2. Determine the signaling schemes NDS or FDS in each bin according to (19) and (20).
3. Choose powers $P_1, \cdots, P_m$ over the m NDS and FDS regions. Note that $\sum_{i=1}^{m} P_i = P_{\max}$. Distribute these powers optimally in each bin by the technique of optimization in the presence of self-interference [8]. Compute channel capacity. Optimize over different choices of powers $P_1, \cdots, P_m$ to yield the maximum channel capacity.
4. Update NDS to FDS (and vice-versa) switch-over bins. □

Repeat steps 3 and 4 until we find the maximum channel capacity. The algorithm has no guaranteed convergence to the global maximum.

In non-monotonic channels, NDS and FDS regions (comprising of one or more bins) could be distributed throughout the transmission bandwidth. The search for the optimal switch-over bins from one signaling scheme to the other could be exceedingly expensive (involving a multi-dimensional search). However we have found that for most cases of bridged taps, the optimal solution yields just two distinct regions of NDS and FDS bins.

D. Optimization under a PSD mask constraint: No self-interference

A peak power constraint in the frequency-domain could arise in a number of ways: (1) peak power limit, (2) a standardized fixed PSD, or (3) from spectral compatibility

[2] Bridged taps can be removed from DSLs, but this is an expensive (labor-intensive) procedure.

considerations. In this section we will impose an additional frequency-domain *peak* power constraint, i.e., a *PSD mask* constraint. This implies that no transmit spectrum can lie above the PSD mask constraint. This constraint is in addition to the *average* power constraint. We obtain optimal transmit spectra for a DSL service under these constraints, in the absence of self-interference.

The problem statement is similar to that in Section III-B without the equal capacity constraint but with additional power constraints. Consider a line carrying a new DSL service in the presence of different-service interference and channel noise. The twisted pair channel can be treated as a Gaussian channel with colored Gaussian noise [2]. The channel capacities (in bps) are given by (7) and (8) [12] with the average power constraints (9) and (10). In addition we have new peak power constraints

$$S^u(f) \leq Q^u(f)\ \forall f \quad \text{and} \quad S^d(f) \leq Q^d(f)\ \forall f. \tag{25}$$

For discussion purposes, we will focus on the upstream transmission. The same analysis can be applied to the downstream channel.

We wish to maximize (7) subject to the constraints (9), (10), and (25). The constraints are linear and differentiable. Further, the objective function to be maximized (7) is concave (the log function is concave). Any solution to this problem must satisfy the Karush-Kuhn-Tucker (KKT) [16] necessary conditions for optimality. For a concave objective function and linear, differentiable constraints, any solution that satisfies the necessary KKT conditions is a unique globally optimal solution [16]. Thus, we seek any solution that satisfies the necessary KKT conditions.

"Peak-constrained water-filling"[3] yields the optimal solution to (7), (9), (10), and (25). The optimal transmit spectrum is given by

$$S^u_{\text{opt}}(f) = \begin{cases} \lambda - \frac{N_o(f)) + DS_N(f) + DS_F(f)}{|H_C(f)|^2} & \text{for } f \in E_{\text{pos}}, \\ Q^u(f) & \text{for } f \in E_{\text{max}}, \\ 0 & \text{otherwise,} \end{cases} \tag{26}$$

with λ a Lagrange multiplier. The spectral regions E_{pos} and E_{max} are specified by

$$E_{\text{pos}} \;=\; \{f : 0 \leq S^u(f) \leq Q^u(f)\}, \text{ and}$$

[3]Peak-constrained water-filling can be likened to filling water in a closed vessel with uneven top and bottom surfaces.

$$E_{\max} = \{f : S^u(f) > Q^u(f)\}. \tag{27}$$

We vary the value of λ to achieve the optimal transmit spectrum $S^u_{\text{opt}}(f)$ that satisfies the average and peak power constraints (9), (10), and (25). It can be easily shown that this solution satisfies the KKT conditions for optimality. Substituting the optimal PSD $S^u_{\text{opt}}(f)$ into (7) yields the capacity C^u under the average and peak power constraints.

Note that if the maximum allowed average power ($P_{\max}$) exceeds the power under the constraining mask then the optimal transmit spectrum is the constraining PSD mask itself. In the absence of an average power constraint (but with a peak power constraint) the optimal transmit spectrum is again the constraining PSD mask.

E. Optimization under a PSD mask constraint: With self-interference

The solution in the previous section applies only in the absence of self-interference. In this section we find optimal transmit spectra in the presence of additional self-NEXT and self-FEXT. The problem statement is the same as that in Section III-B except for the additional PSD mask constraints for the transmit spectra; $Q^u(f)$ for upstream and $Q^d(f)$ for downstream. We consider a line carrying a new DSL service in the presence of self-interference, different-service interference and channel noise.

Algorithm 4: Peak-constrained optimal transmit spectra

1. Choose the constraining PSD mask as

$$Q(f) = \max(Q^u(f), Q^d(f)) \quad \forall f. \tag{28}$$

2. Solve for the intermediate transmit spectrum $S^u_{\text{opt}}(f)$ as:

$$S^u_{\text{opt}}(f) = \begin{cases} Q(f) & \forall f \text{ where } S^u(f) > Q(f), \\ S^u(f) & \text{otherwise,} \end{cases} \tag{29}$$

where $S^u(f)$ is obtained according to the algorithms in Section III-B.3 or IV-A. We refer to this as the peak-constrained optimization solution in the presence of self-interference As argued in the previous section, this solution satisfies the sufficient KKT conditions for optimality and is the unique optimal solution.

3. Denote the spectral region employing FDS signaling as E_{FDS} and the spectral region employing NDS as E_{NDS}. Obtain intermediate spectra $S^d_{\text{opt}}(f)$ from $S^u_{\text{opt}}(f)$ by symmetry, i.e., $S^d_{\text{opt}}(f) = S^u_{\text{opt}}(f)$ in NDS spectral regions and $S^d_{\text{opt}}(f)$ orthogonal to $S^u_{\text{opt}}(f)$ in FDS spectral regions. Merge $S^d_{\text{opt}}(f)$ and $S^u_{\text{opt}}(f)$ to form $S_{\text{opt}}(f)$ as

$$\begin{aligned} S_{\text{opt}}(f) &= S^u_{\text{opt}}(f) = S^d_{\text{opt}}(f) \quad \forall f \text{ in } E_{\text{NDS}}, \\ S_{\text{opt}}(f) &= S^u_{\text{opt}}(f) + S^d_{\text{opt}}(f) \quad \forall f \text{ in } E_{\text{FDS}}. \end{aligned} \tag{30}$$

Group the bins in E_{FDS} and E_{NDS} regions to obtain upstream and downstream spectra as

$$\begin{aligned} S^u_1(f) &= S_{\text{opt}}(f) \quad \forall f \text{ in } E_{\text{FDS}} \cap \{Q^u(f) \geq Q^d(f)\}, \\ S^d_1(f) &= S_{\text{opt}}(f) \quad \forall f \text{ in } E_{\text{FDS}} \cap \{Q^u(f) < Q^d(f)\} \end{aligned} \tag{31}$$

where $\cap$ represents the intersection of the two sets of frequencies.

$$S^u_1(f) = S^d_1(f) = S_{\text{opt}}(f) \quad \forall f \ \text{ in } E_{\text{NDS}}. \tag{32}$$

4. Check if the average power constraint is violated for upstream or downstream transmission. If the average power constraint is violated for direction o (i.e., the total transmit power in the direction o is more than $P_{\max}$)[4] then transfer power from $S^o_1(f)$ to $S^{\overline{o}}_1(f)$. Transfer power first from spectral regions with the least $S^o_1(f) - S^{\overline{o}}_1(f)$ difference. Repeat this successively in spectral regions with increasing $S^o_1(f) - S^{\overline{o}}_1(f)$ difference until the average power in both directions are the same.[5] This power transfer scheme tries to even out the powers between the two directions, with the least loss in the total sum of the transmit powers of the two directions.

If the difference $S^o_1(f) - S^{\overline{o}}_1(f)$ is the same (or marginally varying) for a range of frequencies, then transfer power from direction o to direction $\overline{o}$ in those spectral regions that give the maximum gain in bit rates for direction $\overline{o}$. □

[4]Note that if the total transmit power in direction o is more than $P_{\max}$ then the transmit power in direction $\overline{o}$ is less than $P_{\max}$.

[5]This approach of transferring power from direction o to direction $\overline{o}$ can be likened to "stealing from the rich and giving to the poor."

V. Simulation Results and Discussion

A. Examples

Figures 6 and 7 illustrate the optimal transmit spectra on carrier serving area (CSA) [6] loop 6 for HDSL2 in the presence of 49 HDSL NEXT, and 39 HDSL2 self-NEXT and self-FEXT interferers, respectively.[6] Note that the optimal transmit spectra vary significantly with the interference combination. In the case of different service interferers (HDSL in Figure 6a and T1 in Figure 6b), the optimal upstream and downstream spectra are the same (NDS throughout). In the case of HDSL2 interferers (Figure 7), self-NEXT at high frequencies forces the optimal upstream and downstream spectra to separate in frequency giving rise to an FDS region.

Figure 8 shows the constrained optimal upstream and downstream transmit spectra for HDSL2 under the OPTIS (ANSI T1E1.4's HDSL2 standard for transmit PSD masks) constraining PSD masks in the presence of self-NEXT and self-FEXT from 39 HDSL2 interferers. Note that the optimal upstream and downstream transmit spectra use FDS in a larger spectral region than OPTIS in order to avoid the high self-NEXT.

B. Performance margins

The *performance margin* or *noise margin* of a channel for a fixed bit rate and bit error rate measures the maximum degradation (from noise and interference) in performance that a channel can sustain before being unable to transmit at that bit rate and bit error rate [20].

Table I lists the uncoded (without any coding gains) performance margins[7] of the optimal transmit spectra versus those obtained using the OPTIS transmit spectra (obtained

[6]Simulation Details: Bit rate fixed at 1.552 Mbps. Total average input power in each direction $P_{\max} = 19.78$ dBm. Different service interference models obtained from Annex B of T1.413-1995 (from [9], the ADSL standard), with exceptions as in [17]. Self-NEXT interference modeled as a 2-piece Unger model [10]. Margins calculated according to [18]. OPTIS (ANSI T1E1.4's HDSL2 standard for transmit PSD masks) transmit spectra obtained by tracking 1 dBm/Hz below the OPTIS PSD masks [19]. OPTIS performance margin figures from [19]. AGN of -140 dBm/Hz added to the interference.
DMT modulation scheme: Sampling frequency $f_s = 1000$ kHz. Bin width $W = 2$ kHz. Number of bins $K = 250$. Start frequency = 1 kHz. Bit error rate = 10^{-7}. SNR gap = 9.8 dB. No cyclic prefix. No limitation on maximum number of bits per tone. See [20] for more details.

[7]OPTIS numbers were obtained from [19]. Diff = Difference between worst-case Optimal and worst-case OPTIS.

by tracking 1 dBm/Hz below the OPTIS PSD masks [19]) for CSA loop 6. For different service interferers (HDSL and T1) only the NEXT powers were considered; for HDSL2, "self" comprises both self-NEXT and self-FEXT. Equal performance margins were obtained for upstream and downstream transmissions. The optimal scheme outperforms OPTIS (uses fixed transmit spectra) with large performance gains in all the cases.

Table II compares the uncoded performance margins[7] of the constrained optimal transmit spectra "under OPTIS" versus the OPTIS transmit spectra [19] for CSA loop 6. For different service interferers (HDSL and T1), only the NEXT powers were considered, while for HDSL2 "self" comprises both self-NEXT and self-FEXT. Constrained-optimal transmit spectra always outperform (have higher performance margins) fixed, standard (non-adaptive) spectra. The margins of constrained optimal spectra are much smaller than ones obtained using optimal spectra (from Table I) since OPTIS is a bad constraining PSD mask.

C. Spectral compatibility

Spectral compatibility is measured in terms of noise margins (called spectral compatibility margins) of existing services in the presence of the new optimized service. By design, the optimal transmit spectra achieve good spectral compatibility margins. Through optimal power distribution techniques, we distribute more power of the optimized DSL service in the regions of low DS interference and less power in the regions of high DS interference. Thus, we avoid the higher-power transmission frequencies of existing services and therefore simultaneously reduce the effect of the leakage power from the new optimized DSL service into these existing services.

To illustrate, consider the spectral compatibility between HDSL2 and HDSL. Table III lists the spectral compatibility margins[7] of the optimal transmit spectra versus OPTIS CSA loop 6. We compare the performance margins for HDSL in the presence of two types of interferers; other HDSL lines and HDSL2 lines. Optimal spectra have significantly better spectral compatibility margins than OPTIS. Analysis for other DSL services like T1 and ADSL yields similar results [14].

VI. Conclusions

In this paper, we have derived optimal transmit spectra for symmetric bit-rate communication channels dominated by self-NEXT, in particular for DSLs. We setup and solved an optimization problem to jointly maximize the bit rates of each new-service DSL line in a binder given the channel, noise, and crosstalk characteristics. We developed optimal spectra under an average power constraint and a combined average plus peak frequency-domain power constraint. Optimal spectra and constrained optimal spectra vary significantly with interference combination. Constrained spectra do not necessarily follow the constraining PSD.

The key advantages of our techniques are: (1) Optimal and constrained-optimal transmit spectra yield large performance margin gains (up to 16 dB in our experience) as compared to current fixed-spectra schemes. We can trade these increased performance margins for increased bit rates or decreased average transmission power (which in turn will reduce crosstalk). (2) Optimal transmit spectra are inherently spectrally compatible with existing services. (3) Optimal spectra are not bound to any particular modulation scheme. (4) There exist near-optimal transmit spectra that are extremely easy to compute, even for complicated loops such as those with bridged taps. (5) No echo cancellation is required in frequency bands employing FDS. (6) Equal performance margins can be obtained for upstream and downstream directions using optimal transmit spectra. (7) Transmit spectra can be adapted on-line to changes in line conditions (e.g., temperature variations, etc.). (8) Optimal spectra yield performance bounds on maximum achievable bit rates. (9) Optimal spectra can be used for spectrum management in a telephone cable. (10) Vendor advantage can be achieved by using adaptive optimal spectra underneath fixed, standardized spectra.

Our scheme requires a priori knowledge of the characteristics of the neighboring interfering services. These can either be estimated at start-up or analyzed in a worst-case manner for a particular line under consideration. This information could also be obtained from a central office database that specifies the type of services in each binder group in the telephone cable. We are currently investigating techniques to obtain optimal transmit spectra for crosstalk-dominated MIMO channels with differing channel, self-NEXT, and self-FEXT transfer functions.

References

[1] J. Werner, "The HDSL Environment," *IEEE J. Select. Areas Commun.*, vol. 9, pp. 785–800, Aug. 1991.

[2] K. Kerpez, "Near-End Crosstalk is almost Gaussian," *IEEE Trans. Commun.*, vol. 41, Jan. 1993.

[3] S. Verdu, *Multiuser Detection.* Cambridge University Press, 1998.

[4] A. Sendonaris, V. Veeravalli, and B. Aazhang, "Joint Signaling Strategies for Approaching the Capacity of Twisted Pair Channels," *IEEE Trans. Commun.*, vol. 46, May 1998.

[5] R. Gaikwad and R. Baraniuk, "Optimal Transmit Spectra for HDSL2," *ANSI T1E1 Contribution*, no. T1E1.4/98-162R1, 1998.

[6] W. Chen, *DSL Simulation Techniques and Standards Development for Digital Subscriber Line Systems.* Macmillan Technology Series, 1998.

[7] R. Gaikwad and R. Baraniuk, "Optimal Transmit Spectra for HDSL2 under a Peak Frequency–Domain Power Constraint," *ANSI T1E1 Contribution*, no. T1E1.4/98-188R2, 1998.

[8] J. Aslanis and J. Cioffi, "Achievable Information Rates on Digital Subscriber Loops: Limiting Information Rates with Crosstalk Noise," *ANSI T1E1 Contribution*, vol. 40, Feb. 1992.

[9] American National Standard for Telecommunications, "Network and Customer Installation Interfaces– Asymmetric Digital Subscriber Line (ADSL) Metallic Interface," *ANSI T1E1 Standard*, no. T1.413-1995, 1995.

[10] K. Kerpez, "Full-duplex 2B1Q Single-pair HDSL Performance and Spectral Compatibility," *ANSI T1E1 Contribution*, no. T1E1.4/95-127, 1995.

[11] R. Gaikwad and R. Baraniuk, "Transmit Spectra Adaptive to Noise and Interference at Activation for HDSL2," *ANSI T1E1 Contribution*, no. T1E1.4/98-258, 1998.

[12] R. Gallager, *Information Theory and Reliable Communication.* New York: Wiley, 1968.

[13] R. Gaikwad and R. Baraniuk, "Joint Signaling Strategies for crosstalk dominated channels," *IEEE Trans. Commun.*, To be Submitted 1999.

[14] R. Gaikwad and R. Baraniuk, "Spectral optimization and joint signaling techniques for communication in the presence of crosstalk," Tech. Rep. 9806, Rice University, Electrical and Computer Engineering Department, Houston, Texas, July 1998. Web: `www.dsp.rice.edu/publications/Papers/tr9806.ps`.

[15] W. Press, S. Teukolsky, W. Vellerling, and B. Flannery, *Numerical Recipes in C–The Art of Scientific Computing.* Cambridge University Press, second ed., 1997.

[16] R. Horst, P. Pardalos, and N. Thoai, *Introduction to Global Optimization.* Kluwer Academic Publishers, 1995.

[17] G. Zimmerman, "Performance and Spectral Compatibility of OPTIS HDSL2," *ANSI T1E1 Contribution*, no. T1E1.4/97-237, 1997.

[18] G. Zimmerman, "Normative Text for Spectral Compatibility Evaluations," *ANSI T1E1 Contribution*, no. T1E1.4/97-180R1, 1997.

[19] J. Girardeau, M. Rude, H. Takatori, and G. Zimmerman, "Updated OPTIS PSD Mask and Power Specification for HDSL2," *ANSI T1E1 Contribution*, no. T1E1.4/97-435, 1997.

[20] M. Barton and M. Honig, "Optimization of Discrete Multitone to Maintain Spectrum Compatibility with Other Transmission Systems on Twisted Copper Pairs," *IEEE J. Select. Areas Commun.*, vol. 13, pp. 1558–1563, Dec. 1995.

TABLE I

Uncoded (no coding gains) performance margins (in dB) for HDSL2 on CSA loop 6: OPTIS vs. Optimal.

Crosstalk source	DSL service	OPTIS		**Optimal**		Diff
		Up	Dn	Up	Dn	
49 HDSL	HDSL2	2.7	12.2	**18.5**	**18.5**	15.8
25 T1	HDSL2	19.9	17.5	**21.3**	**21.3**	3.8
39 self	HDSL2	2.1	9.0	**18.3**	**18.3**	16.2
24 self+24 T1	HDSL2	4.3	1.7	**5.4**	**5.4**	3.7

TABLE II

Uncoded (no coding gains) performance margins (in dB) for HDSL2 on CSA loop 6: OPTIS vs. Constrained optimal.

Crosstalk Src	DSL service	OPTIS		Optimal		Diff	
		Up	Dn	**Up**	**Dn**	**Up**	**Dn**
49 HDSL	HDSL2	2.7	12.2	3.7	13.8	1.0	**1.6**
25 T1	HDSL2	19.9	17.5	20.4	18.8	0.5	**1.3**
39 self	HDSL2	2.1	9.0	15.5	17.6	**13.4**	8.6
24 self+24 T1	HDSL2	4.3	1.7	4.5	4.7	0.2	**3.0**

TABLE III

Spectral-compatibility margins (in dB) for HDSL2 on CSA loop 6: OPTIS vs. Optimal.

Crosstalk source	xDSL service	OPTIS	**Optimal**	
			Up	Dn
49 HDSL	HDSL	7.86 (OPTIS and Optimal not involved)		
39 HDSL2	HDSL	7.84	**13.28**	**20.84**
49 HDSL2	HDSL	7.26	**12.71**	**20.15**

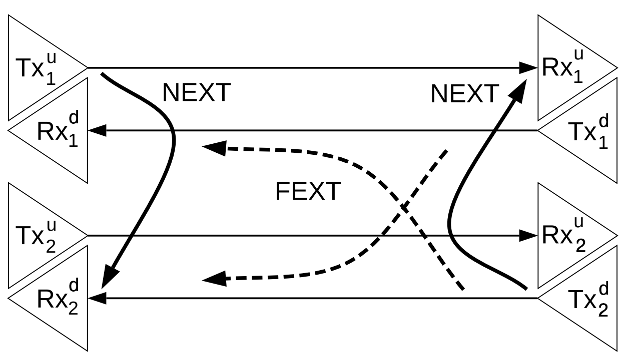


Fig. 1. *Near-end crosstalk (NEXT) and far-end crosstalk (FEXT) between neighboring lines in a binder. Tx's are transmitters and Rx's are receivers. A 4–2 line hybrid circuit connects the transceivers (*$\mathrm{Tx}_1^u/\mathrm{Rx}_1^u$ *pair, for example) to the telephone lines.*

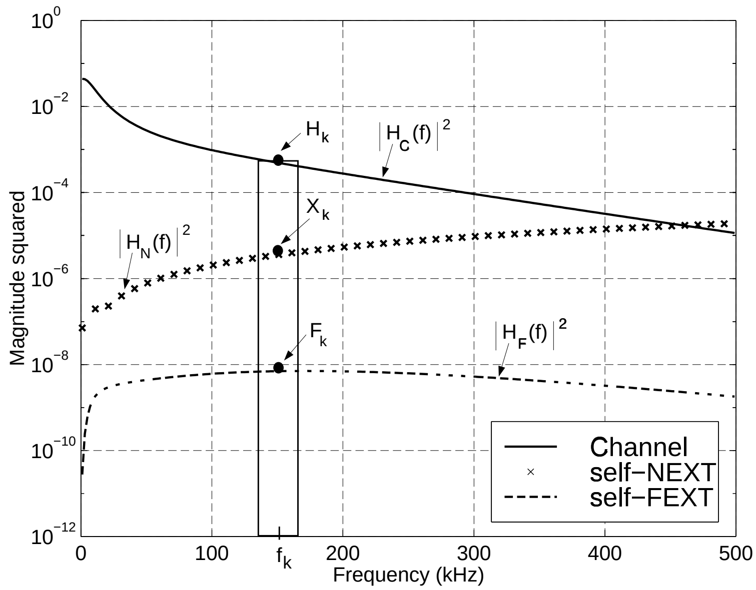


Fig. 2. *Magnitude-squared transfer function of the channel (CSA loop 6),* 39 *self-NEXT interferers, and* 39 *self-FEXT interferers.*

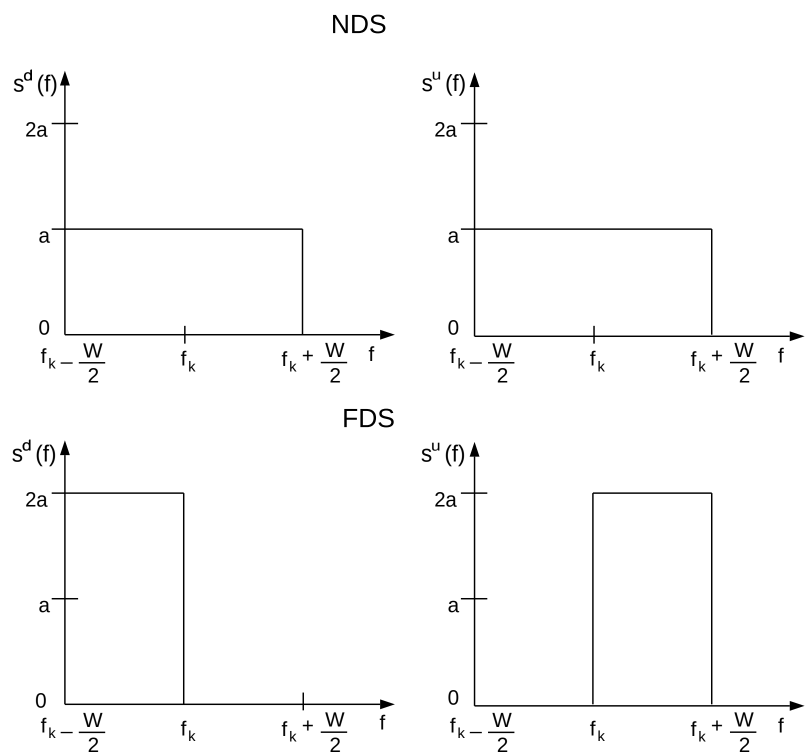


Fig. 3. *Transmit spectra for no division signaling (NDS) and frequency division signaling (FDS) schemes in a single frequency bin k. In the FDS scheme the spectra $s^u(f)$ and $s^d(f)$ share disjoint frequency bands and are orthogonal to each other. This eliminates self-NEXT, but reduces bandwidth by half.*

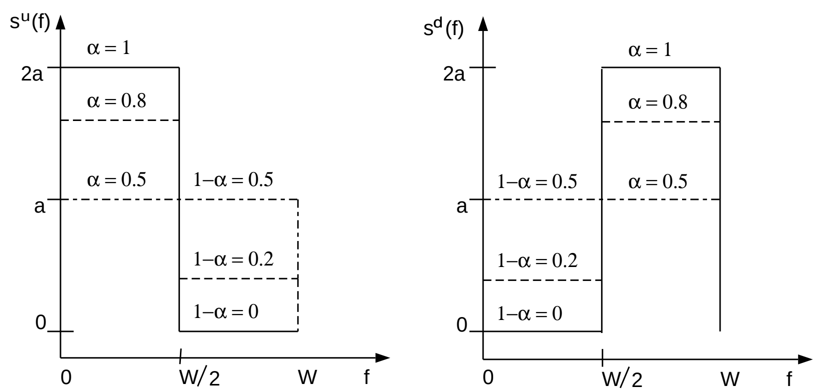


Fig. 4. *Upstream and downstream transmit spectra in a single frequency bin ($\alpha = 0.5 \Rightarrow$ NDS and $\alpha = 1 \Rightarrow$ FDS).*

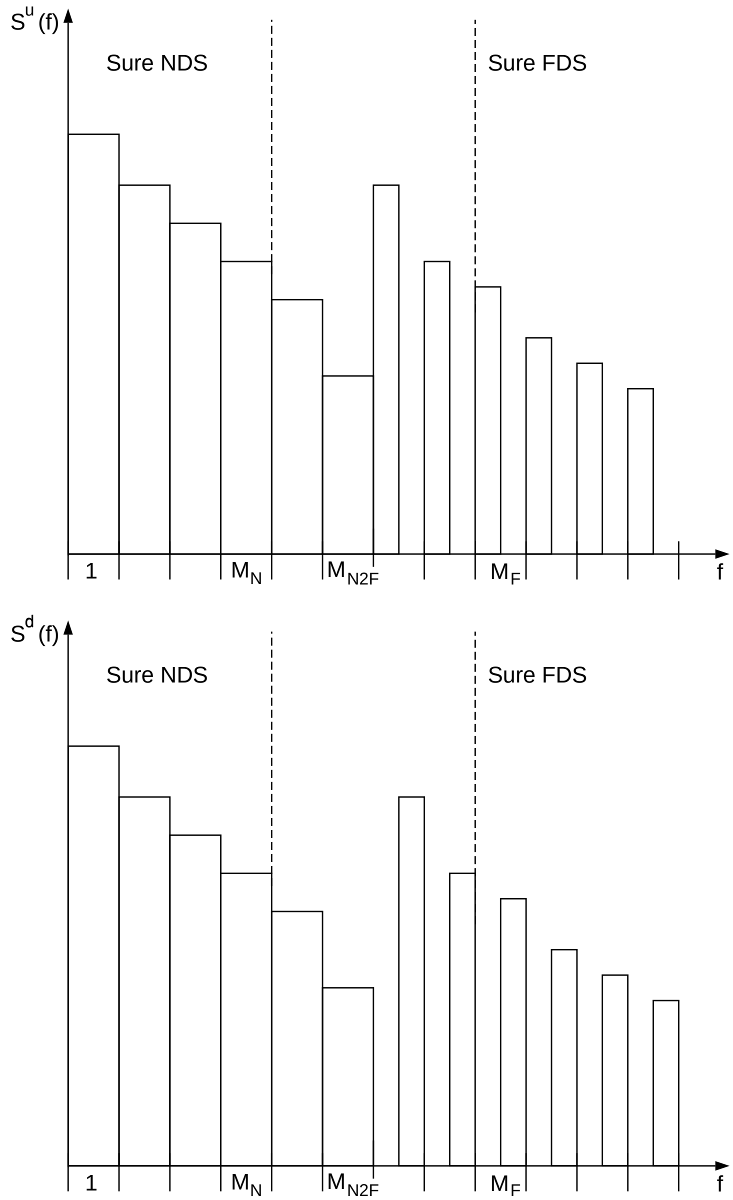


Fig. 5. *Upstream and downstream transmit spectra showing regions employing NDS and FDS. The bins* $[1, M_{\mathrm{N2F}}]$ *employ NDS and the bins* $[M_{\mathrm{N2F}} + 1, K]$ *employ FDS.*

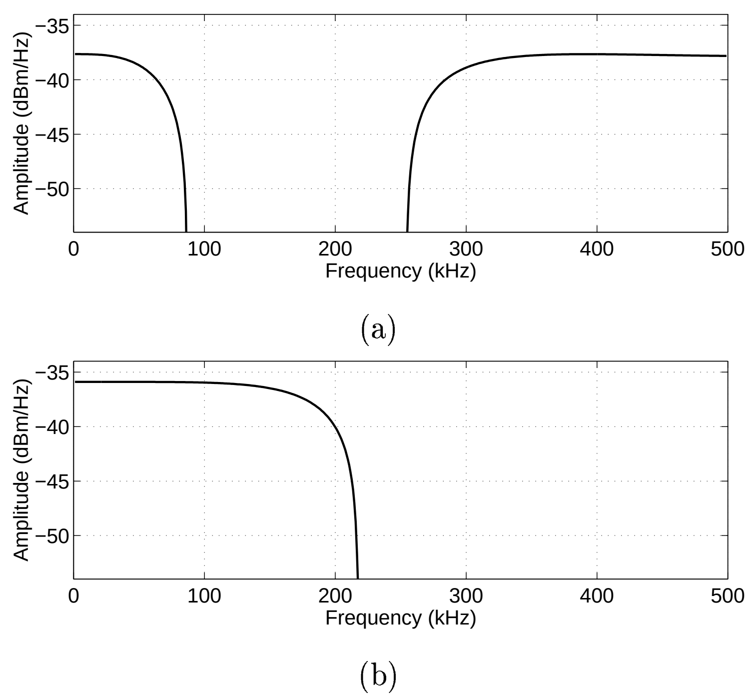
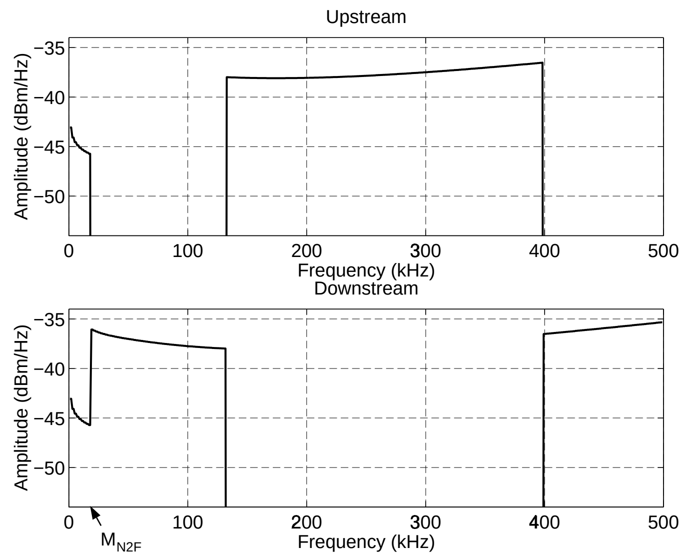
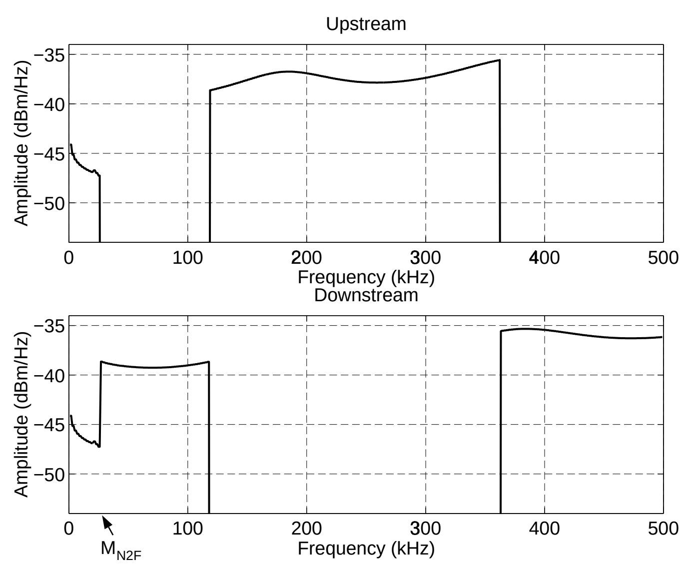


Fig. 6. *Optimal transmit spectra for HDSL2 on CSA loop* 6 *with (a)* 49 *HDSL NEXT interferers and AGN of* -140 *dBm/Hz and (b)* 25 *T1 NEXT interferers and AGN of* -140 *dBm/Hz. Since there is no self-interference, FDS is not required. The upstream and downstream transmissions employ the same spectrum. The optimal spectra vary significantly between interference type HDSL and T1.*



(a)



(b)

Fig. 7. *Optimal upstream and downstream transmit spectra for HDSL2 on (a) CSA loop* 6 *and (b) CSA loop* 4 *(having bridged-taps) with* 39 *self-NEXT and* 39 *self-FEXT interferers and AGN of* -140 *dBm/Hz. NDS takes place to the left of switch-over frequency* M_{N2F} *and FDS to the right.*

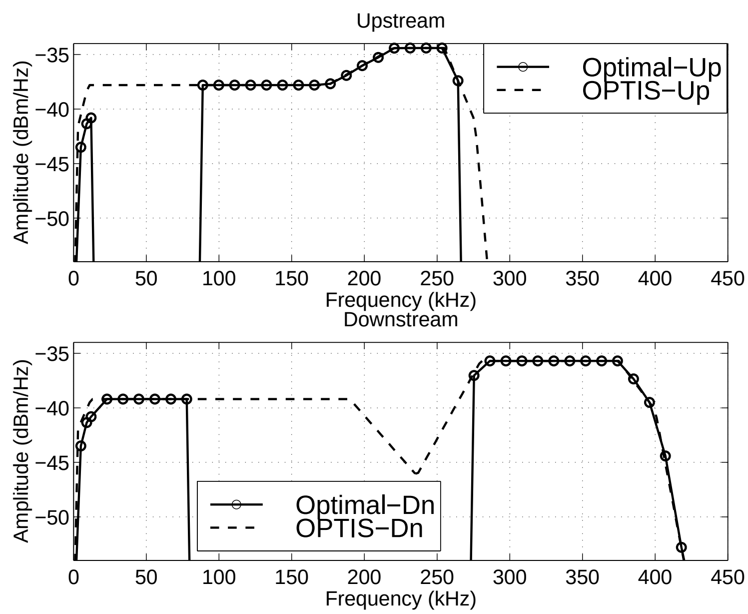


Fig. 8. *Optimal upstream and downstream transmit spectra for HDSL2 on CSA loop* 6 *under the OPTIS upstream and downstream constraining PSD masks with* 39 *self-NEXT and* 39 *self-FEXT interferers and AGN of* -140 *dBm/Hz.*