An Efficient Power-Reduction Technique on DSL Modems

Henry K. Kwok,* Douglas L. Jones
Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
1308 W. Main St., Urbana, IL 61801
email: henry@ifp.uiuc.edu, dl-jones@uiuc.edu

August 2, 2000

Abstract

Discrete multitone (DMT) has been considered for various wireline, broadband communication systems. However, it is well-known that this modulation scheme exhibits high peak-to-average power ratio (PAR). We introduce a novel approach to reduce the peak power using constellation shaping. We propose a method based on a hyper-spherical boundary, to shape ordinary DMT constellations. This method reduces the peak power by 3-6 dB with no loss of data rate and with virtually identical symbol error rate as conventional systems when no clipping is applied to the signal. When clipping is applied, it provides at least 2 dB of peak-power reduction.

1 Introduction

In a discrete multitone (DMT) system, the output time signal, $x(t)$, is generated by

$$x(t) = \sum_k \mathrm{Re}(X_k) \cdot \cos(2\pi f_k t) + \mathrm{Im}(X_k) \cdot \sin(2\pi f_k t) \quad (1)$$

for $0 \leq k < M$ where the X_k are constellation points from QAM constellations of roughly equal average energy[1] and $f_k = \frac{k}{MT}$. The sampled version of the signal can be efficiently computed by an fast Fourier transform.

However, when all the X_k take on similar values, the constellation points interfere constructively to produce a large time-sample magnitude. In fact, the time-domain peak power grows linearly as the number of frequency bands. Thus, the time samples may occasionally have very high output levels, which leads to the requirement of an expensive, highly linear, and power-inefficient analog front end (AFE) and/or a clipping mechanism to limit the time-sample magnitude, which leads to impulsive noise and performance degradation. High peak-to-average ratio (PAR) is arguably the greatest drawback of DMT. In order to maintain an acceptable level of system reliability, the output signal is usually clipped at a high PAR which leads to higher power consumption. In fact, the AFE consumes a substantial amount of power for a typical DSL modem. This power consumption requirement may lead to various difficulties in system designs. For example, a USB DSL modem may need an extra power supply when the total power consumption of the modem exceeds the amount that the bus may supply. Also, the telephone company imposes a maximum spatial density of power consumption in the central office (CO). Lower power consumption allows more DSL modems to be placed more densely in the CO which leads to lower rental cost.

Several authors [1], [2] exploit some unused bandwidth in the DMT system to cancel out the large-amplitude samples generated by the other information-bearing channels. This approach requires frequency spectrum that could otherwise be used for transmission of information. One class of methods clips the transmitted signal intelligently so that the degradation due to the impulsive noise is minimized, or the receiver uses *a priori* knowledge of the clipping mechanism to decode the signal reliably [3], [1]. However, this class of methods degrades the performance of the overall system by introducing artificial noise.

Numerous methods have been proposed to reduce the PAR of another multicarrier modulation scheme - orthogonal frequency division multiplexing (OFDM). Currently, most of the existing schemes approach this problem by modifying the transmitted signal. Various block coding schemes have been used [4], [5], [6]. These schemes lower the overall data rate of the system and essentially trade bandwidth for lower peak amplitude. Others [7], [8] relax the constellation by allowing possibly more than one choice for each constellation point. The signals generated by these methods may occasionally violate the power constraint. Finally, there are methods which adjust the phase of the constellation points in order to avoid phase alignments that lead to large magnitude time samples [9], [10]. In addition, [11] points out that evaluating PAR

*This work is supported by a Tellabs fellowship at the University of Illinois.

[1] Without fine power adjustment used in actual DMT systems, the average energies of these QAM constellations are exactly equal.

reduction techniques in the digital domain often leads to optimistic values. At times, over 3 dB of PAR reduction are fictitious. A phase rotation scheme with over 4 dB of PAR reduction in the discrete-time domain was shown (by [11]) to have less than 1 dB of reduction when the performance measurement is done in the continuous-time domain. We refer to this phenomenon as *peak regrowth* because the peak seems to reappear after going from the discrete-time to the continuous-time domain.

Unfortunately, most of these schemes cannot be easily apply to DMT. Most coding schemes require PSK in each tone. Phase rotation do not perform well as the per-tone constellation become large. In addition, many of these schemes assume identical and fixed constellation in each tone. A DMT modem designs its constellation during the initialization according to the SNR of the subbands. Most in-band signal-modification schemes cannot be initialized in such short time.

In this paper, we propose a method for peak-power reduction in DMT systems based on constellation shaping based on the shell mapping used in the V.34 modems. It shapes the constellation into a hypersphere in order to reduces the peaks, without lowering the data rate or increasing the symbol error rate. It yields up to 6 dB of peak-power reduction. We will show theoretically that the proposed scheme is immune to peak regrowth. For practical channel parameters with clipping, it yields about 2 dB of peak-power reduction.

2 Constellation Shaping

For ordinary DMT systems, we may consider the QAM constellations from all channels jointly as a real-valued $2N$-D lattice in the frequency domain, and we choose a boundary in the $2N$-D space such that all lattice points inside the boundary are considered to belong to the constellation. Typically, the boundary is square or rectangular in nature because the input bits can be easily divided into subgroups, each of which indexes one frequency dimension. However, when a constellation-dependent metric needs to be minimized, a rectangular constellation is often suboptimal [12].[2] Instead, a properly-chosen boundary may lead to better performance in terms of the particular metric without lowering the data rate or increasing the symbol error rate. In the V.34 modem standard, the purpose is to reduce the average power. In our case, the goal is to minimize the peak power, or the ∞-norm, $\|\mathbf{x}\|_\infty$, in the time-domain. We illustrate this idea with an example.

Example 1 *Suppose we use a 2-point inverse DFT to create a two-channel real-valued DMT signal. The modulation matrix is given by:*

$$\mathbf{A}_2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \qquad (2)$$

A 16*-point, square constellation (see Figure 1) is used for the unshaped baseline case. (Note that this is not a QAM constellation, but a* 2*-D joint real-valued cross-constellation of frequencies* 1 *and* 2*.) If* $\mathbf{X} = \begin{bmatrix} X_1 & X_2 \end{bmatrix}$ *is a constellation point belonging to the constellation* $\mathcal{C}$*, we wish to find the quantity* $\min_{\mathcal{C}} \left(\max_{\mathbf{X}\in\mathcal{C}} \|\mathbf{A}_N \mathbf{X}\|_\infty \right) = \min_{C} \left(\max_{\mathbf{x}\in\mathbf{A}_N(\mathcal{C})} \|\mathbf{x}\|_\infty \right)$*. (Note that* $\mathbf{A}_N(C) = \{\mathbf{x} : \mathbf{x} = \mathbf{A}_N\mathbf{X} \text{ for } \forall \mathbf{X} \in \mathcal{C}\}$*.) One choice that leads to better performance is shown in Figure 2. In Figures 3 and 4, the corresponding time-domain sample vector,* $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$*, for each constellation point is plotted. The dotted lines indicate the equi-metric line for the maximum time-sample magnitude for the unshaped and shaped case. The overall PAR reduction is* 1.5 *dB. As we will see later, for higher dimensions, the PAR reduction is larger.*

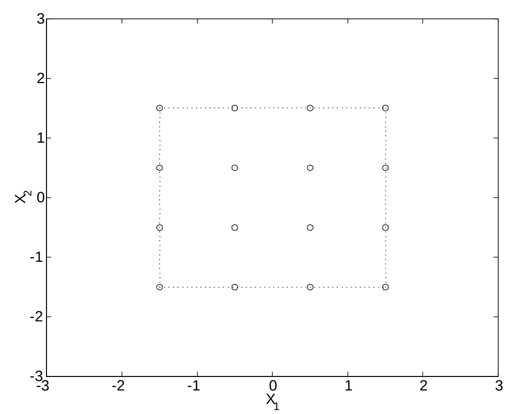

Figure 1: The unshaped, 2-D constellation.

The constellation boundary is usually determined by the metric that we want to optimize. In this paper, we propose a hyper-spherical boundary.

3 Hyper-Spherical Shaping

Spherically-shaped constellations are used in the ITU V.34 modem standard to reduce the average power of the constellation while neither lowering the data rate nor increasing the symbol error rate. For the peak-power reduction problem, a spherical constellation is also desirable because a spherical volume is invariant under rotation and reflection. Therefore, the radius of the spherical boundary is the maximum time-sample magnitude when a unitary transformation is applied to the constellation points. (Otherwise, we must scale by the

[2] In general, constellation shaping is a precoding technique in which a proper boundary for a multi-dimensional constellation is selected to match a certain metric of interest. See [12] for details.

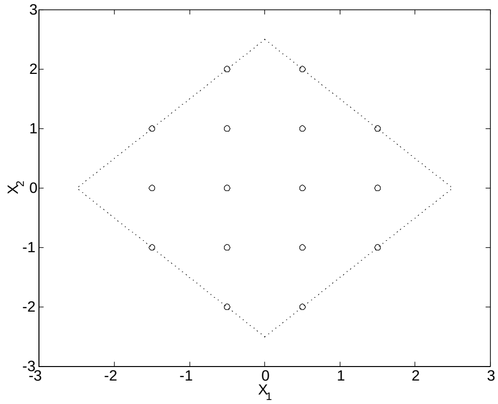

Figure 2: The shaped 2-D constellation in the 2-D real frequency domain..

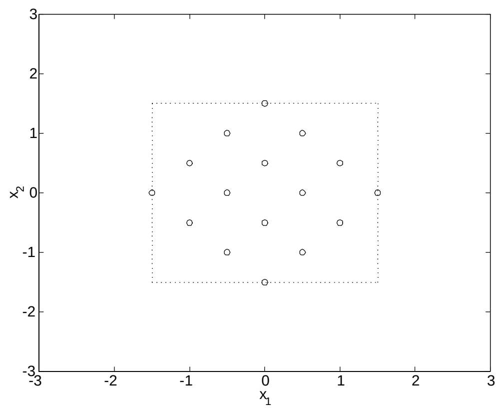

Figure 3: The time samples generated by the unshaped constellation. Note that the worst-case amplitude is 1.5.

matrix norm.) By choosing a spherical boundary, the peak time sample will be no larger than the peak magnitude of the per-channel base constellation. In other words, a hyperspherical constellation is "*amplification free*" under a unitary transformation.

In the time-domain, we may bound the magnitude of the continuous-time signal, $x(t)$, using the Cauchy-Schwartz inequality

$$|x(t)| = \left|\sum_k \mathrm{Re}(X_k) \cdot \cos(2\pi f_k t) + \mathrm{Im}(X_k) \cdot \sin(2\pi f_k t)\right| \tag{3}$$

$$= |\langle \mathbf{X}, \mathbf{d}_t \rangle| \leq \|\mathbf{X}\|_2 \|\mathbf{d}_t\|_2 = \sqrt{N}\|\mathbf{X}\|_2 \tag{4}$$

where $\mathbf{X} = \begin{pmatrix} \cdots & \mathrm{Re}\,X_k & \cdots & \mathrm{Im}\,X_k & \cdots \end{pmatrix}$, $\mathbf{d}_t = \begin{pmatrix} \cdots & \cos(2\pi f_k t) & \cdots & \sin(2\pi f_k t) & \cdots \end{pmatrix}$, and N is the number of transmitting tones. In order words, the maximum magnitude of the complex envelope of the signal is bounded by the l_2-norm of the constellation point. If we limit this upper-bound to a maximum value of β, we have a constellation with each point, $\mathbf{X}$, satisfying

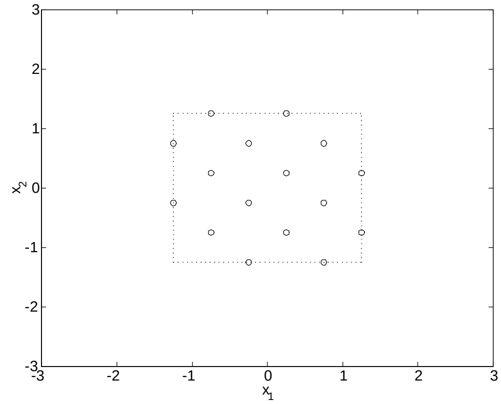

Figure 4: The time samples generated by the shaped constellation. Via shaping, the largest peak amplitude is reduced to 1.25.

$$|x(t)| \leq \sqrt{N}\|\mathbf{X}\|_2 \leq \beta \tag{5}$$

or all constellation points must be inside a $2N$-D hypersphere of radius $\frac{\beta}{\sqrt{N}}$.

3.1 Asymptotic performance

In order to evaluate the theoretical performance of this scheme, we use the fact that the volume of an N-D hypersphere with a radius of ρ is given by [13]

$$vol(S_N) = \frac{\pi^{\frac{N}{2}}}{\Gamma\left(\frac{N}{2}+1\right)}\rho^N \tag{6}$$

For simplicity, we ignore the fine power adjustment used to tune the bit error rate of each individual tone to the desired level. We assume that there are N square, QAM constellations each with unit variance. When the constellation size is large, we may apply the continuous approximation [12] and treat each QAM constellation as uniform random variable over a square region with an edge of $\sqrt{6}$. The maximum magnitude of the unshaped constellation is given by[3]

$$M_{unshaped} = \max_t \max_{\mathbf{X}} \left| \begin{array}{c} \sum_k \mathrm{Re}(X_k) \cdot \cos(2\pi f_k t) + \\ \mathrm{Im}(X_k) \cdot \sin(2\pi f_k t) \end{array} \right| \tag{7}$$

$$= \frac{\sqrt{6}}{2} \max_\theta \left| \sum_k \cos(k\theta) + \sin(k\theta) \right| \tag{8}$$

$$\approx \sqrt{3}N \tag{9}$$

[3]Experimentally, it can be shown that $M_{unshaped} \approx 1.6N$

We require that the data rate of the shaped system remains the same which is equivalent to requiring both the unshaped hypercube and the shaped hypersphere to have the same volume in the N-D space. Therefore, we compute the radius of a $2N$-D sphere with the volume of 6^N. This yields the equation

$$6^N = \frac{\pi^N}{\Gamma(N+1)} \rho^{2N} \tag{10}$$

or the radius of the shaped constellation is given by

$$\rho = M_{shaped} = \sqrt{\frac{6}{\pi}} \left[\Gamma(N+1)\right]^{\frac{1}{2N}} \tag{11}$$

Thus, the overall peak-power reduction, $G_{sphere}(N)$, is

$$G_{sphere}(N) = \frac{M_{unshaped}}{M_{shaped}} = \frac{\sqrt{3}N}{\sqrt{\frac{6}{\pi}} \left[\Gamma(N+1)\right]^{\frac{1}{2N}} \cdot \sqrt{N}} \tag{12}$$

$$= \sqrt{\frac{\pi N}{2}} \left[\frac{1}{\left[\Gamma(N+1)\right]^{\frac{1}{2N}}}\right] \tag{13}$$

$$= \sqrt{\frac{\pi N}{2}} \left[\frac{1}{\left[(N+1)!\right]^{\frac{1}{2N}}}\right] \tag{14}$$

As N increases, we approach asymptotically

$$\lim_{N\to\infty} G_{sphere}(N) = \lim_{N\to\infty} \frac{\sqrt{\frac{\pi N}{2}}}{\left[(N+1)!\right]^{\frac{1}{2N}}} \tag{15}$$

$$= \lim_{N\to\infty} \frac{\sqrt{\frac{\pi N}{2}}}{\left[\sqrt{2\pi(N+1)}\left(\frac{N+1}{e}\right)^{(N+1)}\right]^{\frac{1}{2N}}} \tag{16}$$

$$= \lim_{N\to\infty} \sqrt{\frac{\pi e N}{2(N+1)}} = \sqrt{\frac{\pi e}{2}} \tag{17}$$

Therefore, the peak-power reduction approaches asymptotically to $\sqrt{\frac{\pi e}{2}}$, or 6.304 dB.[4]

4 Results

The peak-power reduction of the hyper-spherical shaping under various numbers of channels and constellation sizes (identical constellation for each tone) are shown in Figure 5.We can see that, for large constellations, it approaches the theoretical limit, $G_{sphere}(N)$, rather rapidly. The peak-power reduction for typical constellation sizes and numbers of channels is about 3 to 6 dB.

Next, we use two channels that one may encounter in an actual ADSL. Figure 6 shows the bit allocation for the 31-channel and 215-channel cases. The amount of peak-power

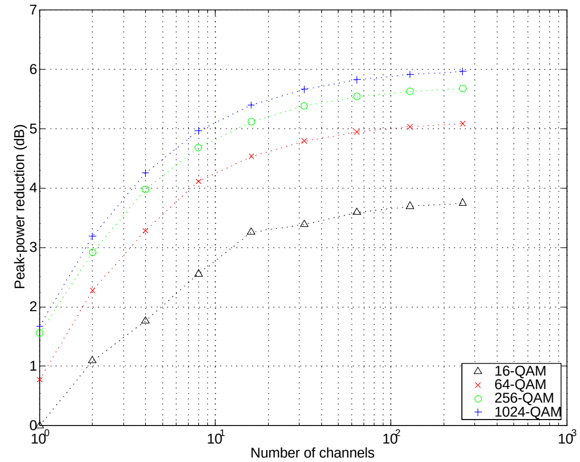

Figure 5: The peak-power reduction vs. the number of channels for several different constellation sizes.

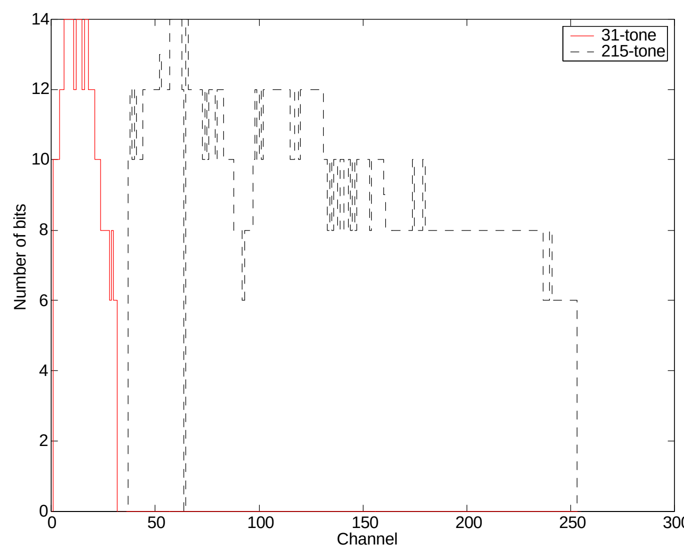

Figure 6: The bit allocation schemes used by the two simulated channels.

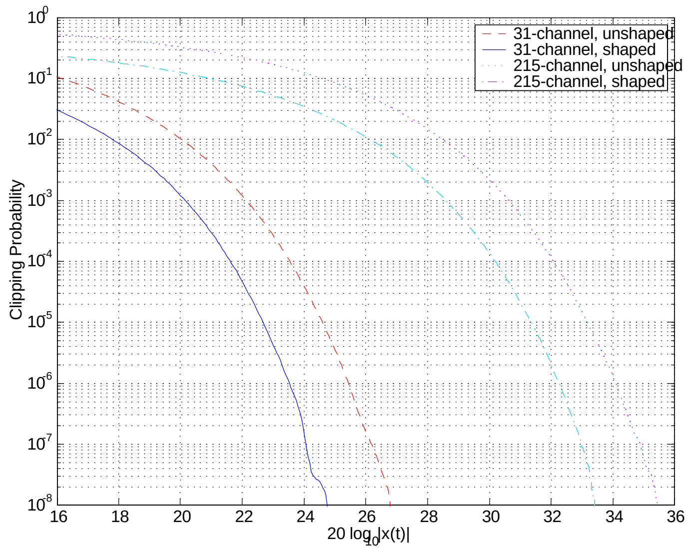

Figure 7: The clipping probability curves for the two channels. The maximum peak-power reduction for the 31-tone and 215-tone cases are 4.79 dB and 5.22 dB respectively.

reduction is 4.8 dB for the 31-channel case and 5.2 dB for the 215-channel case.

In practice, we can tolerate a small amount of clipping as long as it does not increase BER significantly and the signal is compliant with the PSD mask. The ANSI ADSL standard recommends a clipping rate lower than 10^{-7}. To prevent frequent CRC frame errors, a value as small as 10^{-10} is used. This requirement allows the AFE to clip the signal at least 5.2 times of the rms value of the signal. Thus, with clipping, the PAR of a DMT system is upper bounded at 14.3 dB. With an upper limit of 6 dB of peak reduction, any systems with an unshaped PAR of over 20 dB may achieve lower PAR by clipping instead of shaping. However, we may clip the shaped system. When we apply clipping to the shaped constellation, the amount of peak-power reduction decreases. Figure 7 shows the clipping probability curves for the unshaped and shaped systems. From the curves, we see that the amount of peak-power reduction at a clipping rate of 10^{-8} is about 2.1 dB for the 31-tone case and 2.0 dB for the 215-tone case. As the clipping rate decreases asymptotically to 0, the amount of peak-power reduction approaches those without clipping (4.8 and 5.2 dB).

5 Conclusion

We have introduced the constellation shaping approach in the peak-power reduction problem for DMT systems and design a spherically shaped constellation. As the size of the constellation for each channel becomes large, the peak-power reduction achievable by these constellations is about 3-6 dB without clipping. However, with clipping, the gain is reduced to about 2 dB. The hyper-spherical is one example of shaped constellations and is by no means optimal for the peak-power reduction. Recent discovery has shown that other choices of boundary may lead to even more reduction of peak power at the expense of higher computational complexity. [14], [15]

There are certain drawbacks of the shaping-based peak-power reduction methods proposed here. First, they require relatively large constellation size (and thus high data rate) to obtain significant reduction, which may limit their application to shorter loops in the DSL application. However, most DSL systems are designed with high data rate in mind. Moreover, we may use a shaped constellation with more points than required and apply coding to both reduce the data rate and increase the reliability of the system simultaneously. A second concern may be that the channels of the shaped constellation become correlated. We can apply coding on top of the shaped constellation to partially remedy this problem, as convolutional codes are applied on top of the 16-D shaped constellation in V.34 modems. Coding for DMT systems remains a topic for future research.

[4] If the experimental value for $M_{unshaped}$ is used instead, the asymptotical gain is 5.67 dB.

References

[1] A. Gatherer and M. Polley, "Controlling clipping probability in DMT transmission," in *Proceedings of the 31st Asilomar Conference on Signals, Systems, and Computers*, pp. 578–584, 1997.

[2] J. Tellado and J. Cioffi, "Peak power reduction for multicarrier transmission," in *Proceedings of International Symposium on Information Theory*, August 1998.

[3] D. J. G. Mestdagh and P. M. P. Spruyt, "A method to reduce the probability of clipping in dmt-based transceiver," vol. 44, pp. 1234–1238, October 1996.

[4] R. van Nee and A. de Wild, "Reducing the peak-to-average power ratio of OFDM," in *Proceedings of IEEE Vehicular Technology Conference*, vol. 3, pp. 2072–2076, 1998.

[5] J. A. Davis and J. Jedwab, "Peak-to-mean power control in OFDM, Golay complementary sequences, and Reed-Muller codes," *IEEE Transactions on Information Theory*, vol. 45, pp. 2397–2417, November 1999.

[6] K. Patterson, "Generalized Reed-Muller codes and power control in OFDM modulation," vol. 46, pp. 104–120, January 2000.

[7] D. L. Jones, "Peak power reduction in OFDM and DMT via active channel modification," in *Proceedings of Asilomar Conference*, 1999.

[8] J. Tellado and J. Cioffi, "Peak power reduction for multicarrier transmission," in *Proceedings CTMC, Globecom 1998*, November 1998.

[9] S. Muller and J. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *Electronics Letters*, vol. 33, pp. 368–369, February 1997.

[10] M. Friese, "Multicarrier modulation with low peak-to-average power ratio," *Electronics Letters*, vol. 32, pp. 713–714, April 1996.

[11] C. Tellambura, "Coding technique for reducing peak-to-average power ratio in OFDM," in *Proceedings of IEEE Global Telecommunications Conference*, vol. 5, pp. 2783–2787.

[12] G. D. Forney and L. Wei, "Multidimensional constellations - Part I: Introduction, figures of merit, and generalized cross constellation," *IEEE Journal on Selected Areas in Communications*, vol. 7, pp. 877–892, August 1989.

[13] J. H. Conway and N. J. A. Sloane, *Sphere Packings, Lattices and Groups.* Springer, 3rd ed., 1999.

[14] H. Kwok and D. Jones, "PAR reduction for hadamard transform-based ofdm," in *Proceedings of Computer and Information System Symposium*, 2000.

[15] H. Kwok and D. Jones, "PAR reduction via constellation shaping," in *Proceedings of International Symposium of Information Theory*, 2000.