Signal-to-noise ratio of an LVDT amplitude demodulator

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Introduction

Linear variable differential transformers (LVDTs) are used for measuring the position of linear actuators such as industrial hydraulic valves and aircraft control surfaces, among others, to perform precise control actions. The output of an LVDT is an amplitude-modulated sinusoid signal in which position information is embedded in the amplitude of the sine wave. Many techniques are used to perform amplitude demodulation and extract the amplitude information, and thus the position of the LVDT core.

This article presents an amplitude-demodulation technique based on a sigma-delta modulator and a digital-filtering scheme, which results in very good position measurement accuracy. Specifically, the architecture and implementation of determining the amplitude of the secondary-coil sine wave in sensor-signal conditioners is described. Finally, the formula for signal-to-noise ratio (SNR) is derived. This formula is then used to show the accuracy in position measurement.

LVDT characteristics

Position sensors based on the LVDT are commonly used to measure the position of moving components in machines, such as control valves used in agricultural and construction hydraulic systems, and control surfaces used in aircraft.[1] Typically, an LVDT sensor consists of primary coils, secondary coils, and a moving magnetic core. The goal of an LVDT sensor signal conditioner is to determine the position of the moving core.

The electrical functionality of an LVDT is similar to that of a transformer. The primary of the LVDT is excited using a high-frequency sine wave, and this sinusoidal voltage couples to the secondary coils via the moving low-permeability magnetic core. The amplitude of the secondary sine wave changes with the position of the moving core. Therefore, to determine the position of the core, the amplitude of the secondary signal voltage has to be determined.

Amplitude demodulation

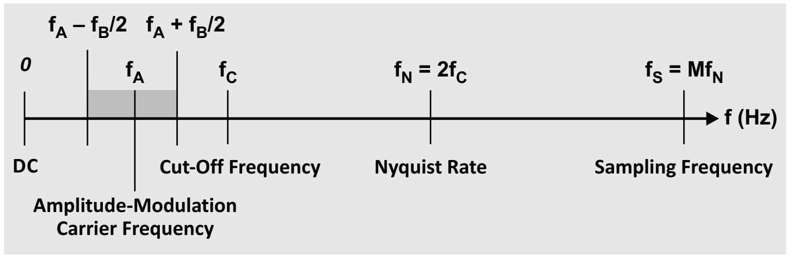
There are many amplitude demodulation techniques to determine the amplitude of a sine wave, including synchronous-to-the-carrier-signal techniques and asynchronous-to-the-carrier-signal techniques. A demodulation architecture based on an analog sigma-delta modulator followed by a digital filtering scheme is an asynchronous-to-the-carrier-signal architecture. The advantage of using this type of architecture for LVDT position measurement is that the measured amplitude is not influenced by the phase difference between the primary excitation signal and the secondary output signal.

Frequency variables

Figure 1 shows the frequency variables used in this article and are defined below.

- f_A—Carrier frequency of the amplitude modulation signal. In the case of LVDT, this corresponds to the primary coil excitation frequency
- f_B—Twice the maximum frequency of amplitude variation (or LVDT core position variation)
- f_C—Cut off frequency, which is maximum frequency of interest
- f_N—Nyquist rate, which is twice the cut of frequency
- f_S—Sampling frequency, which is M times the Nyquist rate. M is called the oversampling ratio

Figure 1. Definitions of various frequency variables



Architecture of a LVDT sensor signal conditioner

Reference 2 describes an architecture used to demodulate the signal from the LVDT secondary coils in order to extract the amplitude information. This amplitude information is then used to infer the position of the LVDT core. With this architecture, the LVDT secondary coil output is an amplitude-modulated signal that is represented by Equation 1:

$$x(t)=\sin(2\pi f_A t)+A_2\sin(2\pi f_x t)\times\sin(2\pi f_A t) \qquad (1)$$

where:

- f_A is the carrier frequency of the amplitude modulation signal. In the case of the LVDT, this corresponds to the primary coil excitation frequency.
- f_x is the frequency of amplitude variation. In the case of the LVDT, this corresponds to the frequency of core movement.
- f_B represents a frequency value that is twice the maximum frequency of amplitude variation. Note that $f_x < f_B / 2$.

Figure 2 shows a possible architecture using an analog-to-digital converter (ADC) to perform asynchronous-to-carrier-signal amplitude demodulation.

Using the average method described in Reference 2, the output of the low-pass filter of the signal conditioning circuit is given by Equation 2.

$$y_{LPF}(t)=LPF\left[y_R(t)\right]\cong\frac{2A_2}{\pi}\sin(2\pi f_x t) \qquad (2)$$

SNR with Nyquist ADC

The well known SNR formula for Nyquist N-bit ADCs can be modified based on Equation 2 to obtain Equation 3:

$$\begin{aligned}\text{SNR} &= 20\log_{10}\left(\frac{\text{RMS of Signal}}{\text{RMS of Quantization Noise}}\right)\\ &= 20\log_{10}\left(2^N\right)+20\log_{10}\left(\frac{2\sqrt{3}}{\pi\sqrt{2}}\right) \qquad (3)\\ &= 6.02\text{ N}-2.16\text{ dB}\end{aligned}$$

Equation 3 shows that the SNR of the amplitude-demodulation circuit is modified because the amplitude of the signal is scaled by pi.

LVDT signal conditioner with sigma-delta modulator ADC

Figure 3 shows a amplitude-demodulation scheme that uses a first-order sigma-delta modulator for the ADC in Figure 2. In this scheme, amplitude demodulation is performed by the rectifier modeled as multiplication of the bandpass filter (BPF) output and square wave (u).

Figure 2. Block diagram of a LVDT signal conditioner

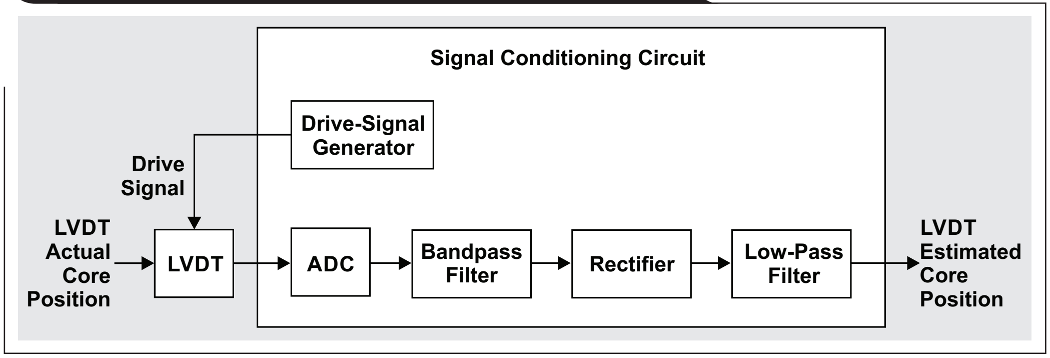
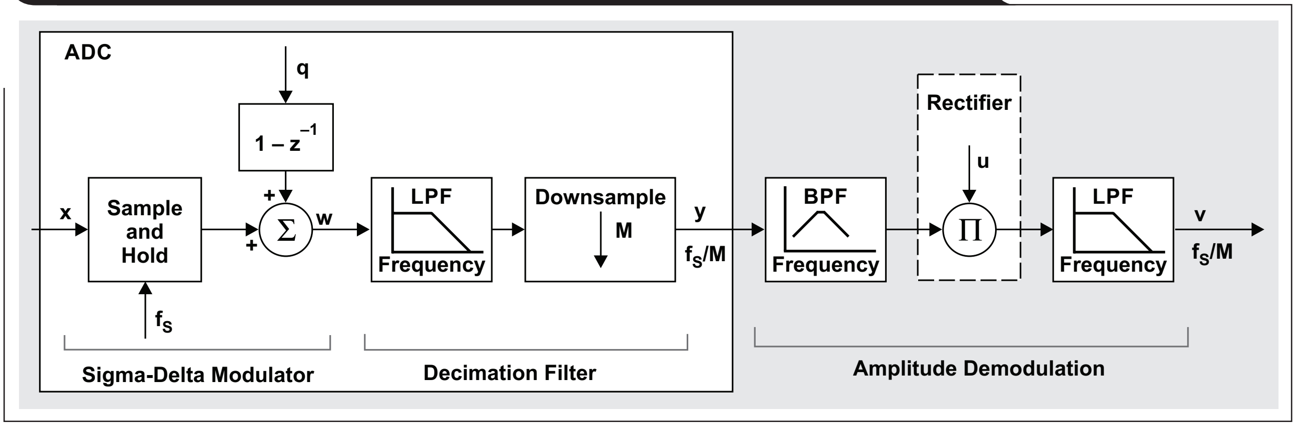


Figure 3. Amplitude-demodulation scheme based on a sigma-delta modulator



While the architecture shown in Figure 3 achieves the goal of amplitude demodulation, it has one clear limitation. Because the demodulation is performed in the decimated frequency domain, the primary excitation frequency has to be limited to frequencies below the decimation filter's cut-off frequency.

To overcome this limitation, the order of the blocks in Figure 3 are changed to the order shown in Figure 4. Notice the difference in the position of the bandpass filter between the two figures.

Note that the blocks can be rearranged because Figure 3 is a linear architecture. This rearranged architecture allows for higher primary excitation frequencies and it eliminates a low-pass filter in the signal chain. The result is a reduced size of the implementation. This is the type of architecture that is implemented in interface devices for LVDT sensors, such as the PGA970.

SNR with sigma-delta modulator ADC

The SNR of the amplitude demodulation technique described in Figure 4 directly affects the accuracy of the LVDT position measurement. One objective is to analyze the quantization noise of the architecture shown in Figure 4 and derive the expression for the SNR of this architecture.

Based on Figure 4, the output, w, of the sigma-delta modulator can be modeled in discrete-time domain by Equation 4:

$$w(z) = x(z) + \left(1 - z^{-1}\right) \times q(z) \qquad (4)$$

where z is the z-transform.

From Equation 4, the noise transfer function (NTF) can be inferred as $(1 - z^{-1})$ and the squared magnitude of the NTF is given by Equation 5:

$$\left|NTF(f)\right|^2 = \left|1 - e^{-\left(\frac{j2\pi f}{f_s}\right)}\right|^2, \text{ where } -\frac{f_S}{2} < f < \frac{f_S}{2} \qquad (5)$$

Using trigonometric identities, Equation 5 can be rewritten as Equation 6:

$$\begin{aligned} \left|NTF(f)\right|^2 &= \left|1 - \cos\left(\frac{2\pi f}{f_S}\right) + j\sin\left(\frac{2\pi f}{f_S}\right)\right|^2 \\ &= 2\left[1 - \cos\left(\frac{2\pi f}{f_S}\right)\right], \text{ where } -\frac{f_S}{2} < f < \frac{f_S}{2} \end{aligned} \qquad (6)$$

The quantization noise variance in the region of interest, assuming the bandpass filter is a "brick-wall" filter, is given by Equation 7:

$$\text{Noise Variance} = \frac{2V_{LSB}^2}{12 f_S} \times \int_{f_A - \frac{f_B}{2}}^{f_A + \frac{f_B}{2}} \left[1 - \cos\left(\frac{2\pi f}{f_S}\right)\right] df \qquad (7)$$

where V_{LSB} is the quantization noise.

The integral value of the noise variance in Equation 7 is given by Equation 8:

$$\frac{2V_{LSB}^2}{12 f_S}\left\{ f_B - \frac{f_s}{2\pi}\left[\sin\left(\frac{2\pi\left(f_A + \frac{f_B}{2}\right)}{f_S}\right) - \sin\left(\frac{2\pi\left(f_A - \frac{f_B}{2}\right)}{f_S}\right)\right]\right\} \qquad (8)$$

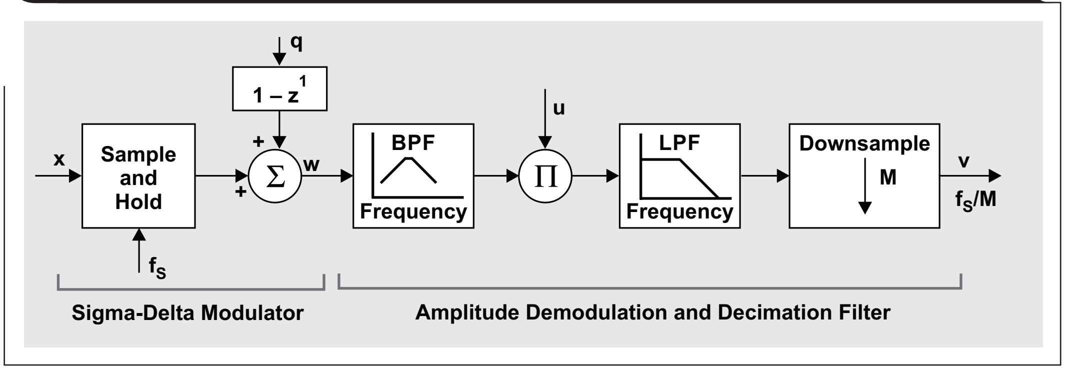
After some algebraic manipulations, the noise variance for the architecture shown in Figure 4 can be simplified to the value shown in Equation 9:

$$\text{Noise Variance} = 4 \times \frac{V_{LSB}^2}{12} \times \frac{f_B}{f_S}\left[\sin\left(\frac{\pi f_A}{f_S}\right)\right]^2 \qquad (9)$$

Based on the noise expression given in Equation 9, we draw the following intuitive conclusions:

1. The smaller the ratio of f_A / f_S, the smaller is the noise variance. This implies that if the primary excitation frequency is smaller than the secondary sampling frequency, the smaller will be the noise variance. This is naturally true because of the quantization noise shaping performed by the sigma-delta modulator.

Figure 4. Linear equivalent of amplitude demodulation scheme shown in Figure 3



2. The noise variance is linearly proportional to f_B. This is intuitively true because the smaller the quantization-noise integration range, the smaller is the noise variance.

Based on Equation 9 and expression for SNR given by Equation 3, the SNR of the architecture shown in Figure 4 is given by Equation 10:

$$\begin{aligned}\text{SNR} &= 6.02\ \text{N} - 2.16\ \text{dB} - 20\log_{10}(2) \\ &\quad - 20\log_{10}\left[\sqrt{\frac{f_B}{f_S}}\sin\left(\frac{\pi f_A}{f_S}\right)\right] \\ &= 6.02\ \text{N} - 8.18\ \text{dB} - 20\log_{10}\left[\sqrt{\frac{f_B}{f_S}}\sin\left(\frac{\pi f_A}{f_S}\right)\right]\end{aligned} \tag{10}$$

Effect of an AFE

The total noise of the analog front-end (AFE), which could include flicker and thermal noise, can be treated as an additional noise source in the analysis.[3] Because of the downstream bandpass filter in the signal chain, the noise requirements on the AFE can be relaxed to some level.

Example

Consider an LVDT that has a displacement range of 10 cm. Assume that this linear displacement produces a secondary amplitude of 1 V. Next, assume that the AFE, including the gain resistors, has an equivalent input referred noise of e_{IN} of 1 μV_{RMS} in a 20-Hz bandwidth and a gain setting of 1 V/V.

Also for this example, assume a 3-bit quantizer with f_S = 1 MHz, f_A = 5 kHz, and f_B = 20 Hz. Based on Equation 10, the achievable SNR of the ADC is 93 dB. Assuming that the ADC full-scale voltage is 1 V, the equivalent quantization noise for the ADC is:

$$e_{ADC} = \frac{\tfrac{1}{2}\sqrt{2}^{\,V}}{10^{93/20}} = 8\ \mu V_{RMS} \tag{11}$$

Therefore, the total system noise is given by:

$$e_{sys} = \sqrt{e_{in}^2 + e_{ADC}^2} = 8.03\ \mu V_{RMS} \tag{12}$$

This implies that the system-level SNR is:

$$\text{SNR}_{sys} = 20\log_{10}\frac{\tfrac{1}{2}\sqrt{2}^{\,V}}{8.03\ \mu V_{RMS}} = 92.9\ \text{dB} \tag{13}$$

Therefore, the effective resolution is:

$$\text{Effective Resolution} = \frac{\text{SNR}_{sys}/20}{\log_{10}2} \cong 15.4\ \text{bits} \tag{14}$$

Using this resolution of 15.4 bits, the LVDT position measurement accuracy is 10 cm / $2^{15.4}$ = 23 µm.

Conclusions

The efficient amplitude-demodulation scheme described in this article shows that designers can easily fine tune bandpass filter parameters for improved flexibility and increased performance. This implementation yields an amplitude resolution of greater than 15 bits for amplitude-demodulation applications. Rather than using the traditional approach of combining standalone analog and digital solutions that typically are developed in isolation, the architecture described in this article is a state-of-the-art, end-to-end solution developed at Texas Instrument.

References

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