Analytic Expressions for currents in the CCM PFC stage

Colin Gillmor, HVPS

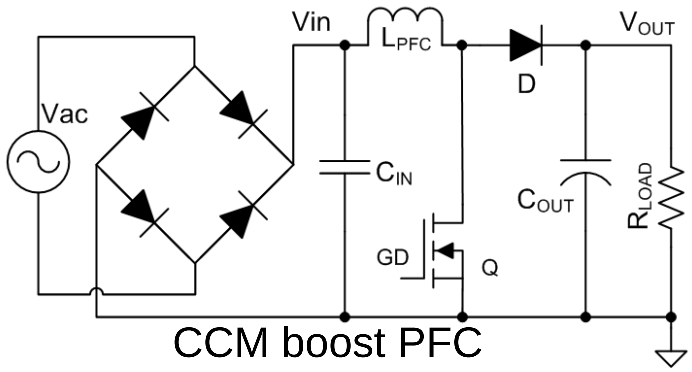
SLYY131

Agenda

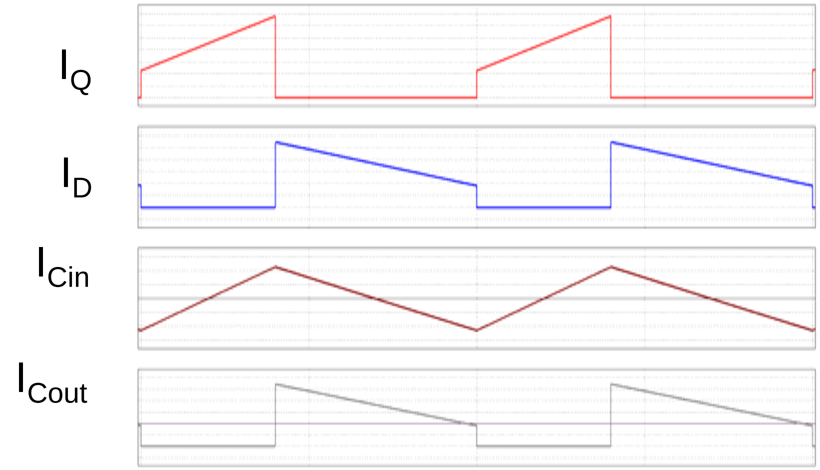
- CCM Boost PFC stage with current waveforms
  - RMS calculation
  - RMS current expressions, Diode, MOSFET, C_{OUT}, C_{IN}
- Two Phase Interleaved CCM Boost PFC
  - RMS current expressions, Diode, MOSFET
  - RMS current calculation, C_{OUT}
  - RMS current expression, C_{OUT}
- Results
- Conclusions
- References

Note: currents are rms unless otherwise stated

CCM Boost PFC Waveforms



CCM boost PFC



Typical currents over two switching cycles

Currents are functions of –

Parameters: L_{PFC}, F_{sw}, V_{OUT}, I_{OUT} ← Assumed to be constant:

Variables: Vac, Duty Cycle (depends on Line phase angle)

Analytic Expressions for currents in the CCM PFC stage

CCM Boost PFC Waveforms

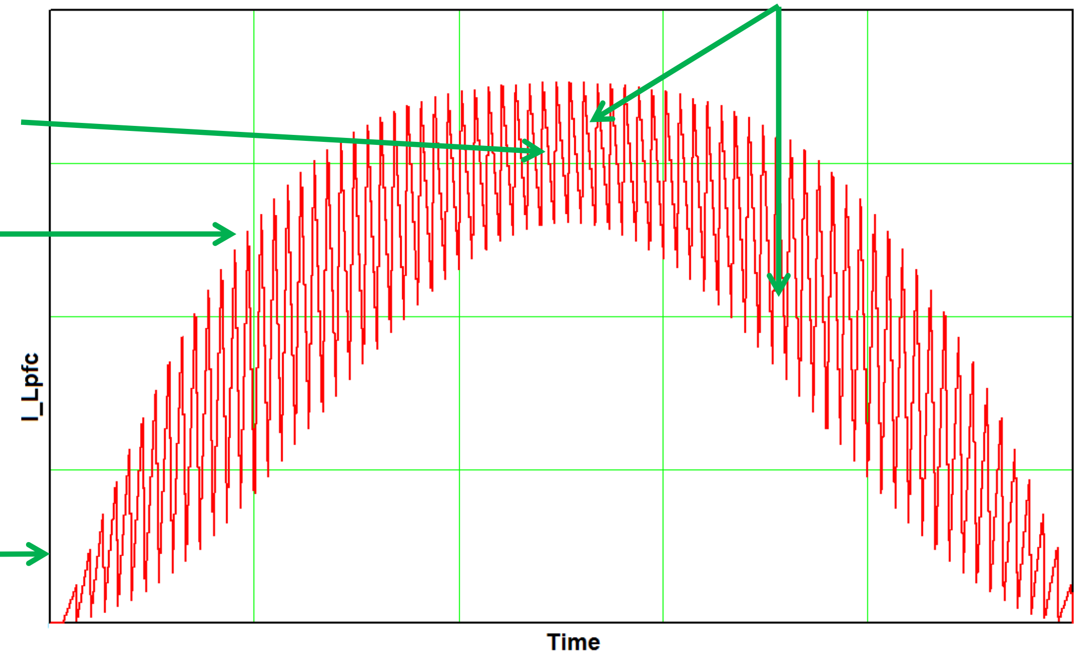
Peak to Peak Ripple current varies through the line cycle

Duty Cycle varies

Ripple reduces as Vin reaches peak

Max ripple at Vin = 50% Vout

Min ripple at low Vin



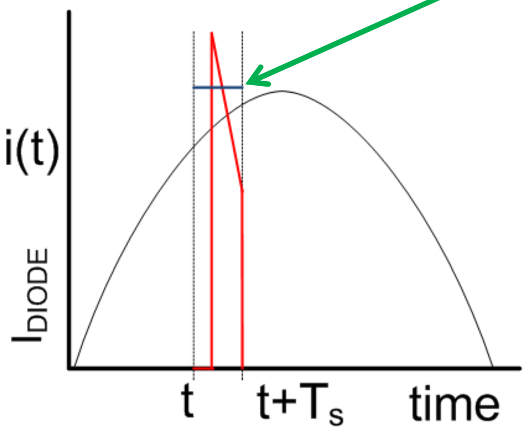
How to calculate the RMS value ?

Typical Inductor current over half a line cycle

RMS Calculation – PFC Diode Current

$I^2(t)$ is first averaged over a switching cycle then averaged over the AC line period

1/ The Mean Square (MS) current in each switching cycle is given by



$$\left[i^2(t)\right]_{Ts} = \frac{1}{T_s} \cdot \int_t^{t+T_s} i^2(t)\, dt$$

2/ Integrate the MS current over a half line cycle, average it and take the square root

$$I_{rms} = \frac{2}{T_{ac}} \cdot \sqrt{\int_0^{\frac{T_{ac}}{2}} \left[i^2(t)\right]_{Ts} dt}$$

RMS Currents, MOSFET and Diode

$$I_Q = \frac{\sqrt{2}\cdot P_{in}}{V_{in\_pk}}\cdot\sqrt{1-\frac{8}{3\pi}\cdot\frac{V_{in\_pk}}{V_{out}}+\frac{T_s^{\,2}\cdot V_{in\_pk}^{\,4}}{48L_{pfc}^{\,2}\cdot P_{in}^{\,2}}\cdot\left(1-\frac{8}{\pi}\cdot\frac{V_{in\_pk}}{V_{out}}+\frac{9}{4}\frac{V_{in\_pk}^{\,2}}{V_{out}^{\,2}}-\frac{32}{15\cdot\pi}\cdot\frac{V_{in\_pk}^{\,3}}{V_{out}^{\,3}}\right)}$$

$$I_D = \sqrt{\frac{16}{3\pi}\cdot\frac{P_{in}^{\,2}}{V_{pk}\cdot V_{out}}+\frac{1}{9\cdot\pi}\cdot\frac{T_s^{\,2}\cdot V_{pk}^{\,3}}{L_{pfc}^{\,2}\cdot V_{out}}-\frac{1}{16}\cdot\frac{T_s^{\,2}\cdot V_{pk}^{\,4}}{L_{pfc}^{\,2}\cdot V_{out}^{\,2}}+\frac{4}{45\cdot\pi}\cdot\frac{T_s^{\,2}\cdot V_{pk}^{\,5}}{L_{pfc}^{\,2}\cdot V_{out}^{\,3}}}$$

and then:

$$I_{LPFC} = \sqrt{I_Q^{\,2}+I_D^{\,2}}$$

$$I_{Cin} = \sqrt{I_{LPFC}^{\,2}-I_{ac}^{\,2}}$$

$$I_{Cout} = \sqrt{I_D^{\,2}-I_{Out}^{\,2}}$$

Erickson and Maksimovic
give the part in red (Ref 1)

Analytic Expressions for currents in the CCM PFC stage

RMS Currents, C_{OUT}

The total current in C_{OUT} has two components.

A LF component at twice line frequency Ref (3)

$$I_{Cout\_LF} = \frac{P_{in}}{\eta \cdot \sqrt{2} \cdot V_{out}}$$

A HF component at the switching frequency and its harmonics. This current is I_{Diode} – AVG(I_{Diode}) – the calculation will be outlined later (slide 16)

$$I_{Cout\_HF\_1Ph} := \sqrt{\frac{16}{3\cdot\pi}\cdot\frac{P_{in}^{2}}{V_{in\_pk}\cdot V_{out}} - \frac{3}{8}\cdot\frac{4\cdot P_{in}^{2}}{V_{out}^{2}} + \frac{4}{3\cdot\pi}\cdot\frac{T_s^{2}\cdot V_{in\_pk}^{3}}{12\cdot L_{pfc}^{2}\cdot V_{out}} - \frac{3}{8}\cdot\frac{T_s^{2}\cdot V_{in\_pk}^{4}}{6\cdot L_{pfc}^{2}\cdot V_{out}^{2}} + \frac{16}{15\cdot\pi}\cdot\frac{T_s^{2}\cdot V_{in\_pk}^{5}}{12\cdot L_{pfc}^{2}\cdot V_{out}^{3}}}$$

The total current in C_{OUT} is then

$$I_{Cout} = \sqrt{I_{Cout\_HF}^{2} + I_{Cout\_LF}^{2}}$$

RMS Currents, C_{IN}

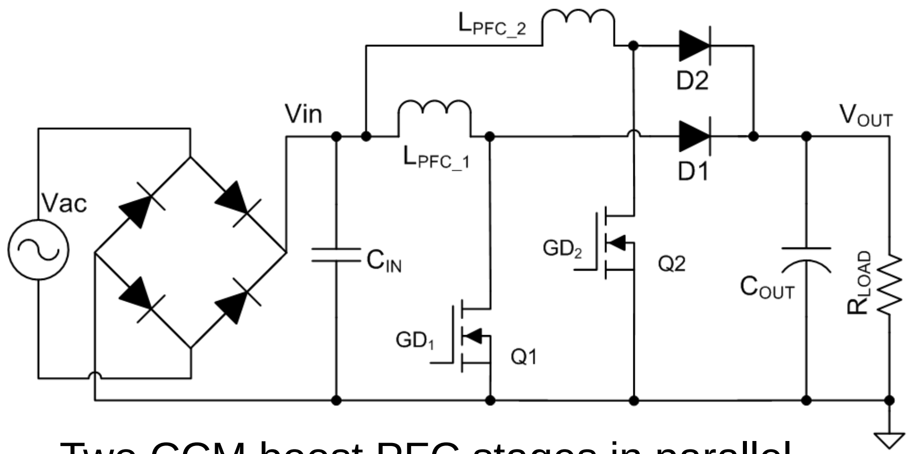
The total ripple current in C_{IN} is

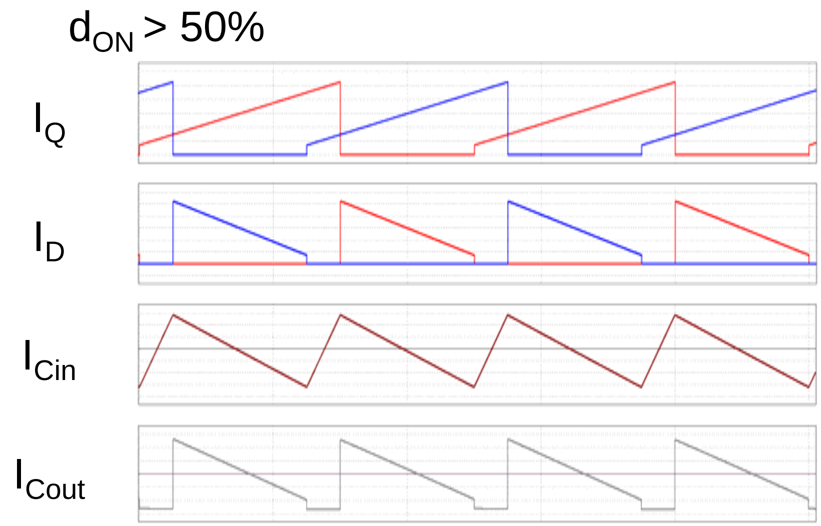
$$I_{Cin} = \sqrt{I_{LPFC}^{2} - I_{ac}^{2}}$$

C_{IN} is usually small and there is effectively no line frequency current in this capacitor

The RMS current in this capacitor is high frequency only.

2 Phase Interleaved boost PFC





Two CCM boost PFC stages in parallel

- Each carries half the power
- 180° phase shift between GD_1 and GD_2
- $L_{PFC\_1} = L_{PFC\_2} = 2*L_{PFC}$ (for equivalent proportion of inductor ripple current)

I_{Cout} Ripple at twice line frequency is unaffected by interleaving

HF ripple is at twice the switching frequency

Switching frequency ripple reduction in C_{IN} and C_{OUT} is a function of the duty cycle

Analytic Expressions for currents in the CCM PFC stage

RMS Currents, 2 Phase, MOSFET and Diode

Calculation for MOSFET and diode currents is the same as in the 1 Phase case

The input power is shared between the two phases so P_{in} is replaced by $P_{in}/2$

There is no ripple current cancellation in the MOSFET or Diodes

$$I_Q = \frac{\sqrt{2}\cdot\frac{P_{in}}{2}}{V_{in\_pk}}\cdot\sqrt{1 - \frac{8}{3\pi}\cdot\frac{V_{in\_pk}}{V_{out}} + \frac{T_s^{\,2}\cdot V_{in\_pk}^{\,4}}{48L_{pfc}^{\,2}\cdot P_{in}^{\,2}}\cdot\left(1 - \frac{8}{\pi}\cdot\frac{V_{in\_pk}}{V_{out}} + \frac{9}{4}\,\frac{V_{in\_pk}^{\,2}}{V_{out}^{\,2}} - \frac{32}{15\cdot\pi}\cdot\frac{V_{in\_pk}^{\,3}}{V_{out}^{\,3}}\right)}$$

$$I_D = \sqrt{\frac{16}{3\pi}\cdot\frac{\left(\frac{P_{in}}{2}\right)^2}{V_{pk}\cdot V_{out}} + \frac{1}{9\cdot\pi}\cdot\frac{T_s^{\,2}\cdot V_{pk}^{\,3}}{L_{pfc}^{\,2}\cdot V_{out}} - \frac{1}{16}\cdot\frac{T_s^{\,2}\cdot V_{pk}^{\,4}}{L_{pfc}^{\,2}\cdot V_{out}^{\,2}} + \frac{4}{45\cdot\pi}\cdot\frac{T_s^{\,2}\cdot V_{pk}^{\,5}}{L_{pfc}^{\,2}\cdot V_{out}^{\,3}}}$$

2 Phase, I_{Cout_rms} Calculation Outline

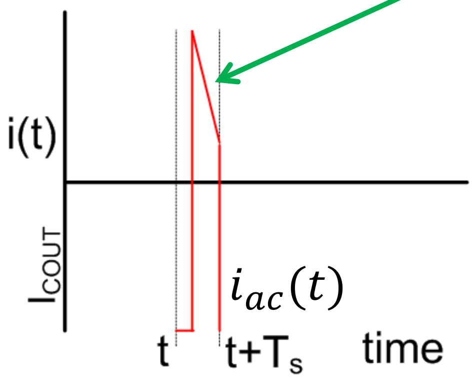
$$I_{Cout\_LF} = \frac{P_{in}}{\eta \cdot \sqrt{2} \cdot V_{out}}$$

- The LF current ripple in C_{OUT} is unchanged by interleaving
- Calculate the 1 Ph HF current in C_{OUT}
- Determine a correction factor ($K_{MS}(t)$)
- Apply the correction factor to the 1 Ph HF solution to get the 2 Ph HF solution
- Add the 2 Ph HF current to the LF current to get the total

RMS Calculation – Calculate the HF ripple in C_{out}

$I^2(t)$ is first averaged over a switching cycle then averaged over the AC line period

The Mean Square (MS) current in C_{OUT} in each switching cycle is given by



$$i_{avg}(t) = i_{ac}(t) \cdot d_{off}(t)$$

$$\left[i^2(t)\right]_{Ts} = K_{MS}(t) \cdot \frac{1}{T_s} \cdot \int_t^{t+T_s} i^2(t)\, dt$$

Integrate the MS current over a half line cycle, average it and take the square root

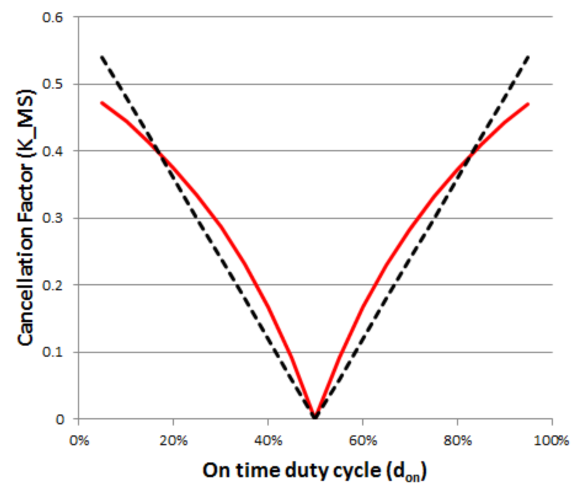
$$I_{rms} = \sqrt{\frac{2}{T_{ac}} \cdot \int_0^{\frac{T_{ac}}{2}} \left[i^2(t)\right]_{Ts} dt}$$

Correction Factor

1 Ph and 2 Ph I_{cout} currents simulated (PSIM)
Mean Square values taken across d_{on} range

Plot $\frac{I_{2Ph}}{I_{1Ph}}$ results against d_{on}

Linearise $K_{MS}(d_{on}) = md\ d_{on} + cd$



$md = 1.2$ $cd = -0.6$ If $d_{on} > 50\%$

$md = -1.2$ $cd = 0.6$ If $d_{on} < 50\%$

Duty cycle d_{on} is a function of time
Restate in terms of t

$$d_{on}(t) = 1 - \frac{V_{pk}}{V_{out}} \cdot \sin(\omega \cdot t)$$

$$K_{MS}(t) = mt\ \frac{V_{in\_pk}}{V_{out}} \sin(\omega t) + ct$$

$mt = -1.2$ $ct = 0.6$ If $d_{on} > 50\%$

$mt = 1.2$ $ct = -0.6$ If $d_{on} < 50\%$

C_{OUT} – Switching Cycle HF Mean Square Current

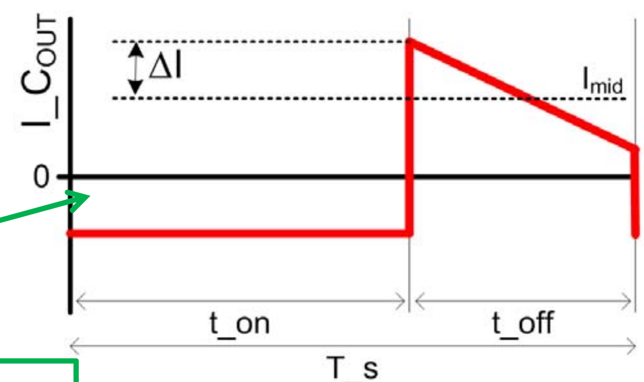
Diode and MOSFET currents are trapezoidal

During Ton current is $-i_{avg}(t)$

$$I_{MS\_Trap} = d\cdot I_{mid}^2 + \frac{d}{3}\cdot \Delta I(t)^2$$

During Toff current is $i_{ac}(t) - i_{avg}(t)$

So
$$\left[i(t)^2\right]_{Ts} = i_{avg}(t)^2\cdot\left(1 - d_{off}\right) + d_{off}\cdot i_{mid}(t)^2 + \frac{d_{off}}{3}\cdot\Delta i(t)^2$$



Multiply by $K_{MS}(t)$ for the interleaved case

$$\left[i_{Cout}{}^2(t)\right]_{Ts} = K_{MS}(t)\cdot\left[\left(i_{avg}(t)\right)^2\cdot\left(1 - d_{off}(t)\right) + d_{off}(t)\cdot\left(i_{mid}(t)\right)^2 + \frac{d_{off}(t)}{3}\cdot\Delta i(t)^2\right]$$

Now we need expressions for the functions of t in this equation

Substitutions

- We have the following

$$d_{off}(t) = \frac{V_{in\_pk}}{V_{out}} \cdot \sin(\omega \cdot t)$$

$$\Delta i\,(t) = \frac{T_s \cdot V_{in\_pk}}{2 \cdot L_{pfc}} \cdot \left(1 - \frac{V_{in\_pk}}{V_{out}} \cdot \sin(\omega \cdot t)\right) \cdot \sin(\omega \cdot t)$$

$$i_{ac}(t) = \frac{\sqrt{2} \cdot P_{in}}{V_{in\_pk}} \cdot \sin(\omega \cdot t)$$

$$I_{mid}(t) = I_{ac}(t) - I_{avg}(t)$$

$$K_{MS}(t) = mt \cdot \frac{V_{pk}}{V_{out}} \cdot \sin(\omega \cdot t) + ct$$

C_{OUT} – Switching Cycle HF Mean Square Current

$$\left[i_{Cout}{}^2(t) \right]_{Ts} = K_{MS}(t) \cdot \left[\left(i_{avg}(t) \right)^2 \cdot \left(1 - d_{off}(t) \right) + d_{off}(t) \cdot \left(i_{mid}(t) \right)^2 + \frac{d_{off}(t)}{3} \cdot \Delta i(t)^2 \right]$$

Set $K_{MS}(t)$ to 1 for the 1Ph solution

- Making the substitutions and simplifying gives

$$\left[i_{Cout}{}^2(t) \right] = mt \cdot \frac{4 P_{in}{}^2}{V_{out}{}^2} \cdot \left(\sin(\omega \cdot t)^4 - \frac{V_{pk} \cdot \sin(\omega \cdot t)^5}{V_{out}{}^2} \right) + ct \cdot \frac{4 \cdot P_{in}{}^2}{V_{out}} \cdot \left(\frac{\sin(\omega \cdot t)^3}{V_{pk}} - \frac{\sin(\omega \cdot t)^4}{V_{out}} \right)$$

- Integrating gives the rms value

$$I_{RMS} = \sqrt{\frac{2}{T_{AC}} \int_0^{\frac{T_{AC}}{2}} [i^2(t)]_{T_{SW}} \, dt}$$

$$I_{Cout\_HF\_2Ph} = \sqrt{mt \cdot \left(\frac{3}{2} \cdot \frac{P_{in}{}^2}{V_{out}{}^2} - \frac{64}{15 \cdot \pi} \cdot \frac{P_{in}{}^2 \cdot V_{in\_pk}}{V_{out}{}^3} \right) + ct \cdot \left(\frac{16}{3 \cdot \pi} \cdot \frac{P_{in}{}^2}{V_{in\_pk} \cdot V_{out}} - \frac{3}{2} \cdot \frac{P_{in}{}^2}{V_{out}{}^2} \right)}$$

C_{OUT} – RMS Current

We have

$$I_{Cout\_HF\_2Ph} = \sqrt{mt \cdot \left(\frac{3}{2} \cdot \frac{P_{in}^{2}}{V_{out}^{2}} - \frac{64}{15 \cdot \pi} \cdot \frac{P_{in}^{2} \cdot V_{in\_pk}}{V_{out}^{3}} \right) + ct \cdot \left(\frac{16}{3 \cdot \pi} \cdot \frac{P_{in}^{2}}{V_{in\_pk} \cdot V_{out}} - \frac{3}{2} \cdot \frac{P_{in}^{2}}{V_{out}^{2}} \right)}$$

And $$I_{Cout\_LF} = \frac{P_{in}}{\eta \cdot \sqrt{2} \cdot V_{out}}$$

So

$$I_{Cout\_2Ph} = \sqrt{I_{Cout\_HF}^{2} + I_{Cout\_LF}^{2}}$$

Valid for $V_{in\_pk} <$ Vout/2

If $V_{in\_pk} >$ Vout/2 then the MS calculation and the integral have to be split.
One for 0 < Vin < Vout/2
One for Vout/2 < Vin < Vout/2
And the results added

Results*

Taken on a Texas Instruments UCC28070 EVM - 2Ph interleaved CCM Boost PFC

ICout_2Ph				
Vin	Pin	Meas, mA	Calc, mA	Error
120V	300W	817	732	10.4%
120V	200W	702	610	13.1%
120V	100W	595	488	18.0%
90V	300W	976	946	3.1%
90V	200W	821	788	4.0%
90V	100W	684	631	7.7%

IQ_2Ph				
Vin	Pin	Meas, mA	Calc, mA	Error
120V	300W	1185	1170	1.3%
120V	200W	1029	1035	0.6%
120V	100W	880	910	3.4%
90V	300W	1581	1525	3.5%
90V	200W	1355	1309	3.4%
90V	100W	1128	1102	2.3%

C_{OUT}, Ripple reduction example

Icout (Calc, mA)			
	1Ph	2Ph	Ratio
300W, 120V	1407	732	52%
300W, 90V	1633	946	58%

ID_2Ph				
V_in	Pin	Meas, mA	Calc, mA	Error
120V	300W	788	895	13.6%
120V	200W	670	788	17.6%
120V	100W	555	690	24.3%
90V	300W	880	949	7.8%
90V	200W	741	814	9.9%
90V	100W	602	684	13.6%

* With thanks to Sonal Singh for taking the measurements

Conclusions

- Equations for the RMS currents in the CCM boost PFC developed and presented
  - Diode
  - MOSFET
  - Input Capacitor
  - Output Capacitor
- Significant reduction in RMS currents in the interleaved CCM Boost PFC
- Output capacitor High Frequency ripple current significantly reduced by interleaving
- Output capacitor Low Frequency ripple current unaffected by interleaving
- Output capacitor total ripple current reduced by interleaving
- Good agreement with experimental results

References

Ref (1): 'Fundamentals of Power Electronics, Erickson and Maksimovic; Springer 2001, Table 18.3, summary of rectifier current stresses.

Ref (2): SLUP279 An Interleaving PFC Pre-Regulator for High-Power Converters. Mike O'Loughlin

Ref (3): Capacitor Ripple current in an interleaved PFC converter, Pratt and Jinsong,

IEEE transactions on Power Electronics, Vol 24, No 6 June 2009.

UCC28070: http://www.ti.com/product/UCC28070

UCC28180: http://www.ti.com/product/UCC28180

Useful Integrals

$$\frac{1}{\pi}\cdot\int_0^{\pi} \sin(\theta)\, d\theta = \frac{2}{\pi} \qquad \frac{1}{\pi}\cdot\int_0^{\pi} (\sin(\theta))^2\, d\theta = \frac{1}{2} \qquad \frac{1}{\pi}\cdot\int_0^{\pi} (\sin(\theta))^3\, d\theta = \frac{4}{3\cdot\pi}$$

$$\frac{1}{\pi}\cdot\int_0^{\pi} (\sin(\theta))^4\, d\theta = \frac{3}{8} \qquad \frac{1}{\pi}\cdot\int_0^{\pi} (\sin(\theta))^5\, d\theta = \frac{16}{15\cdot\pi} \qquad \frac{1}{\pi}\cdot\int_0^{\pi} (\sin(\theta))^6\, d\theta = \frac{15}{48}$$

Thank You

Colin Gillmor, HVPS