

Compensation Methodology for Error in Sallen-Key Low-Pass Filter, Caused by Limited Gain-Bandwidth of Operational Amplifiers

Vito Shen, Thomas Kuehl

ABSTRACT

Analog active filters are essential signal-processing circuits, widely used to modify the frequency spectrum of an analog signal. The Sallen-Key (SK) filter is a widely applied topology used to realize a low-pass filter. During the design of the SK filter, the active element is always modeled as an ideal operational amplifier (op amp), however all real op amps have a limited gain-bandwidth (GBW) product. The limited bandwidth of the op amp introduces errors into the amplitude and phase responses of the filter.

To diminish these errors, the normal solution is to choose an op amp with a GBW of more than 100x the –3 dB bandwidth of the filter. Only by choosing an op amp with a high GBW, relative to the bandwidth of the filter, can it be treated as an ideal op amp. However, the cost of a high-GBW op amp may significantly increase the cost of the filter. For some applications, the GBW requirement can be so high that it can be difficult to identify a suitable, cost-effective op amp.

This application report provides a tutorial description of the error analysis for the SK active low-pass filter. Additionally, the report introduces a new compensation method to reduce the errors caused by the limited GBW of an op amp. Simulation and test data, which demonstrate the performance achieved with this compensation technique are presented.

Contents

1	Introduction	2
2	Ideal Transfer Function and Error Analysis	2
3	Simulation and Test Results.....	5
4	Conclusions.....	8
5	References	8

List of Figures

1	Low-Pass Sallen-Key Architecture	2
2	Modified Sallen-Key Topology Incorporating Op Amp GBW Compensation	4
3	Schematic for 300-kHz, Sallen-Key, Low-Pass Filter	5
4	TINA Spice AC Simulations for Ideal Op Amp and TL081	6
5	Modified Schematic for 300-kHz Sallen-Key Low-Pass Filter	6
6	TINA Spice AC Simulations of Sallen-Key Filter Using Ideal Op Amp, Standard TL081 Circuit and Compensated TL081 Circuit	7
7	S21 for Modified Sallen-Key Filter Using TL081.....	7

Trademarks

FilterPro is a trademark of Texas Instruments.
 WEBENCH is a registered trademark of Texas Instruments.
 All other trademarks are the property of their respective owners.

1 Introduction

The Sallen-Key (SK) topology is an electronic filter topology used to implement a second-order response. The topology is particularly valued for its simplicity. The SK is a noninverting filter topology, which can make it preferable over the Multiple Feed Back (MFB) filter, which is an inverting topology. However, this is not the only potential advantage. The SK topology may be a better choice if the following apply:

- Gain accuracy is important.
- A unity-gain filter is needed.
- The pole-pair Q is low (for example, $Q < 3$).

Specific filter performance can be achieved by cascading one or more SK stages. The poles of the SK are sensitive to the circuit elements, especially to the GBW of the op amp.

This application report develops the ideal SK low-pass transfer function, and then analyzes the frequency response error contributed by the limited bandwidth of the op amp. The report then introduces a new compensation methodology to reduce the error caused by the limited gain-bandwidth. Lastly, a real-case example of an SK low-pass filter is provided. Simulation and testing results confirm the effectiveness of this new compensation method.

2 Ideal Transfer Function and Error Analysis

Figure 1 shows a circuit diagram for low-pass SK architecture.

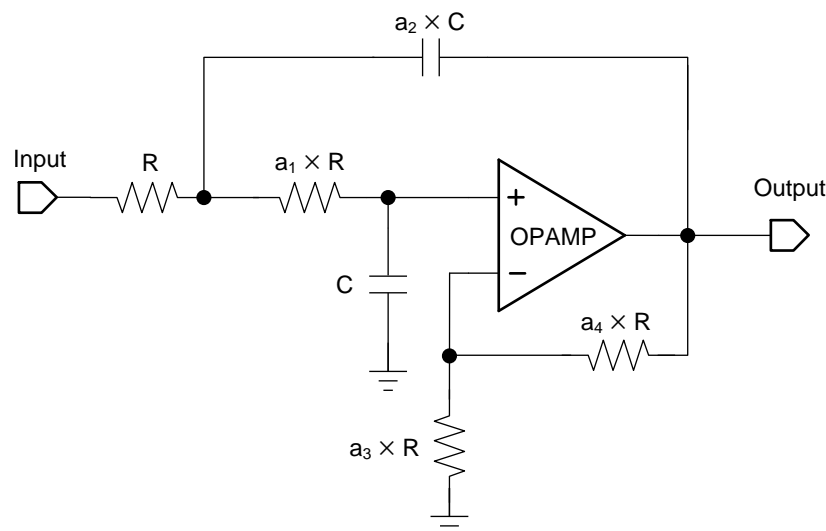


Figure 1. Low-Pass Sallen-Key Architecture

Equation 1 provides the Laplace transfer function for the circuit of Figure 1.

$$F(s) = \frac{a_3 + a_4}{a_3} \frac{1}{s^2 \times (RC)^2 \times a_1 a_2 + s \times RC \left(1 + a_1 - \frac{a_2 a_4}{a_3} \right) + 1} \quad (1)$$

DC gain (where $s = 0$, see Equation 2):

$$A_v(\text{DC}) = \frac{a_3 + a_4}{a_3} \quad (2)$$

Characteristic frequency (see Equation 3):

$$\omega_0 = \frac{1}{\sqrt{a_1 a_2}} \times \frac{1}{RC} \quad (3)$$

The quality factor (see [Equation 4](#)):

$$Q = \frac{\sqrt{a_1 a_2}}{1 + a_1 - \frac{a_2 a_4}{a_3}} \quad (4)$$

The ideal Laplace transfer function of the SK can also be expressed as shown in [Equation 5](#):

$$F(s) = \frac{\frac{a_3 + a_4}{a_3}}{s^2 \times \left(\frac{1}{\omega_0}\right)^2 + s \times \frac{1}{\omega_0 Q} + 1} \quad (5)$$

When the filter performance requirements have been established, the resistance and capacitance values are determined by calculations or by using filter synthesis software. In the real-filter case, the op amps used to construct the active filter are not ideal. These op amps have limited GBW which alters the response of the filter, thereby causing it to deviate from the ideal.

If the limited GBW of the op amp is considered, the frequency response can be modeled as a 1-pole system.

[Equation 6](#) shows the Laplace transfer function for an actual op amp.

$$A(s) = \frac{GBW}{s} \quad (6)$$

NOTE: GBW is specified in radians.

When the transfer function of the op amps has been introduced into the transfer function of the SK, the equation becomes more complex (see [Equation 7](#) and [Equation 8](#)).

$$F(s) = \frac{\frac{a_3 + a_4}{a_3}}{b_3 s^3 + b_2 s^2 + b_1 s + b_0} \quad (7)$$

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 1 \\ CRa_1 + CR - \frac{CRa_2 a_4}{a_3} - \frac{1}{GBW} - \frac{a_4}{GBW a_3} \\ C^2 R^2 a_1 a_2 - \frac{CR}{GBW} - \frac{CRa_1}{GBW} - \frac{CRa_2}{GBW} - \frac{CRa_4}{GBW a_3} - \frac{CRa_1 a_4}{GBW a_3} - \frac{CRa_2 a_4}{GBW a_3} \\ \frac{C^2 R^2 a_1 a_2}{GBW} - \frac{C^2 R^2 a_1 a_2 a_4}{GBW a_3} \end{pmatrix} \quad (8)$$

This result indicates that when the transfer function of the op amp has been included, the filter order increases to that of a third-order response. Additionally, the quality factor (Q) and the characteristic frequency of the filter have been affected by the GBW of the op amp. These changes alter the response of the filter.

It has been determined that the response errors caused by the limited-GBW op amp can be effectively compensated for by adding a resistor, R_{comp} , in series with the capacitor, C.

Figure 2 shows the resulting revised SK topology.

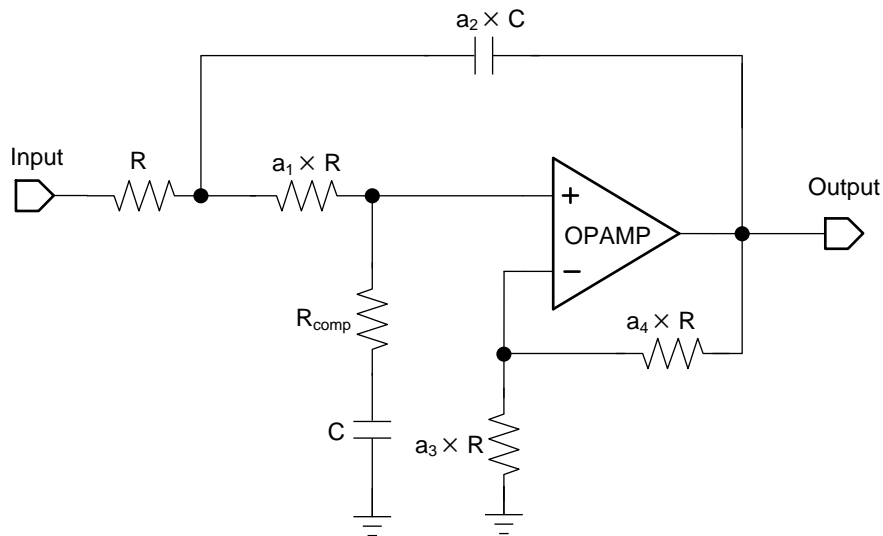


Figure 2. Modified Sallen-Key Topology Incorporating Op Amp GBW Compensation

The R_{comp} value is selected so that it, combined with capacitor C, results in a corner frequency equal to the op amp GBW multiplied by the DC feedback factor (see Equation 9):

$$R_{\text{comp}} = \frac{1}{C \frac{a_3}{a_3 + a_4} \text{GBW}} \quad (9)$$

When R_{comp} is added to the standard SK topology, the transfer function can be simplified, as shown in Equation 10 and Equation 11.

$$F(s) = \frac{a_3 + a_4}{a_3} \frac{a_3}{b_2 s^2 + b_1 s + b_0} \quad (10)$$

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{a_4}{a_3 \text{GBW}} + \frac{1}{\text{GBW}} + \text{CR}a_1 + \text{CR} - \frac{\text{CR}a_2 a_4}{a_3} \\ \frac{\text{CR}a_2 a_4}{a_3 \text{GBW}} + \frac{\text{CR}a_2}{\text{GBW}} + \text{C}^2 \text{R}^2 a_1 a_2 \end{pmatrix} \quad (11)$$

Characteristic frequency (see Equation 12):

$$\omega_0 = \frac{1}{\text{RC} \sqrt{\frac{a_2 a_4}{a_3 \text{CR} \times \text{GBW}} + \frac{a_2}{\text{CR} \times \text{GBW}} + a_1 a_2}} \quad (12)$$

The quality factor (see Equation 13):

$$Q = \frac{\sqrt{\frac{a_2 a_4}{a_3 \text{CR} \times \text{GBW}} + \frac{a_2}{\text{CR} \times \text{GBW}} + a_1 a_2}}{\frac{a_4}{a_3 \text{CR} \times \text{GBW}} + \frac{1}{\text{CR} \times \text{GBW}} + a_1 + 1 + \frac{a_2 a_4}{a_3}} \quad (13)$$

R_{comp} , in conjunction with capacitor C, adds a zero into the response that cancels the pole associated with the limited GBW of the op amp.

3 Simulation and Test Results

To verify the response of the revised SK topology, a filter with a second-order, Butterworth low-pass response, DC gain of 4 V/V (12 dB), and a 300-kHz, -3 dB bandwidth was selected. The Butterworth response has a Q factor of 0.71 and a characteristic frequency equal to the -3 dB bandwidth of the filter.

The response requirements were entered into the WEBENCH® Filter Designer tool from TI, which generates the resistor and capacitor component values. FilterPro™, from TI, could have been used as well. Both tools produce a schematic similar to that shown in Figure 3.

Figure 3 shows the SK filter which realizes a Butterworth, second-order low-pass response, with a -3 dB bandwidth of 300 kHz, when an ideal op amp is used. However, Filter Designer indicates that to assure correct filter response with an actual op amp, it should have a GBW of at least 85 MHz. That is a high GBW requirement and would require the using a high-speed op amp. High-speed op amps tend to cost more than their lower-GBW counterparts, so the goal here is to use a low-GBW op amp and incorporate the proposed GBW compensation. The filter responses with and without the compensation can then be compared.

The TL081 device is a popular JFET input op amp. Its GBW is 3 MHz—much less than the 85 MHz indicated by the filter program. Despite this, the TL081 is selected to test the new compensation method.

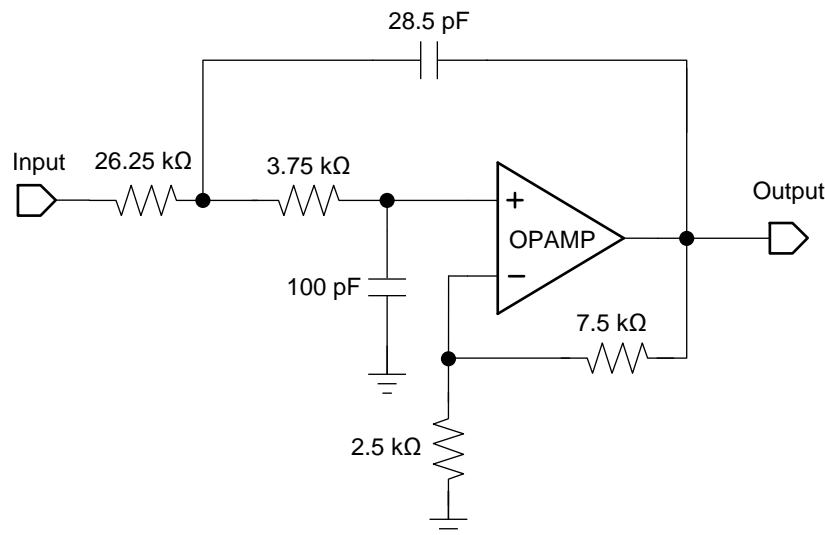


Figure 3. Schematic for 300-kHz, Sallen-Key, Low-Pass Filter

Figure 4 shows the AC transfer curves of the SK low-pass filter using the TL081 device along with an ideal op amp. Using the AC Analysis of TINA Spice, the AC Transfer Characteristic function makes it easy to determine the -3 dB bandwidth of the filter. When the TL081 device is used, the simulation -3 dB bandwidth is 239 kHz, about 20 % below the correct bandwidth achieved by the ideal op amp.

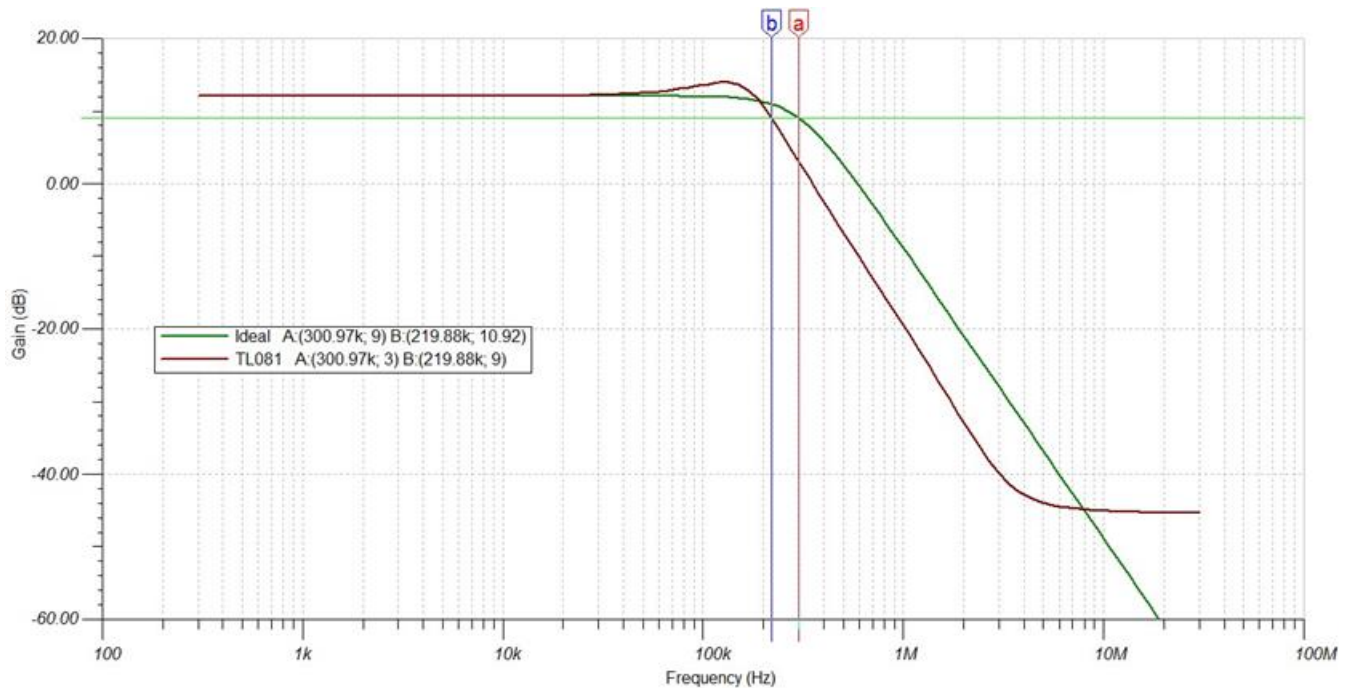


Figure 4. TINA Spice AC Simulations for Ideal Op Amp and TL081

Next, the GBW compensation method is applied to the filter version using the TL081. Applying the 3-MHz GBW of the TL081 to Equation 9 results in a 1.63-k Ω R_{comp} resistor.

Lastly, the resistance connected directly in series with the op amp inverting input is recalculated using Equation 12 and Equation 13. Originally, in Figure 3, the resistor had a value of 3.75 k Ω , but after recalculation it decreases to 1.63 k Ω .

Figure 5 shows the original SK filter schematic now modified to incorporate the two resistor changes.

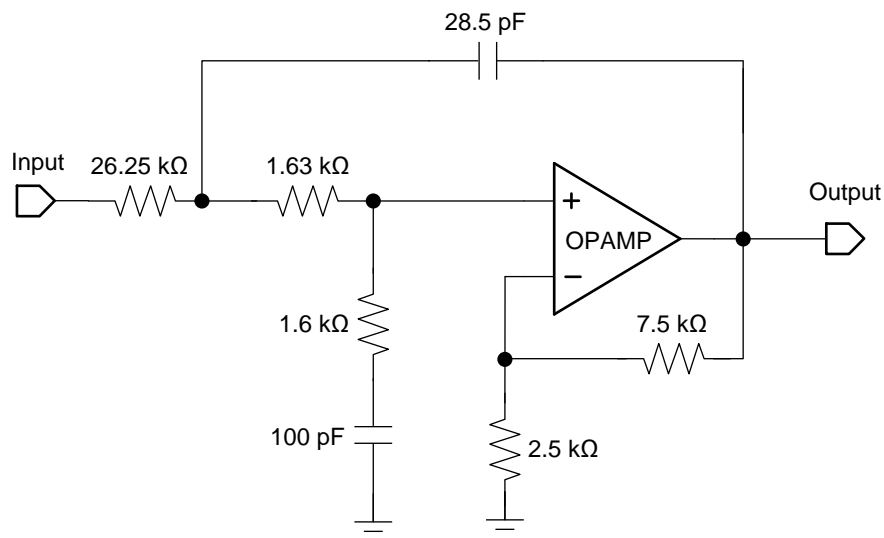
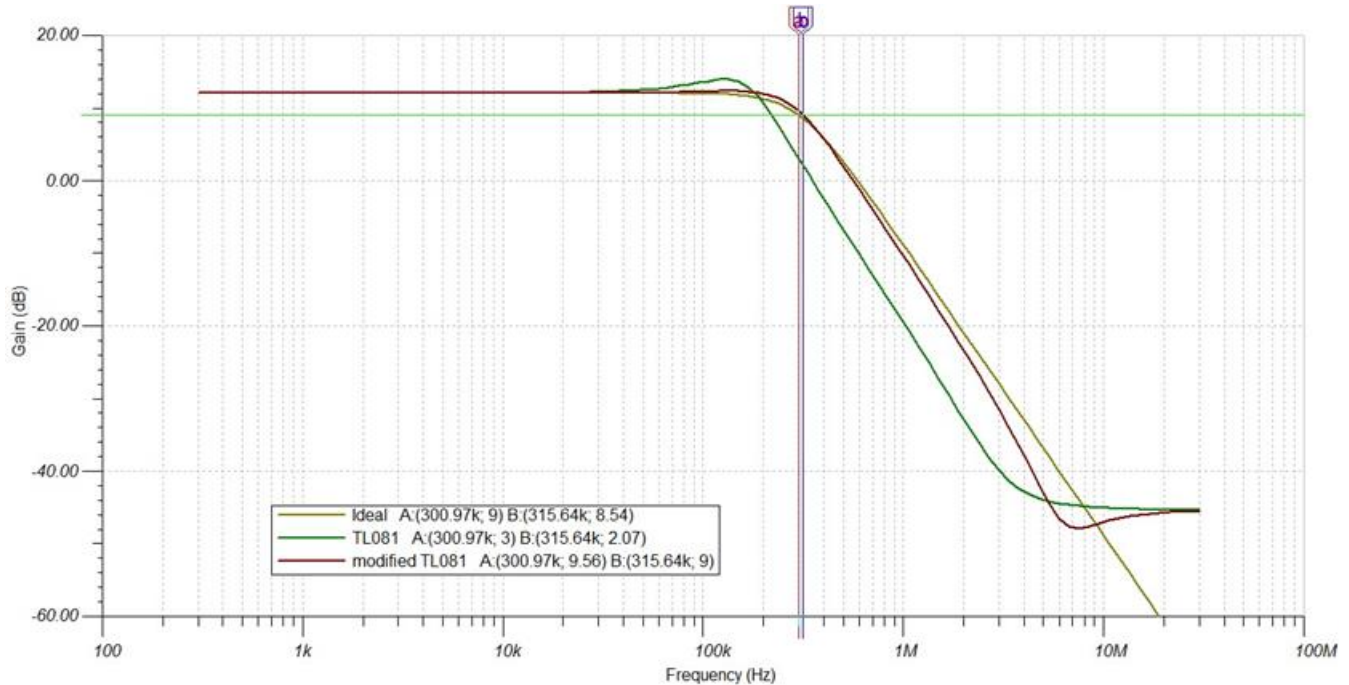


Figure 5. Modified Schematic for 300-kHz Sallen-Key Low-Pass Filter

Figure 6 shows the simulation results for an ideal op amp, an uncompensated TL081, and the compensated TL081. These plots show evidence that the ideal and compensated TL081 SK circuits produce nearly identical gain versus frequency responses.

To further validate the compensation method, a network analyzer was used to determine S21, the forward voltage gain parameter of the filter, which is equivalent to the AC transfer function of the filter.



(1) The ideal and compensated curves are nearly coincident.

Figure 6. TINA Spice AC Simulations of Sallen-Key Filter Using Ideal Op Amp, Standard TL081 Circuit and Compensated TL081 Circuit

Figure 7 shows the measured S21 result. The -3 dB bandwidth is 309 kHz, indicating that the original bandwidth target is supported by the TL081 device with the new compensation technique applied.

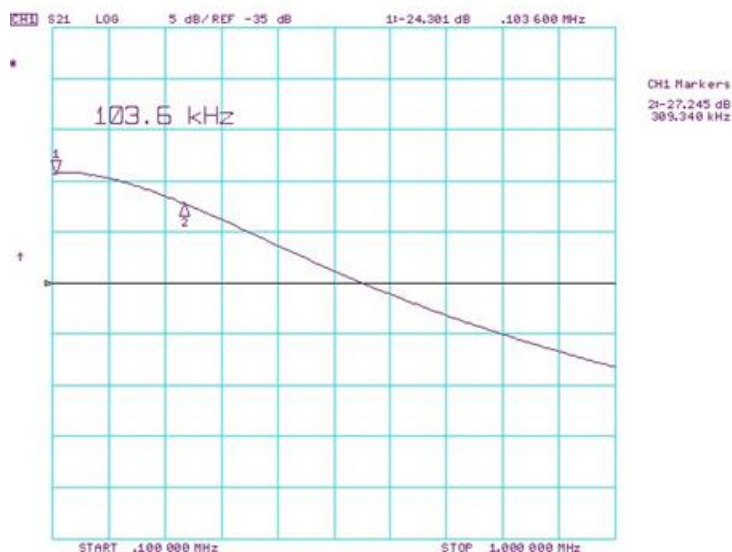


Figure 7. S21 for Modified Sallen-Key Filter Using TL081

4 Conclusions

The response errors of the SK low-pass filter, introduced by the limited GBW of an operational amplifier, were analyzed. A new compensation method that reduces these errors was proposed. The method allows successful use of an op amp with lower GBW than that recommended by filter synthesis programs or other means. The simulation and test results are in a close agreement with the theoretical analysis. Using this compensation method, designers can develop a SK active-low-pass filter for many applications with lower cost, lower bandwidth op amps.

5 References

- Texas Instruments, [Analysis of the Sallen-Key Architecture](#), Application Report
- Texas Instruments, [FilterPro™](#), User's Guide
- Texas Instruments, [TL081](#), Data Sheet

IMPORTANT NOTICE FOR TI DESIGN INFORMATION AND RESOURCES

Texas Instruments Incorporated ("TI") technical, application or other design advice, services or information, including, but not limited to, reference designs and materials relating to evaluation modules, (collectively, "TI Resources") are intended to assist designers who are developing applications that incorporate TI products; by downloading, accessing or using any particular TI Resource in any way, you (individually or, if you are acting on behalf of a company, your company) agree to use it solely for this purpose and subject to the terms of this Notice.

TI's provision of TI Resources does not expand or otherwise alter TI's applicable published warranties or warranty disclaimers for TI products, and no additional obligations or liabilities arise from TI providing such TI Resources. TI reserves the right to make corrections, enhancements, improvements and other changes to its TI Resources.

You understand and agree that you remain responsible for using your independent analysis, evaluation and judgment in designing your applications and that you have full and exclusive responsibility to assure the safety of your applications and compliance of your applications (and of all TI products used in or for your applications) with all applicable regulations, laws and other applicable requirements. You represent that, with respect to your applications, you have all the necessary expertise to create and implement safeguards that (1) anticipate dangerous consequences of failures, (2) monitor failures and their consequences, and (3) lessen the likelihood of failures that might cause harm and take appropriate actions. You agree that prior to using or distributing any applications that include TI products, you will thoroughly test such applications and the functionality of such TI products as used in such applications. TI has not conducted any testing other than that specifically described in the published documentation for a particular TI Resource.

You are authorized to use, copy and modify any individual TI Resource only in connection with the development of applications that include the TI product(s) identified in such TI Resource. NO OTHER LICENSE, EXPRESS OR IMPLIED, BY ESTOPPEL OR OTHERWISE TO ANY OTHER TI INTELLECTUAL PROPERTY RIGHT, AND NO LICENSE TO ANY TECHNOLOGY OR INTELLECTUAL PROPERTY RIGHT OF TI OR ANY THIRD PARTY IS GRANTED HEREIN, including but not limited to any patent right, copyright, mask work right, or other intellectual property right relating to any combination, machine, or process in which TI products or services are used. Information regarding or referencing third-party products or services does not constitute a license to use such products or services, or a warranty or endorsement thereof. Use of TI Resources may require a license from a third party under the patents or other intellectual property of the third party, or a license from TI under the patents or other intellectual property of TI.

TI RESOURCES ARE PROVIDED "AS IS" AND WITH ALL FAULTS. TI DISCLAIMS ALL OTHER WARRANTIES OR REPRESENTATIONS, EXPRESS OR IMPLIED, REGARDING TI RESOURCES OR USE THEREOF, INCLUDING BUT NOT LIMITED TO ACCURACY OR COMPLETENESS, TITLE, ANY EPIDEMIC FAILURE WARRANTY AND ANY IMPLIED WARRANTIES OF MERCHANTABILITY, FITNESS FOR A PARTICULAR PURPOSE, AND NON-INFRINGEMENT OF ANY THIRD PARTY INTELLECTUAL PROPERTY RIGHTS.

TI SHALL NOT BE LIABLE FOR AND SHALL NOT DEFEND OR INDEMNIFY YOU AGAINST ANY CLAIM, INCLUDING BUT NOT LIMITED TO ANY INFRINGEMENT CLAIM THAT RELATES TO OR IS BASED ON ANY COMBINATION OF PRODUCTS EVEN IF DESCRIBED IN TI RESOURCES OR OTHERWISE. IN NO EVENT SHALL TI BE LIABLE FOR ANY ACTUAL, DIRECT, SPECIAL, COLLATERAL, INDIRECT, PUNITIVE, INCIDENTAL, CONSEQUENTIAL OR EXEMPLARY DAMAGES IN CONNECTION WITH OR ARISING OUT OF TI RESOURCES OR USE THEREOF, AND REGARDLESS OF WHETHER TI HAS BEEN ADVISED OF THE POSSIBILITY OF SUCH DAMAGES.

You agree to fully indemnify TI and its representatives against any damages, costs, losses, and/or liabilities arising out of your non-compliance with the terms and provisions of this Notice.

This Notice applies to TI Resources. Additional terms apply to the use and purchase of certain types of materials, TI products and services. These include; without limitation, TI's standard terms for semiconductor products (<http://www.ti.com/sc/docs/stdterms.htm>), [evaluation modules](#), and [samples](http://www.ti.com/sc/docs/sampterm.htm) (<http://www.ti.com/sc/docs/sampterm.htm>).

Mailing Address: Texas Instruments, Post Office Box 655303, Dallas, Texas 75265
Copyright © 2017, Texas Instruments Incorporated