

# Application Note

## Active Low-Pass Filter Design



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### ABSTRACT

This report focuses on active low-pass filter design using operational amplifiers. Low-pass filters are commonly used to implement anti-aliasing filters in data acquisition systems. Design of second-order filters is the main topic of consideration.

Filter tables are developed to simplify circuit design based on the idea of cascading lower-order stages to realize higher-order filters. The tables contain scaling factors ( $FSF$ ) for the corner frequency ( $f_c$ ) and the required quality factor ( $Q$ ) of each of the stages for the particular filter being designed. The tables enable designers to skip straight to calculating required circuit component values.

To illustrate an actual circuit implementation, six circuits, separated into three types of filters (Bessel, Butterworth, and Chebyshev) and two filter configurations (Sallen-Key and Multiple Feedback), are simulated using a TLV9062 operational amplifier. Limiting factors in the high-frequency performance of the filters are also examined. The tables in this document are used in the [Analog Engineer's Circuit Cookbook: Amplifiers](#).

### Table of Contents

<b>1 Introduction</b> .....	3
<b>2 Filter Characteristics</b> .....	3
<b>3 Second-Order Low-Pass Filter Standard Form</b> .....	4
<b>4 Math Review</b> .....	4
<b>5 Examples</b> .....	6
5.1 Second-Order Low-Pass Butterworth Filter.....	6
5.2 Second-Order Low-Pass Bessel Filter.....	7
5.3 Second-Order Low-Pass Chebyshev Filter with 3-dB Ripple.....	8
<b>6 Low-Pass Sallen-Key Architecture</b> .....	8
<b>7 Low-Pass Multiple Feedback (MFB) Architecture</b> .....	9
<b>8 Cascading Filter Stages</b> .....	10
<b>9 Filter Tables</b> .....	11
<b>10 Example Circuit Simulated Results</b> .....	13
<b>11 Non-ideal Circuit Operation</b> .....	16
11.1 Non-ideal Circuit Operation: Sallen-Key.....	16
11.2 Non-ideal Circuit Operation: MFB.....	17
<b>12 Comments About Component Selection</b> .....	18
<b>13 Conclusion</b> .....	18
<b>A Filter Design Specifications</b> .....	19
A.1 Sallen-Key Design Simplifications.....	19
A.2 MFB Design Simplifications.....	19
<b>B Higher-Order Filters</b> .....	20
B.1 Fifth-Order Low-Pass Butterworth Filter.....	20
B.2 Sixth-Order Low-Pass Bessel Filter.....	21
<b>Revision History</b> .....	21

### List of Figures

Figure 5-1. Butterworth Low-Pass Filter With TLV9062 Circuit.....	6
Figure 5-2. Butterworth Low-Pass Filter With TLV9062 PSpice® Results.....	7
Figure 5-3. Butterworth Low-Pass Filter With TLV9062 Measured Results.....	7
Figure 6-1. Low-Pass Sallen-Key Architecture.....	8
Figure 7-1. Low-Pass MFB Architecture.....	9

Figure 8-1. Building Even-Order Filters by Cascading Second-Order Stages.....	10
Figure 8-2. Building Odd-Order Filters by Cascading Second-Order Stages and Adding a Single Real Pole.....	10
Figure 10-1. Sallen-Key Circuit and Component Values ( $f_c = 1$ kHz).....	13
Figure 10-2. Second-Order Butterworth Filter Frequency Response.....	14
Figure 10-3. Second-Order Bessel Filter Frequency Response.....	14
Figure 10-4. Second-Order 3-dB Chebyshev Filter Frequency Response.....	15
Figure 10-5. Second-Order Butterworth, Bessel, and 3-dB Chebyshev Filter Frequency Response.....	15
Figure 11-1. Second-Order Low-Pass Sallen-Key High-Frequency Model.....	16
Figure 11-2. Sallen-Key Butterworth Filter With RC Added in Series With the Output.....	16
Figure 11-3. Second-Order Low-Pass MFB High-Frequency Model.....	17
Figure 11-4. MFB Butterworth Filter With RC Added in Series With the Output.....	17
Figure B-1. Fifth-Order Low-Pass Filter Topology Cascading Two Sallen-Key Stages and an RC.....	20
Figure B-2. Sixth-Order Low-Pass Filter Topology Cascading Three MFB Stages.....	21

### List of Tables

Table 1-1. Summary of Filter Type Trade-Offs.....	3
Table 1-2. Summary of Architecture Trade-Offs.....	3
Table 9-1. Butterworth Filter Table.....	11
Table 9-2. Bessel Filter Table.....	11
Table 9-3. 1-dB Chebyshev Filter Table.....	12
Table 9-4. 3-dB Chebyshev Filter Table.....	12
Table 10-1. Sallen-Key Circuit and Component Values ( $f_c = 1$ kHz).....	13
Table 10-2. MFB Circuit and Component Values ( $f_c = 1$ kHz).....	13

### Trademarks

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## 1 Introduction

There are many books that provide information on popular filter types like the Butterworth, Bessel, and Chebyshev filters. This application note examines how to implement these three types of filters.

The mathematics used to transform standard filter table data into the transfer functions required to build filter circuits is examined. Using the same method, filter tables are developed that enable the designer to skip straight to the calculation of the required circuit component values. Actual filter implementation is shown for two circuit topologies: the Sallen-Key and the Multiple Feedback (MFB). The Sallen-Key circuit is sometimes referred to as a voltage-controlled voltage source (VCVS) filter.

Circuits are often referred to as Butterworth filters, Bessel filters, or a Chebyshev filters because their transfer function has the same coefficients as the Butterworth, Bessel, or the Chebyshev polynomial. The MFB or Sallen-Key circuits are also often referred to as filters. The difference is that the Butterworth filter defines a transfer function that can be realized by many different circuit topologies (both active and passive), while the MFB or Sallen-Key circuit defines an architecture or a circuit topology that can be used to realize various second-order transfer functions.

The choice of circuit topology depends on performance requirements. The MFB is generally preferred because the MFB has better sensitivity to component variations and better high-frequency behavior. The unity-gain Sallen-Key inherently has the best gain accuracy because the gain is not dependent on component values.

Table 1-1 and Table 1-2 give a brief summary of the overall trade-offs.

**Table 1-1. Summary of Filter Type Trade-Offs**

Filter Type	Advantages	Disadvantages
Butterworth	Maximum pass-band flatness	Slight overshoot in response to pulse input and moderate rate of attenuation above $f_c$
Bessel	Constant group delay – no overshoot with pulse input	Slow rate of attenuation above $f_c$
3-dB Chebyshev	Fast rate of attenuation above $f_c$	Large overshoot and ringing in response to pulse input

**Table 1-2. Summary of Architecture Trade-Offs**

Architecture	Advantages	Disadvantages
Sallen-Key	Not sensitive to component variation at unity gain	High-frequency response limited by the frequency response of the amplifier
MFB	Less sensitive to component variations and excellent high-frequency response	Less simplifications available to ease design

## 2 Filter Characteristics

If an ideal low-pass filter existed, it would completely eliminate signals above the cutoff frequency and perfectly pass signals below the cutoff frequency. In real filters, various trade-offs are made to get optimum performance for a given application.

**Butterworth** filters are termed maximally-flat-magnitude-response filters, optimized for gain flatness in the pass band. The attenuation is  $-3$  dB at the cutoff frequency. Above the cutoff frequency, the attenuation is  $-20$  dB/decade/order. The transient response of a Butterworth filter to a pulse input shows moderate overshoot and ringing.

**Bessel** filters are optimized for maximally-flat time delay (or constant-group delay). This means that they have linear phase response and excellent transient response to a pulse input. This comes at the expense of flatness in the pass-band and rate of rolloff. The cutoff frequency is defined as the  $-3$  dB point.

**Chebyshev** filters are designed to have ripple in the pass band, but steeper rolloff after the cutoff frequency. Cutoff frequency is defined as the frequency at which the response falls below the ripple band. For a given filter order, a steeper cutoff can be achieved by allowing more pass-band ripple. The transient response of a Chebyshev filter to a pulse input shows more overshoot and ringing than a Butterworth filter.

When constructing a filter, there are two topologies that can be used: the Sallen-Key topology, which is a non-inverting circuit, or the Multiple Feedback (MFB) topology, which creates an inverting second-order stage

circuit. See the [Filter Designer](#) tool or the [filtering cookbooks](#) for more information on the Sallen-Key and MFB filters.

### 3 Second-Order Low-Pass Filter Standard Form

The transfer function  $H_{LP}$  of a second-order low-pass filter can be expressed as a function of frequency ( $f$ ) as shown in [Equation 1](#), the Second-Order Low-Pass Filter Standard Form.

$$H_{LP}(f) = -\frac{K}{\left(\frac{f}{FSF \times f_c}\right)^2 + \left(\frac{1}{Q} \times \frac{jf}{FSF \times f_c}\right) + 1} \quad (1)$$

In this equation,  $f$  is the frequency variable,  $f_c$  is the cutoff frequency,  $FSF$  is the frequency scaling factor, and  $Q$  is the quality factor. [Equation 1](#) has three regions of operation: below cutoff, in the area of cutoff, and above cutoff. For each area, [Equation 1](#) reduces to:

- $f \ll f_c \Rightarrow H_{LP}(f) \approx K$ 
  - The circuit passes signals multiplied by the gain factor  $K$ .
- $\frac{f}{f_c} = FSF \Rightarrow H_{LP}(f) = jKQ$ 
  - Signals are phase-shifted  $90^\circ$  and modified by the  $Q$  factor.
- $f \gg f_c \Rightarrow H_{LP}(f) \approx -K\left(\frac{FSF \times f_c}{f}\right)^2$ 
  - Signals are phase-shifted  $180^\circ$  and attenuated by the square of the frequency ratio.

With attenuation at frequencies above  $f_c$  increasing by a power of two, the last formula describes a second-order low-pass filter.

The frequency scaling factor  $FSF$  is used to scale the cutoff frequency of the filter so that it follows the definitions given before.

### 4 Math Review

A second-order polynomial using the variable  $s$  can be given in two equivalent forms.

The coefficient form:

$$s^2 + a_1s + a_0 \quad (2)$$

Or the factored form:

$$(s - z_1)(s - z_2) \quad (3)$$

In summary,

$$P(s) = s^2 + a_1s + a_0 = (s - z_1)(s - z_2) \quad (4)$$

where  $z_1$  and  $z_2$  are the locations in the  $s$ -plane where the polynomial is zero.

The three filters being discussed here are all-pole filters, meaning that their transfer functions contain all poles and no zeros. The polynomial, which characterizes the response of the filter, is used as the denominator of the transfer function of the filter. The zeros of the polynomial are thus the poles of the filter.

All even-order Butterworth, Bessel, or Chebyshev polynomials contain complex-conjugate zero pairs. This means that [Equation 5](#) and [Equation 6](#) are true, where  $\text{Re}$  is the real part and  $\text{Im}$  is the imaginary part.

$$z_1 = \text{Re} + \text{Im} \quad (5)$$

$$z_2 = \text{Re} - \text{Im} \quad (6)$$

In typical mathematical notation,  $z_1$  indicates the conjugate zero with the positive imaginary part and  $z_1^*$  indicates the conjugate zero with the negative imaginary part. Odd-order filters have a real pole in addition to the complex-conjugate pairs.

Some filter books provide tables of the zeros of the polynomial which describes the filter, others provide the coefficients, and some provide both. Since the zeros of the polynomial are the poles of the filter, some books use the term poles. Zeros and poles are used with the factored form of the polynomial, and coefficients are used with the coefficient form. No matter how the information is given, conversion between the two forms is routine.

Expressing the transfer function of a filter in factored form makes it easy to quickly see the location of the poles. Conversely, a second-order polynomial in coefficient form makes it easier to correlate the transfer function with circuit components. This is seen later when examining the filter-circuit topologies. Therefore, an engineer typically wants to use the factored form, but needs to scale and normalize the polynomial first.

The coefficient form of the second-order equation shows that when  $s \ll a_0$ , the equation is dominated by  $a_0$ ; when  $s \gg a_0$ ,  $s$  dominates.  $a_0$  is the break point where the equation transitions between dominant terms. To normalize and scale to other values, divide each term by  $a_0$  and divide the  $s$  terms by  $\omega c$ . The result is shown in [Equation 7](#):

$$P(s) = \left( \frac{s}{\omega c \sqrt{a_0}} \right)^2 + \frac{a_1 s}{a_0 \omega c} + 1 \quad (7)$$

This scales and normalizes the polynomial so that the break point is at  $s = \omega c \sqrt{a_0}$ .

By making the substitutions  $s = j2\pi f$ ,  $\omega c = 2\pi f c$ ,  $a_1 = \frac{1}{Q}$ , and  $\sqrt{a_0} = FSF$ , the equation becomes:

$$P(f) = - \left( \frac{f}{FSF \times f_c} \right)^2 + \left( \frac{1}{Q} \times \frac{jf}{FSF \times f_c} \right) + 1 \quad (8)$$

This is the denominator of [Equation 1](#)—standard form for low-pass filters. Throughout the rest of this article, the substitution  $s = j2\pi f$  is used.

## 5 Examples

The following examples illustrate how to take standard filter table information and process that information into standard form.

### 5.1 Second-Order Low-Pass Butterworth Filter

The Butterworth polynomial requires the least work out of the three types of filters because the frequency scaling factor is always equal to one.

Referring to a table listing the zeros of the second-order Butterworth polynomial:

$$z_1 = -0.707 + j0.707 \quad (9)$$

$$z_1^* = -0.707 - j0.707 \quad (10)$$

This is used with the factored form of the polynomial. Alternatively, the coefficients of the polynomial  $a_0 = 1$  and  $a_1 = 1.414$  can be found. The formula in [Equation 11](#) can be confirmed:

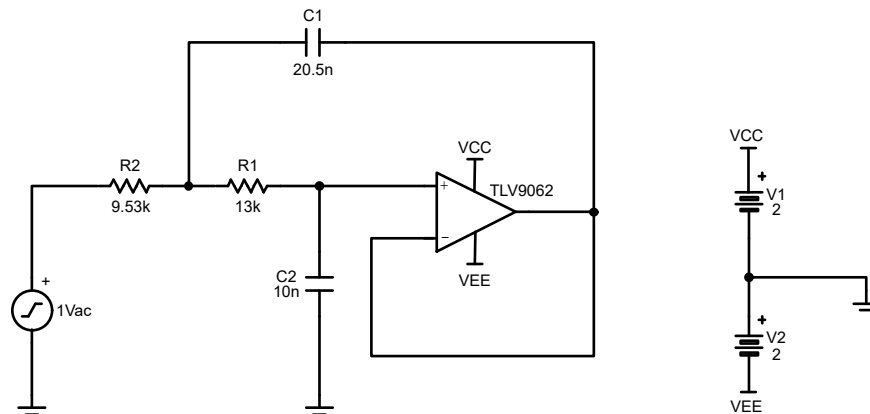
$$(s + 0.707 + j0.707)(s + 0.707 - j0.707) = s^2 + 1.414s + 1 \quad (11)$$

To translate the polynomial into standard form, use the coefficient form of the polynomial in the denominator of the transfer function. The realization of a second-order low-pass Butterworth filter is made by a circuit with the following transfer function:

$$H_{LP}(f) = \frac{K}{-\left(\frac{f}{f_c}\right)^2 + 1.414\frac{jf}{f_c} + 1} \quad (12)$$

[Equation 12](#) is the same as [Equation 1](#) with  $FSF = 1$  and  $Q = \frac{1}{1.414} = 0.707$ .

[Figure 5-1](#) is an example of a Second-Order Butterworth Low-Pass Filter using the Sallen-Key topology and TLV9062 created with the [Filter Design Tool](#). This circuit has a gain of 1 V/V and a pass-band frequency at 1 kHz. The circuit was built in PSpice and then constructed and measured with a gain-phase analyzer.



**Figure 5-1. Butterworth Low-Pass Filter With TLV9062 Circuit**

Figure 5-2 and Figure 5-3 show the Butterworth low-pass filter with TLV9062 PSpice and Measured results, respectively.

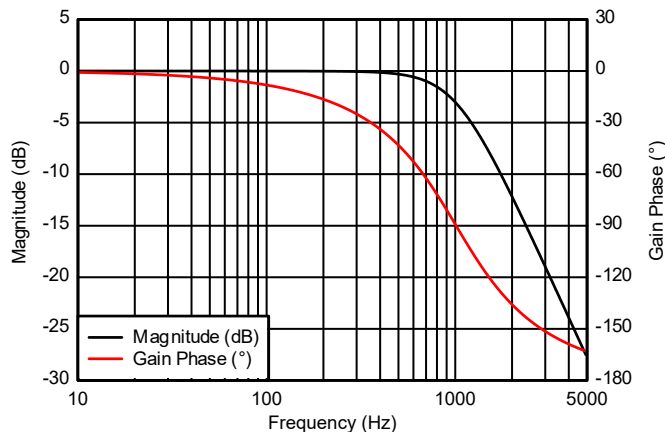


Figure 5-2. Butterworth Low-Pass Filter With TLV9062 PSpice® Results

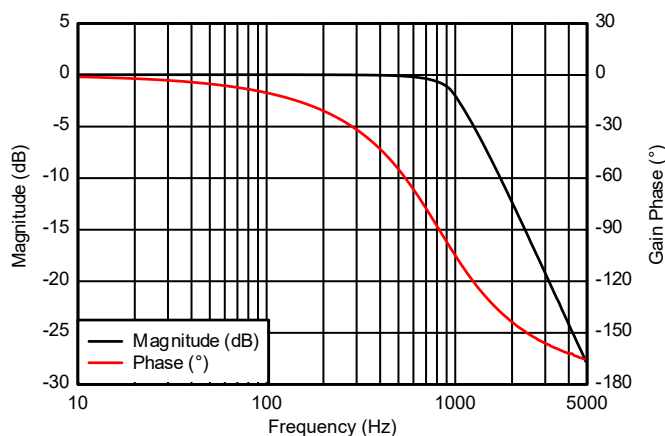


Figure 5-3. Butterworth Low-Pass Filter With TLV9062 Measured Results

## 5.2 Second-Order Low-Pass Bessel Filter

Referring to a table listing the zeros of the second-order Bessel polynomial:

$$z_1 = -1.103 + j0.6368 \quad (13)$$

$$z_1^* = -1.103 - j0.6368 \quad (14)$$

A table of coefficients provides  $a_0 = 1.622$  and  $a_1 = 2.206$ .

Again, coefficients directly appear in standard form, so the realization of a second-order low-pass Bessel filter is made by a circuit with the transfer function:

$$H_{LP}(f) = \frac{K}{-\left(\frac{f}{f_c}\right)^2 + 2.206\frac{jf}{f_c} + 1.622} \quad (15)$$

Normalize Equation 15 so that it is in standard form. Dividing both the numerator and denominator by 1.622 scales the gain factor  $K$  (which is arbitrary) and gives the normalized form:

$$H_{LP}(f) = \frac{K}{-\left(\frac{f}{1.274f_c}\right)^2 + 1.360\frac{jf}{f_c} + 1} \quad (16)$$

Equation 16 is the same as Equation 1 with  $FSF = 1.274$  and  $Q = \frac{1}{1.360 \times 1.274} = 0.577$ .

### 5.3 Second-Order Low-Pass Chebyshev Filter with 3-dB Ripple

Referring to a table listing the zeros of the 3-dB second-order Chebyshev polynomial:

$$z_1 = -0.3224 + j0.7772 \quad (17)$$

$$z_1^* = -0.3224 - j0.7772 \quad (18)$$

A table of coefficients provides  $a_0 = 0.7080$  and  $a_1 = 0.6448$ .

Again, coefficients directly appear in standard form, so the realization of a second-order low-pass Chebyshev filter with 3-dB ripple is made by a circuit with the transfer function:

$$H_{LP}(f) = \frac{K}{-\left(\frac{f}{f_c}\right)^2 + 0.6448\frac{jf}{f_c} + 0.7080} \quad (19)$$

Again, normalize Equation 19 so that it is in standard form by dividing both the numerator and denominator by 0.7080 to get:

$$H_{LP}(f) = \frac{K}{-\left(\frac{f}{0.8414f_c}\right)^2 + 0.9107\frac{jf}{f_c} + 1} \quad (20)$$

Equation 20 is the same as Equation 1 with  $FSF = 0.8414$  and  $Q = \frac{1}{0.8414 \times 0.9107} = 1.3050$ .

The previous work is the first step in designing any of the filters. The next step is to determine which circuit topology to use to implement these filters.

## 6 Low-Pass Sallen-Key Architecture

Figure 6-1 shows the low-pass Sallen-Key filter architecture and the transfer function.

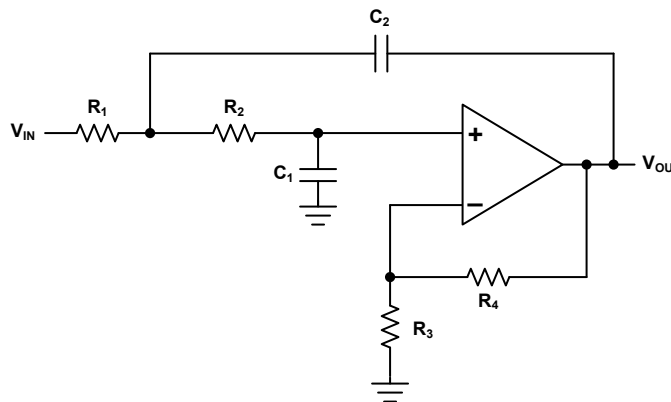


Figure 6-1. Low-Pass Sallen-Key Architecture

$$H(f) = \frac{\frac{R_3 + R_4}{R_3}}{(j2\pi f)^2(R_1R_2C_1C_2) + j2\pi f\left(R_1C_1 + R_2C_1 + R_1C_2\left(-\frac{R_4}{R_3}\right)\right) + 1} \quad (21)$$

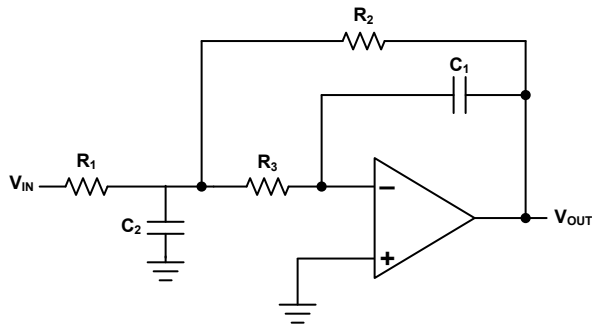
At first glance, the transfer function looks very different from standard form in Equation 1. However, with the substitutions  $K = \frac{R_3 + R_4}{R_3}$ ,  $FSF \times f_c = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}$ , and  $Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1C_1 + R_2C_1 + R_1C_2(1 - K)}$ , the functions become the same.



Depending on how you use the previous equations, the design process can be simple or tedious. [Appendix A](#) shows simplifications that help to ease this process.

## 7 Low-Pass Multiple Feedback (MFB) Architecture

Figure 7-1 shows the low-pass MFB filter architecture and the transfer function.



**Figure 7-1. Low-Pass MFB Architecture**

$$H(f) = \frac{\frac{-R_2}{R_1}}{(j2\pi f)^2 \left( R_2 R_3 C_1 C_2 \right) + j2\pi f \left( R_3 C_1 + R_2 C_1 + \frac{R_2 R_3 C_1}{R_1} \right) + 1} \quad (22)$$

Again, the transfer function looks different from standard form in [Equation 1](#). By substituting  $K = \frac{-R_2}{R_1}$ ,

$FSF \times f_c = \frac{1}{2\pi\sqrt{R_2 R_3 C_1 C_2}}$ , and  $Q = \frac{\sqrt{R_2 R_3 C_1 C_2}}{R_3 C_1 + R_2 C_1 + R_3 C_1 (-K)}$ , the functions become the same.

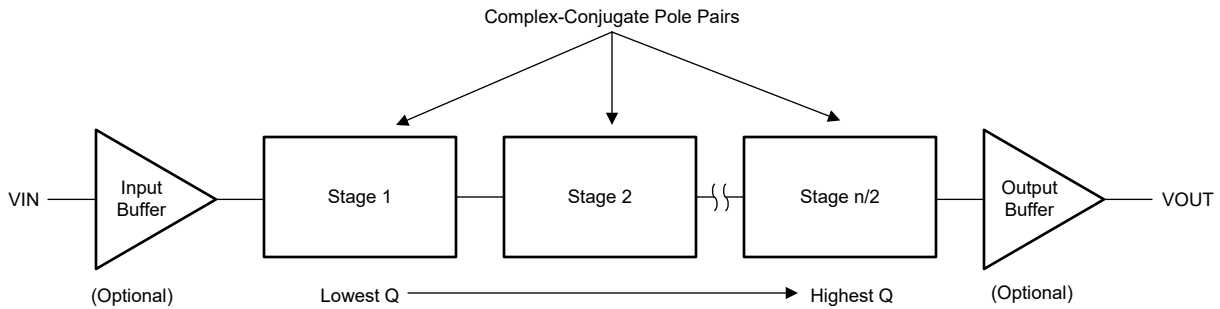
Depending on how you use the previous equations, the design process can be simple or tedious. [Appendix A](#) shows simplifications that help to ease this process.

The Sallen-Key and MFB circuits shown are second-order low-pass stages that can be used to realize one complex pole pair in the transfer function of a low-pass filter. To make a Butterworth, Bessel, or Chebyshev filter, use the previous substitutions with the standard form equations that come from filter coefficients to find circuit component values.

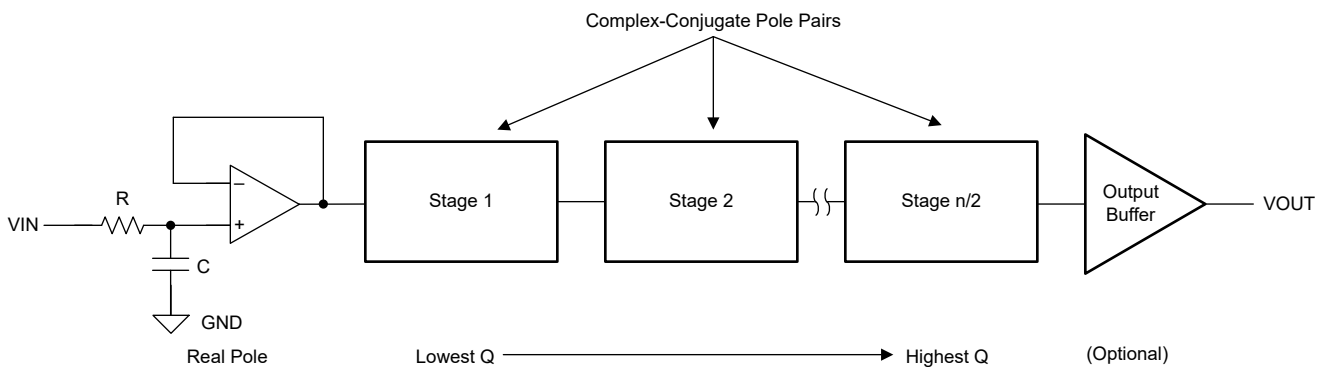
## 8 Cascading Filter Stages

The concept of cascading second-order filter stages to realize higher-order filters is illustrated in Figure 8-1. The filter is broken into complex-conjugate pole pairs that can be realized by Sallen-Key, MFB, or a combination of the architectures. To implement an  $n$ th-order filter,  $\frac{n}{2}$  stages are required. Figure 8-2 extends the concept to odd-order filters by adding a first-order real pole.

Theoretically, the order of the stages makes no difference, but to help avoid saturation, the stages are normally arranged with the lowest  $Q$  near the input and the highest  $Q$  near the output. Appendix B shows detailed circuit examples using cascaded stages for higher-order filters.



**Figure 8-1. Building Even-Order Filters by Cascading Second-Order Stages**



**Figure 8-2. Building Odd-Order Filters by Cascading Second-Order Stages and Adding a Single Real Pole**

## 9 Filter Tables

Typically, filter books list the zeros or the coefficients of the particular polynomial being used to define the filter type. As seen previously in this document, a certain amount of mathematical manipulation is required to turn this information into a circuit realization. The relationships between the zeros and the frequency scaling factor

(  $FSF$  ) and quality factor (  $Q$  ) are given by  $FSF = \sqrt{\text{Re}^2 + |\text{Im}|^2}$  and  $Q = \frac{\sqrt{\text{Re}^2 + |\text{Im}|^2}}{2\text{Re}}$ , where  $\text{Re}$  and  $\text{Im}$  are the real and imaginary parts of the complex-conjugate zero pair. Table 9-1 through Table 9-4 are generated in this way. Higher-order filters are constructed by cascading second-order stages for even-order filters (one for each complex-conjugate zero pair). A first-order stage is then added if the filter order is odd. With the filter tables arranged this way, the preliminary mathematical work is done and the designer is only left with calculating the circuit component values based on three formulas.

For a low-pass Sallen-Key filter with cutoff frequency  $f_c$  and pass-band gain  $K$ , set  $K = \frac{R_3 + R_4}{R_3}$ ,

$FSF \times f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$ , and  $Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 C_1 + R_2 C_1 + R_1 C_2(1 - K)}$  for each second-order stage. If an odd order is required, set  $FSF \times f_c = \frac{1}{2\pi RC}$  for that stage.

For a low-pass MFB filter with cutoff frequency  $f_c$  and pass-band gain  $K$ , set  $K = \frac{-R_2}{R_1}$ ,  $FSF \times f_c = \frac{1}{2\pi\sqrt{R_2 R_3 C_1 C_2}}$ , and  $Q = \frac{\sqrt{R_2 R_3 C_1 C_2}}{R_3 C_1 + R_2 C_1 + R_3 C_1(-K)}$  for each second-order stage. If an odd order is required, set  $FSF \times f_c = \frac{1}{2\pi RC}$  for that stage.

The tables are arranged so that increasing  $Q$  is associated with increasing stage order. High-order filters are normally arranged in this manner to help prevent clipping.

**Table 9-1. Butterworth Filter Table**

Filter Order	Stage 1		Stage 2		Stage 3		Stage 4		Stage 5	
	$FSF$	$Q$	$FSF$	$Q$	$FSF$	$Q$	$FSF$	$Q$	$FSF$	$Q$
2	1.000	0.7071								
3	1.000	1.0000	1.000							
4	1.000	0.5412	1.000	1.3065						
5	1.000	0.6180	1.000	1.6181	1.000					
6	1.000	0.5177	1.000	0.7071	1.000	1.9320				
7	1.000	0.5549	1.000	0.8019	1.000	2.2472	1.000			
8	1.000	0.5098	1.000	0.6013	1.000	0.8999	1.000	2.5628		
9	1.000	0.5321	1.000	0.6527	1.000	1.0000	1.000	2.8802	1.000	
10	1.000	0.5062	1.000	0.5612	1.000	0.7071	1.000	1.1013	1.000	3.1969

**Table 9-2. Bessel Filter Table**

Filter Order	Stage 1		Stage 2		Stage 3		Stage 4		Stage 5	
	$FSF$	$Q$	$FSF$	$Q$	$FSF$	$Q$	$FSF$	$Q$	$FSF$	$Q$
2	1.2736	0.5773								
3	1.4524	0.6910	1.3270							
4	1.4192	0.5219	1.5912	0.8055						
5	1.5611	0.5635	1.7607	0.9165	1.5069					
6	1.6060	0.5103	1.6913	0.6112	1.9071	1.0234				
7	1.7174	0.5324	1.8235	0.6608	2.0507	1.1262	1.6853			
8	1.7837	0.5060	2.1953	1.2258	1.9591	0.7109	1.8376	0.5596		
9	1.8794	0.5197	1.9488	0.5894	2.0815	0.7606	2.3235	1.3220	1.8575	
10	1.9490	0.5040	1.9870	0.5380	2.0680	0.6200	2.2110	0.8100	2.4850	1.4150

**Table 9-3. 1-dB Chebyshev Filter Table**

Filter Order	Stage 1		Stage 2		Stage 3		Stage 4		Stage 5	
	<i>FSF</i>	<i>Q</i>	<i>FSF</i>	<i>Q</i>	<i>FSF</i>	<i>Q</i>	<i>FSF</i>	<i>Q</i>	<i>FSF</i>	<i>Q</i>
2	1.0500	0.9565								
3	0.9971	2.0176	0.4942							
4	0.5286	0.7845	0.9932	3.5600						
5	0.6552	1.3988	0.9941	5.5538	0.2895					
6	0.3532	0.7608	0.7468	2.1977	0.9953	8.0012				
7	0.4800	1.2967	0.8084	3.1554	0.9963	10.9010	0.2054			
8	0.2651	0.7530	0.5838	1.9564	0.5538	2.7776	0.9971	14.2445		
9	0.3812	1.1964	0.6623	2.7119	0.8805	5.5239	0.9976	18.0069	0.1593	
10	0.2121	0.7495	0.4760	1.8639	0.7214	3.5609	0.9024	6.9419	0.9981	22.2779

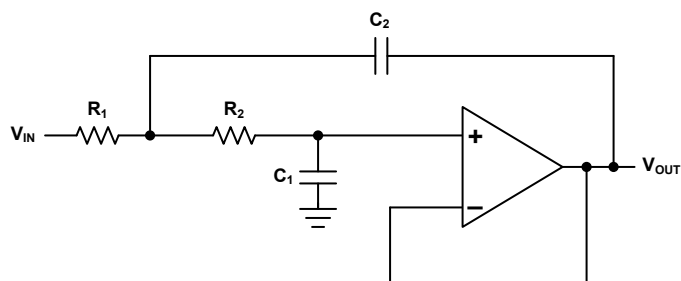
**Table 9-4. 3-dB Chebyshev Filter Table**

Filter Order	Stage 1		Stage 2		Stage 3		Stage 4		Stage 5	
	<i>FSF</i>	<i>Q</i>	<i>FSF</i>	<i>Q</i>	<i>FSF</i>	<i>Q</i>	<i>FSF</i>	<i>Q</i>	<i>FSF</i>	<i>Q</i>
2	0.8414	1.3049								
3	0.9160	3.0678	0.2986							
4	0.4426	1.0765	0.9503	5.5770						
5	0.6140	2.1380	0.9675	8.8111	0.1775					
6	0.2980	1.0441	0.7224	3.4597	0.9771	12.7899				
7	0.4519	1.9821	0.7920	5.0193	0.9831	17.4929	0.1265			
8	0.2228	1.0558	0.5665	3.0789	0.8388	6.8302	0.9870	22.8481		
9	0.3559	1.9278	0.6503	4.3179	0.8716	8.8756	0.9897	28.9400	0.0983	
10	0.1796	1.0289	0.4626	2.9350	0.7126	5.7012	0.8954	11.1646	0.9916	35.9274

## 10 Example Circuit Simulated Results

To further show how to use the previous information, component values are calculated and the filter circuits are simulated in PSpice.

Table 10-1 and Table 10-2 show typical component values computed for the three different filters discussed using the Sallen-Key architecture and the MFB architecture. The equivalent simplification (see Appendix A) is used for each circuit: setting the filter components as ratios, the gain equal to 1 for the Sallen-Key, and the gain equal to  $-1$  for the MFB. The circuits and simplifications are shown for convenience. A corner frequency  $f_c$  of 1 kHz is chosen. The values used for  $m$  and  $n$  are shown. The values shown for  $R_1$ ,  $R_2$ ,  $C_1$ , and  $C_2$  are chosen to be standard values or the nearest standard values to those computed by using the formulas given.



**Figure 10-1. Sallen-Key Circuit and Component Values ( $f_c = 1$  kHz)**

$$R_1 = mR, R_2 = R, C_1 = C, C_2 = nC, \text{ and } K = 1 \text{ result in } FSF \times f_c = \frac{1}{2\pi RC\sqrt{mn}} \text{ and } Q = \frac{\sqrt{mn}}{m+1}.$$

**Table 10-1. Sallen-Key Circuit and Component Values ( $f_c = 1$  kHz)**

Filter Type	$n$	$m$	$C_1$	$C_2$	$R_1$	$R_2$
Butterworth	3.3	0.229	0.01 $\mu$ F	33 nF	4.22 k $\Omega$	18.2 k $\Omega$
Bessel	1.5	0.42	0.01 $\mu$ F	15 nF	7.15 k $\Omega$	14.3 k $\Omega$
3-dB Chebyshev	6.8	1.0	0.01 $\mu$ F	68 nF	7.32 k $\Omega$	7.32 k $\Omega$

$$R_1 = R_2 = R, R_3 = mR, C_1 = C, C_2 = nC, \text{ and } K = 1 \text{ result in } FSF \times f_c = \frac{1}{2\pi RC\sqrt{mn}} \text{ and } Q = \frac{\sqrt{mn}}{1+2m}.$$

**Table 10-2. MFB Circuit and Component Values ( $f_c = 1$  kHz)**

Filter Type	$n$	$m$	$C_1$	$C_2$	$R_1$ and $R_2$	$R_3$
Butterworth	4.7	0.222	0.01 $\mu$ F	47 nF	15.4 k $\Omega$	3.48 k $\Omega$
Bessel	3.3	0.195	0.01 $\mu$ F	33 nF	15.4 k $\Omega$	3.01 k $\Omega$
3-dB Chebyshev	15	10.268	0.01 $\mu$ F	150 nF	9.53 k $\Omega$	2.55 k $\Omega$

The circuits are simulated using a TLV9062 operational amplifier, 1%-tolerance resistors, and 2%-tolerance capacitors. Figure 10-2 through Figure 10-5 show the frequency response of the circuits.

Figure 10-2 compares the frequency response of Sallen-Key and MFB second-order Butterworth filters. The frequency response of the filters is almost identical from 10 Hz to about 80 kHz.

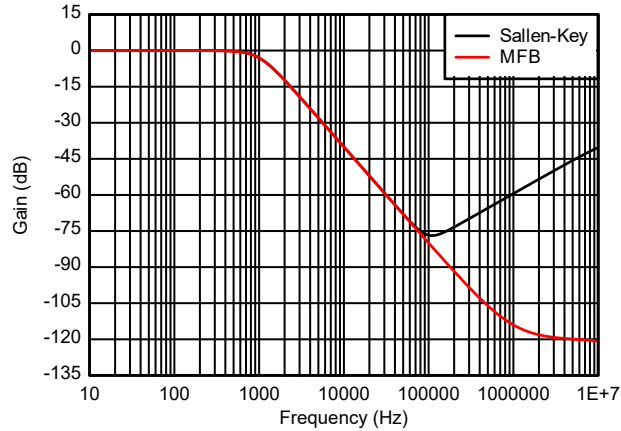


Figure 10-2. Second-Order Butterworth Filter Frequency Response

Figure 10-3 compares the frequency response of Sallen-Key and MFB second-order Bessel filters. The frequency response of the filters is almost identical from 10 Hz to about 90 kHz. Above 90 kHz, the MFB has excellent performance.

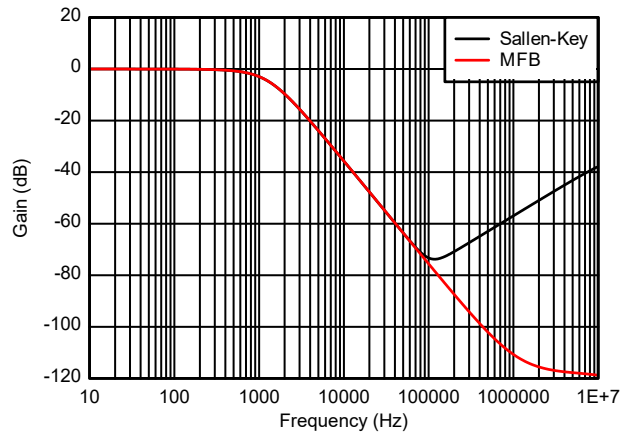


Figure 10-3. Second-Order Bessel Filter Frequency Response

Figure 10-4 compares the frequency response of Sallen-Key and MFB second-order 3-dB Chebyshev filters. The frequency response of the filters is almost identical from 10 Hz to about 60 kHz. Above 60 kHz, the MFB shows better performance.

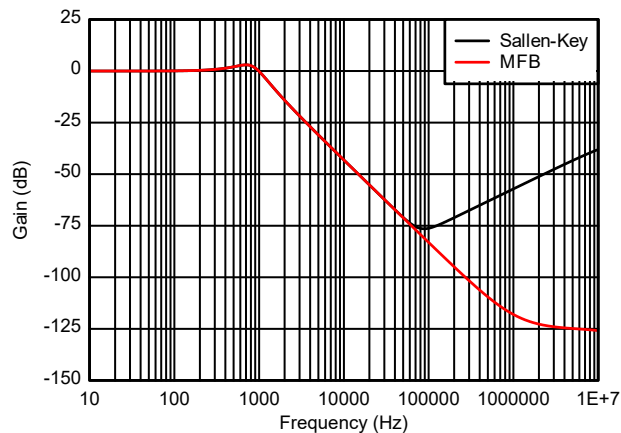


Figure 10-4. Second-Order 3-dB Chebyshev Filter Frequency Response

Figure 10-5 is an expanded view of the frequency response of the three filters in the MFB topology, near  $f_c$  (the Sallen-Key circuits are almost identical). The graph shows the increased rate of attenuation near the cutoff frequency, with the Bessel filter being the slowest and the 3-dB Chebyshev filter being the fastest.

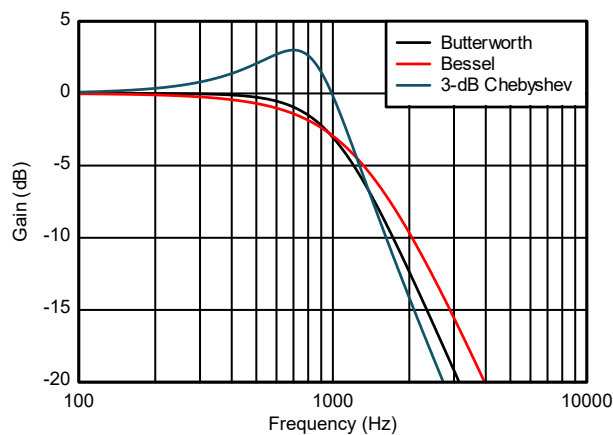


Figure 10-5. Second-Order Butterworth, Bessel, and 3-dB Chebyshev Filter Frequency Response

## 11 Non-ideal Circuit Operation

Up to now, non-ideal operation of the circuits has not been detailed. The following section examines both Sallen-Key and MFB high-frequency behavior and PSpice simulation examples using the TLV9062.

### 11.1 Non-ideal Circuit Operation: Sallen-Key

At frequencies well above the cutoff, simplified high-frequency models help show the expected behavior of the circuits. Figure 11-1 is used to show the expected circuit operation for a second-order low-pass Sallen-Key circuit at high frequency. The assumption made here is that  $C_1$  and  $C_2$  are effective shorts when compared to the impedance of  $R_1$  and  $R_2$ , so the input of the amplifier is at AC ground. In response, the amplifier generates an AC ground at the output, limited only by the closed-loop output impedance  $Z_{OUT}$ . Different amplifiers have different closed-loop output impedances  $Z_{OUT}$  and can impact the high frequency filter response based on  $Z_{OUT}$ .

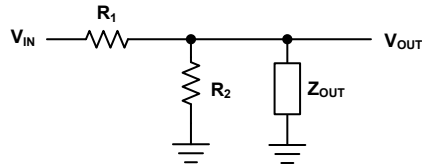


Figure 11-1. Second-Order Low-Pass Sallen-Key High-Frequency Model

Placing a low-pass RC filter at the output of the amplifier can help nullify the feedthrough of high-frequency signals. Figure 11-2 shows a comparison between the original Sallen-Key Butterworth filter and one using an RC filter on the output. A 100- $\Omega$  resistor is placed in series with the output and a 47-nF capacitor is connected from the output to ground. This places a passive pole in the transfer function at about 40 kHz that improves the high-frequency response.

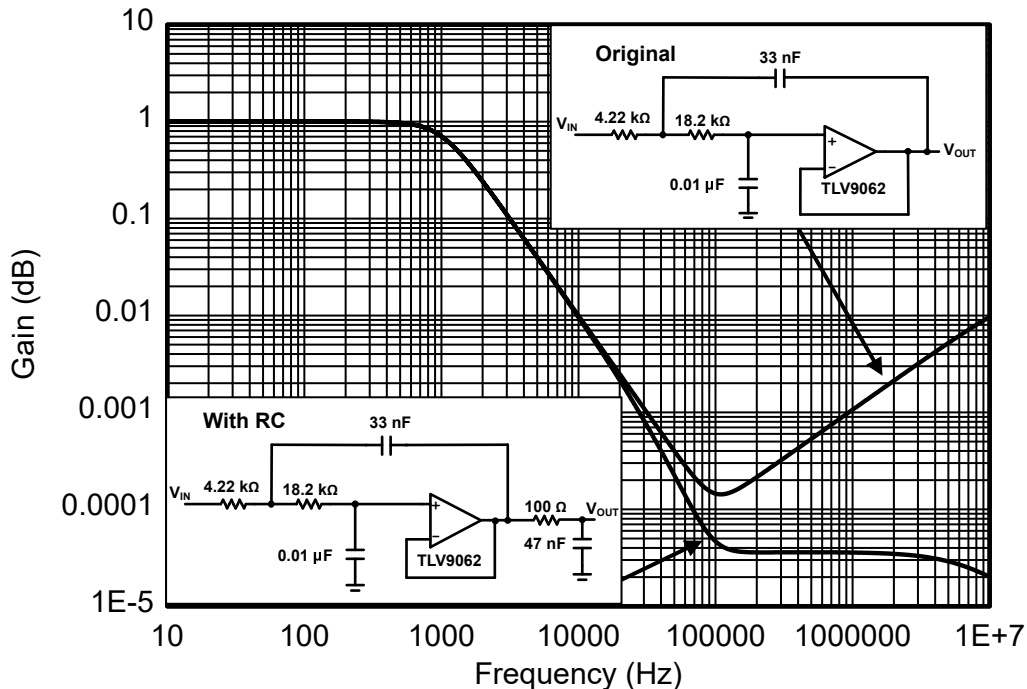


Figure 11-2. Sallen-Key Butterworth Filter With RC Added in Series With the Output



## 11.2 Non-ideal Circuit Operation: MFB

Figure 11-3 is used to show the expected circuit operation for a second-order low-pass MFB circuit at high frequency. The assumption made here is that  $C_1$  and  $C_2$  are effective shorts when compared to the impedance of  $R_1$ ,  $R_2$ , and  $R_3$ . Again, the input of the amplifier is at AC ground, and generates an AC ground at the output limited only by the closed-loop output impedance  $Z_{OUT}$ . Capacitor  $C_P$  represents the parasitic capacitance from  $V_{IN}$  to  $V_{OUT}$ . The ability of the circuit to attenuate high-frequency signals is dependent on  $C_P$  and  $Z_{OUT}$ . Different amplifiers have different closed-loop output impedances  $Z_{OUT}$  and can impact the high frequency filter response based on  $Z_{OUT}$ .

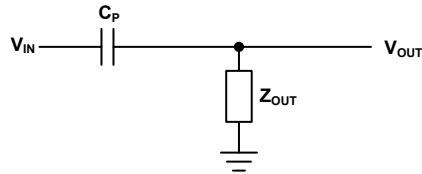


Figure 11-3. Second-Order Low-Pass MFB High-Frequency Model

When routing the input and output signals make sure to keep capacitive coupling to a minimum.

Placing a low-pass RC filter at the output of the amplifier can help nullify the feedthrough of high-frequency signals. Figure 11-4 shows a comparison between the original MFB Butterworth filter and one using an RC filter on the output. A 100- $\Omega$  resistor is placed in series with the output and a 47-nF capacitor is connected from the output to ground. This places a passive pole in the transfer function at about 40 kHz that improves the high-frequency response.

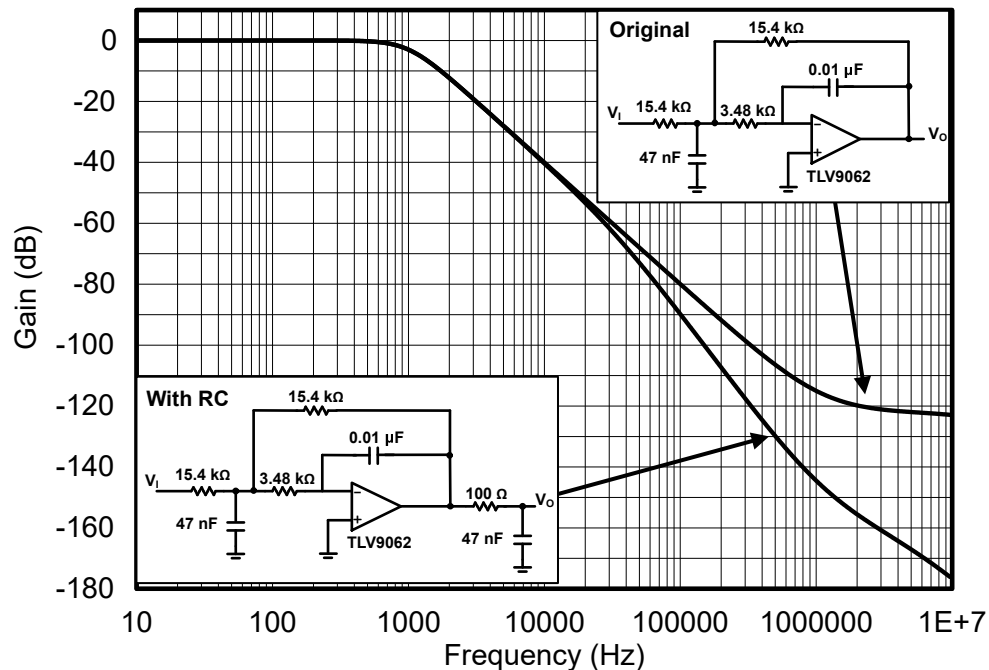


Figure 11-4. MFB Butterworth Filter With RC Added in Series With the Output

## 12 Comments About Component Selection

Theoretically, any values of R and C which satisfy the equations can be used, but practical considerations call for certain guidelines to be followed.

Given a specific corner frequency, the values of R and C are inversely proportional to each other. By making C larger, R becomes smaller, and vice versa.

Making R large can make C so small that parasitic capacitances cause errors. However, smaller values of R makes the current in the circuit larger, which consumes more power as a trade-off.

The best choice of component values depends on the particular circuit and the tradeoffs the designer is willing to make. Adhering to the following general recommendations helps reduce errors:

- **Capacitors**
  - Avoid values less than 10 pF
  - Use C0G (NP0) dielectrics
  - Use 1%-tolerance components
  - Surface mount is preferred
- **Resistors**
  - Values in the range of a few hundred ohms to a few thousand ohms are best
  - Use metal film with low temperature coefficients
  - Use 1% tolerance (or better)
  - Surface mount is preferred

## 13 Conclusion

This document has explored building second-order low-pass Butterworth, Bessel, and 3-dB Chebyshev filters using the Sallen-Key and MFB architectures. The same techniques are extended to higher-order filters by cascading second-order stages for even order and adding a first-order stage for odd order.

The advantages of each filter type come at the expense of other characteristics. The Butterworth can be considered to offer the best all-around filter response. The filter has maximum flatness in the pass band with moderate rolloff past cutoff and shows only slight overshoot in response to a pulse input.

The Bessel is important when signal-conditioning square wave signals. The constant group delay means that the square wave signal is passed with minimum distortion (overshoot). This comes at the expense of a slower rate of attenuation above cutoff.

The 3-dB Chebyshev sacrifices pass-band flatness for a high rate of attenuation near cutoff. This filter also exhibits the largest overshoot and ringing in response to a pulse input of the three filter types discussed.

The Sallen-Key and MFB architectures also have some trade-offs. The simplifications that can be used when designing the Sallen-Key provide for easier selection of circuit components, and at unity gain, Sallen-Key has no gain sensitivity to component variations. The MFB shows less overall sensitivity to component variations and has better high-frequency performance.

## A Filter Design Specifications

### A.1 Sallen-Key Design Simplifications

Filter design can be simple or tedious, depending on the method used to solve the equations. The following simplifications are ordered from harder to easier, but note that the easier the design becomes, the more the design freedom is limited.

#### A.1.1 Sallen-Key Simplification 1: Set Filter Components as Ratios

Letting  $R_1 = mR$ ,  $R_2 = R$ ,  $C_1 = C$ , and  $C_2 = nC$  results in  $FSF \times f_c = \frac{1}{2\pi RC\sqrt{mn}}$  and  $Q = \frac{\sqrt{mn}}{m+1+mn(1-K)}$ . Start the design by determining the ratios  $m$  and  $n$  required for the gain and  $Q$  of the filter, and then selecting  $C$  and calculating  $R$  to set  $f_c$ .

#### A.1.2 Sallen-Key Simplification 2: Set Filter Components as Ratios and Gain = 1

Letting  $R_1 = mR$ ,  $R_2 = R$ ,  $C_1 = C$ ,  $C_2 = nC$ , and  $K = 1$  results in  $FSF \times f_c = \frac{1}{2\pi RC\sqrt{mn}}$  and  $Q = \frac{\sqrt{mn}}{m+1}$ . This sets the gain to 0 dB in the pass band. Start the design by determining the ratios  $m$  and  $n$  for the required  $Q$  of the filter, and then selecting  $C$  and calculating  $R$  to set  $f_c$ .

#### A.1.3 Sallen-Key Simplification 3: Set Resistors as Ratios and Capacitors Equal

Letting  $R_1 = mR$ ,  $R_2 = R$ , and  $C_1 = C_2 = C$  results in  $FSF \times f_c = \frac{1}{2\pi RC\sqrt{m}}$  and  $Q = \frac{\sqrt{m}}{1+m(2-K)}$ . The main motivation behind setting the capacitors instead of resistors equal is the limited selection of values in comparison to resistors.

There is interaction between setting  $f_c$  and  $Q$ . Start the design by choosing  $m$  and  $K$  to set the  $Q$  of the circuit, and then choosing  $C$  and calculating  $R$  to set  $f_c$ .

#### A.1.4 Sallen-Key Simplification 4: Set Filter Components Equal

Letting  $R_1 = R_2 = R$  and  $C_1 = C_2 = C$  results in  $FSF \times f_c = \frac{1}{2\pi RC}$  and  $Q = \frac{1}{3-K}$ . With this simplification,  $f_c$  and  $Q$  are now independent.  $Q$  is now determined solely by the gain of the circuit.  $f_c$  is set by the choice of  $RC$ —choose  $C$  and calculate the corresponding  $R$ . Since the gain controls the  $Q$  of the circuit, further gain or attenuation is necessary to achieve the desired signal level in the pass band.

### A.2 MFB Design Simplifications

The MFB does not have as many simplifications as the Sallen-Key, but the following simplifications provide guidance for two MFB situations.

#### A.2.1 MFB Simplification 1: Set Filter Components as Ratios

Letting  $R_2 = R$ ,  $R_3 = mR$ ,  $C_1 = C$ , and  $C_2 = nC$  results in  $FSF \times f_c = \frac{1}{2\pi RC\sqrt{mn}}$  and  $Q = \frac{\sqrt{mn}}{1+m(1-K)}$ . Start the design by determining the ratios  $m$  and  $n$  required for the gain and  $Q$  of the filter, and then selecting  $C$  and calculating  $R$  to set  $f_c$ .

#### A.2.2 MFB Simplification 2: Set Filter Components as Ratios and Gain = -1

Letting  $R_2 = R$ ,  $R_3 = mR$ ,  $C_1 = C$ ,  $C_2 = nC$ , and  $K = -1$  results in  $FSF \times f_c = \frac{1}{2\pi RC\sqrt{mn}}$  and  $Q = \frac{\sqrt{mn}}{1+2m}$ . This sets the gain to 0 dB in the pass band. Start the design by determining the ratios  $m$  and  $n$  for the required  $Q$  of the filter, and then selecting  $C$  and calculating  $R$  to set  $f_c$ .

## B Higher-Order Filters

This application note previously stated that higher-order filters can be constructed by cascading second-order stages for even-order, and adding a first-order stage for odd-order. To show how this is accomplished, two examples are considered: constructing a fifth-order Butterworth filter and a sixth-order Bessel filter.

By breaking higher than second-order filters into complex-conjugate zero pairs, second-order stages are constructed that, when cascaded, realize the overall polynomial. For example, a sixth-order filter has three complex-zero pairs and can be written as:

$$P_{6\text{th}}(s) = (s + z_1)(s + z_1^*)(s + z_2)(s + z_2^*)(s + z_2)(s + z_3^*) \quad (23)$$

Each of the complex-conjugate-zero pairs can be multiplied out and written as:

$$(s + z_1)(s + z_1^*) = s^2 + a_{1,1}s + a_{0,1} \quad (24)$$

$$(s + z_2)(s + z_2^*) = s^2 + a_{1,2}s + a_{0,2} \quad (25)$$

$$(s + z_3)(s + z_3^*) = s^2 + a_{1,3}s + a_{0,3} \quad (26)$$

The overall polynomial is then reconstructed in the following form:

$$P_{6\text{th}}(s) = (s^2 + a_{1,1}s + a_{0,1})(s^2 + a_{1,2}s + a_{0,2})(s^2 + a_{1,3}s + a_{0,3}) \quad (27)$$

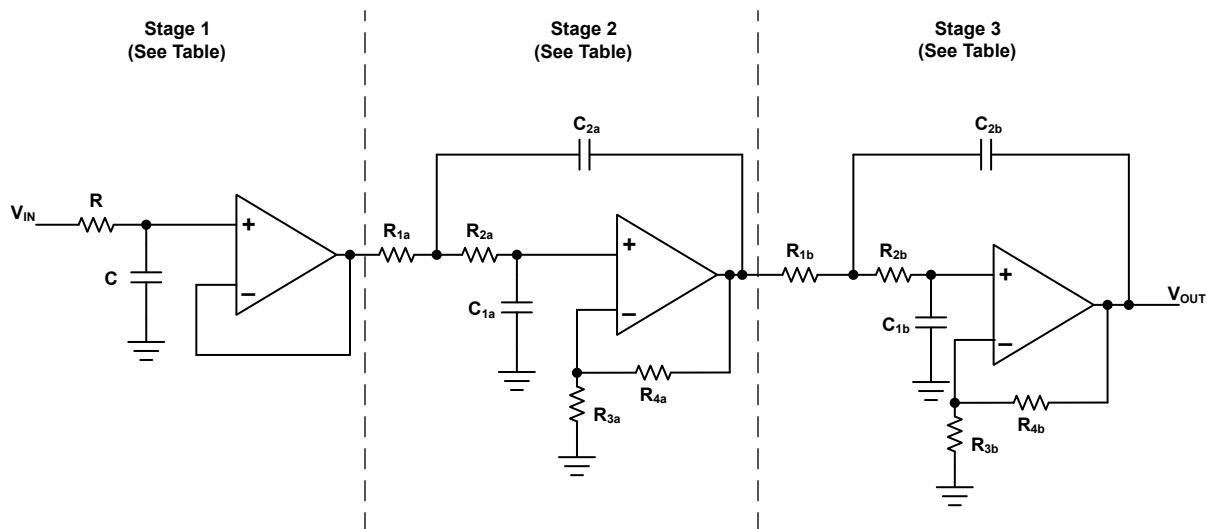
The circuit implementation consists of three second-order stages cascaded to form the overall response.

### B.1 Fifth-Order Low-Pass Butterworth Filter

Referring to [Table 9-1](#), for a fifth-order Butterworth filter, the required circuit transfer function can be written as:

$$H_{\text{LP}}(f) = \frac{K}{\left(\frac{jf}{f_c} + 1\right) \left(-\left(\frac{f}{f_c}\right)^2 + \frac{1}{0.6180} \times \frac{jf}{f_c} + 1\right) \left(-\left(\frac{f}{f_c}\right)^2 + \frac{1}{0.6180} \times \frac{jf}{f_c} + 1\right)} \quad (28)$$

[Figure B-1](#) shows a Sallen-Key circuit implementation and the required component values.  $f_c$  is the  $-3$ -dB point. The overall gain of the circuit in the pass band is  $K = K_a \times K_b$ .



**Figure B-1. Fifth-Order Low-Pass Filter Topology Cascading Two Sallen-Key Stages and an RC**

Stage	$f_c$	$Q$	$K$
1	$\frac{1}{2\pi RC}$	N/A	1
2	$\frac{1}{2\pi\sqrt{R_{1a}R_{2a}C_{1a}C_{2a}}}$	$\frac{\sqrt{R_{1a}R_{2a}C_{1a}C_{2a}}}{R_{1a}C_{1a} + R_{2a}C_{1a} + R_{1a}C_{2a}(1 - K_a)} = 0.618$	$K_a = \frac{R_{3a} + R_{4a}}{R_{3a}}$
3	$\frac{1}{2\pi\sqrt{R_{1b}R_{2b}C_{1b}C_{2b}}}$	$\frac{\sqrt{R_{1b}R_{2b}C_{1b}C_{2b}}}{R_{1b}C_{1b} + R_{2b}C_{1b} + R_{1b}C_{2b}(1 - K_b)} = 1.618$	$K_b = \frac{R_{3b} + R_{4b}}{R_{3b}}$

### B.2 Sixth-Order Low-Pass Bessel Filter

Referring to Table 9-2, for a sixth-order Bessel filter, we can write the required circuit transfer function as:

$$H_{LP}(f) = \frac{K}{\left(-\left(\frac{f}{1.6060 f_c}\right)^2 + 1.2202 \times \frac{jf}{f_c} + 1\right)\left(-\left(\frac{f}{1.6913 f_c}\right)^2 + 0.9674 \times \frac{jf}{f_c} + 1\right)\left(-\left(\frac{f}{1.9071 f_c}\right)^2 + 0.5124 \times \frac{jf}{f_c} + 1\right)} \quad (29)$$

Figure B-2 shows an MFB circuit implementation and the required component values.  $f_c$  is the  $-3$ -dB point. The overall gain of the circuit in the pass band is  $K = K_a \times K_b \times K_c$ .

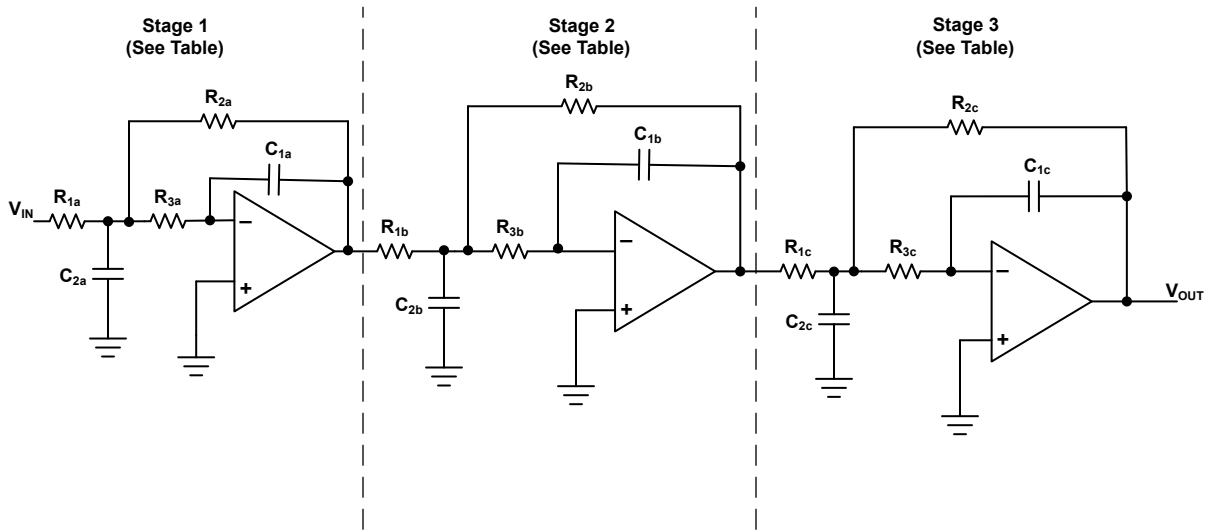


Figure B-2. Sixth-Order Low-Pass Filter Topology Cascading Three MFB Stages

Stage	$f_c$	$Q$	$K$
1	$\frac{1}{1.6060 \times 2\pi\sqrt{R_{2a}R_{3a}C_{1a}C_{2a}}}$	$\frac{\sqrt{R_{2a}R_{3a}C_{1a}C_{2a}}}{R_{3a}C_{1a} + R_{2a}C_{1a} + R_{3a}C_{1a}(-K_a)} = 0.5103$	$K_a = \frac{-R_{2a}}{R_{1a}}$
2	$\frac{1}{1.6913 \times 2\pi\sqrt{R_{2b}R_{3b}C_{1b}C_{2b}}}$	$\frac{\sqrt{R_{2b}R_{3b}C_{1b}C_{2b}}}{R_{3b}C_{1b} + R_{2b}C_{1b} + R_{3b}C_{1b}(-K_b)} = 0.6112$	$K_b = \frac{-R_{2b}}{R_{1b}}$
3	$\frac{1}{1.9071 \times 2\pi\sqrt{R_{2c}R_{3c}C_{1c}C_{2c}}}$	$\frac{\sqrt{R_{2c}R_{3c}C_{1c}C_{2c}}}{R_{3c}C_{1c} + R_{2c}C_{1c} + R_{3c}C_{1c}(-K_c)} = 1.0234$	$K_c = \frac{-R_{2c}}{R_{1c}}$

### Revision History

NOTE: Page numbers for previous revisions may differ from page numbers in the current version.

#### Changes from Revision B (September 2002) to Revision C (November 2022)

Page

- Updated nomenclature and new op amp devices..... 1

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