

AC Error Equations for Operational Amplifiers

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ABSTRACT

This report develops the transfer function and loop gain of nonideal operational-amplifier (op-amp) circuits so the ac errors of the op amp can be analyzed. It describes inverting, noninverting, and differential op-amp circuits.

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1 Introduction

There are two types of error sources in op amps, and they fall under the general classification of dc and ac errors. Examples of dc errors are input offset voltage and input bias current. The dc errors stay constant over the usable op-amp frequency range; therefore, the input bias current is 10 pA at 1 kHz and it is 10 pA at 10 kHz. Because of their constant and controlled behavior, dc errors are not considered.

AC errors are flighty, so they are addressed here by developing a set of nonideal equations that account for ac errors. The ac errors may show up under dc conditions, but they get worse as the operating frequency increases. A good example of an ac error is common-mode rejection ratio (CMRR). Most op amps have a guaranteed CMRR specification, but this specification is only valid at dc or very low frequencies. Further inspection of the data sheet reveals that CMRR decreases as operating frequency increases. Several other specifications that fall into the category of ac specifications are output impedance, power-supply rejection-ratio, peak-to-peak output voltage, differential gain, differential phase, and phase margin.

Differential gain is the most important ac specification because the other ac specifications are derived from it. Until now, differential gain has been called op-amp gain or op-amp open-loop gain, and we shall continue with that terminology. (Data sheets call it differential gain.)

When frequency increases, the op-amp gain decreases and errors increase. This application note develops the equations that illustrate the effects of the gain changes. It starts with a review of the basic canonical feedback system stability because the op-amp equations are developed using the same techniques.

Amplifiers are built with active components such as transistors. Pertinent transistor parameters like transistor gain are subject to drift and initial inaccuracies from many sources, so the amplifiers being built from these components are also subject to drift and inaccuracies. The drift and inaccuracy is minimized or eliminated by using negative feedback. The op-amp circuit configuration employs feedback to make the transfer equation of the circuit independent of the amplifier parameters (well, almost), and while doing this, the circuit transfer function is made dependent on external passive components. The external passive components can be purchased to meet almost any drift or accuracy specification; only the cost and size of the passive components limit their use.

Once feedback is applied to the op amp, it is possible for the op-amp circuit to become unstable. Certain amplifiers belong to a family called internally-compensated op amps; they contain internal capacitors that are sometimes advertised as precluding instabilities. Although internally-compensated op amps should not oscillate when operated under specified conditions, many have relative stability problems that manifest themselves as poor phase response, ringing, and overshoot. The only absolutely stable internally-compensated op amp is the one lying on the workbench without power applied! All other internally-compensated op amps oscillate under some external circuit conditions. See reference [1] (SLOA020) for more details.

Noninternally-compensated or *externally*-compensated op amps are unstable without the addition of external stabilizing components. This situation is a disadvantage in many cases because they require additional components, but the lack of internal compensation enables the top-drawer circuit designer to squeeze the last drop of performance from the op amp. There are two options: op amps internally compensated by the IC manufacturer, or op amps externally compensated by you. Compensation, except that done by the op-amp manufacturer, must be done external to the IC. Surprisingly enough, internally-compensated op amps require external compensation for demanding applications.

Compensation is achieved by adding external components that modify the circuit transfer function so that it becomes unconditionally stable. There are several different methods of compensating an op amp, and as you might suspect, there are pros and cons associated with each method of compensation. After the op-amp circuit is compensated, it must be analyzed to determine the effects of compensation. The modifications that compensation have on the closed-loop transfer function often determine which compensation scheme is most profitably employed.

2 Review of the Canonical Equations

A block diagram for a generalized feedback system is shown in Figure 1. This simple block diagram is sufficient to determine the stability of any system. See reference [2] (SLOA017) for more details.

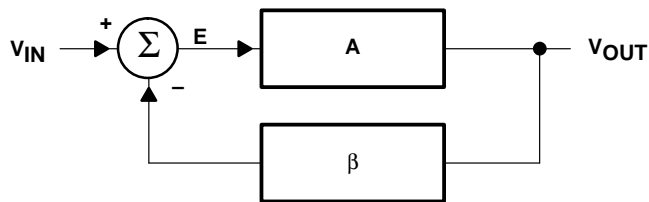


Figure 1. Feedback System Block Diagram

The output and error equation development is as follows.

$$V_{OUT} = EA \quad (1)$$

$$E = V_{IN} - \beta V_{OUT} \quad (2)$$

Combining Equations 1 and 2 yields Equation 3:

$$\frac{V_{OUT}}{A} = V_{IN} - \beta V_{OUT} \quad (3)$$

Collecting terms gives Equation 4:

$$V_{OUT} \left(\frac{1}{A} + \beta \right) = V_{IN} \quad (4)$$

Rearranging terms yields the classic form of the feedback equation.

$$\frac{V_{OUT}}{V_{IN}} = \frac{A}{1 + A\beta} \quad (5)$$

Notice that Equation 5 reduces to Equation 6 when the quantity $A\beta$ in Equation 5 becomes very large with respect to one. Equation 6 is called the ideal feedback equation because it depends on the assumption that $A\beta \gg 1$, and it finds extensive use when amplifiers are assumed to have ideal qualities. Under the conditions that $A\beta \gg 1$, the system gain is determined by the feedback factor, β . Stable passive circuit components are used to implement the feedback factor; thus, the ideal closed-loop gain is predictable and stable because β is predictable and stable.

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{\beta} \quad (6)$$

The quantity $A\beta$ is so important that it has been given a special name, *loop gain*. Consider Figure 2. When the voltage inputs are grounded (current inputs are opened) and the loop is broken, the calculated gain is the loop gain, $A\beta$. Now, keep in mind that this is the arithmetic of complex numbers, which have magnitude and direction. When the loop gain approaches negative one, or to express it mathematically $1 \angle -180^\circ$, the denominator on the right hand side of Equation 5 approaches zero so the right hand side itself approaches infinity. The circuit output heads for infinity. If the output were not energy limited the circuit would explode the world, but it is energy limited by the power supplies, so the world stays intact.

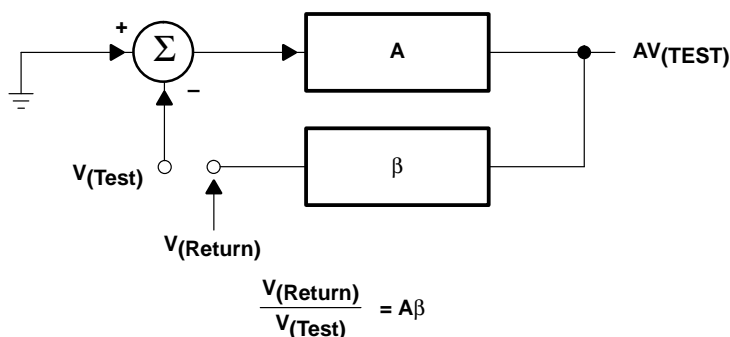


Figure 2. Feedback Loop Broken to Calculate Loop Gain

Active devices in electronic circuits exhibit nonlinear behavior when their output approaches a power-supply rail, and the nonlinearity reduces the amplifier gain until the loop gain is no longer $1 \angle -180^\circ$. Now the circuit could do two things: first, it could become stable at the power supply limit, or second, it could reverse direction (because stored charge keeps the output voltage changing) and head for the negative power-supply rail.

The first state, where the circuit becomes stable at a power supply limit, is called *lockup*; the circuit will remain in the locked up state until power is removed. The second state, where the circuit bounces between power supply limits is called *oscillatory*. Remember, the loop gain, $A\beta$, is the sole factor that determines stability for a circuit or system. Inputs are grounded or disconnected when the loop gain is calculated, so they have no effect on stability.

Equations 1 and 2 are combined and rearranged to yield Equation 7, which gives an indication of system or circuit error.

$$E = \frac{V_{IN}}{1 + A\beta} \quad (7)$$

First, notice that the error is proportional to the input signal. This is the expected result because a bigger input signal results in a bigger output signal, and bigger output signals require more drive voltage. Second, the loop gain is inversely proportional to the error. As the loop gain increases the error decreases, thus large loop gains are attractive for minimizing errors. However, large loop gains also decrease stability; thus, there is always a tradeoff between error and stability.

3 Noninverting Op Amps

A noninverting op amp is shown in Figure 3. The dummy variable, V_B , is inserted to make the calculations easier and a is the op amp gain. See reference [3] (SLOA025) for more details.

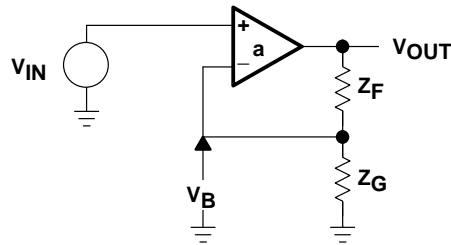


Figure 3. Noninverting Op Amp

Equation 8 is the amplifier transfer equation.

$$V_{OUT} = a(V_{IN} - V_B) \quad (8)$$

The output equation is developed with the aid of the voltage divider rule. Using the voltage divider rule assumes that the op-amp impedance is low.

$$V_B = \frac{V_{OUT}Z_G}{Z_F + Z_G} \text{ for } I_B = 0 \quad (9)$$

Combining Equations 8 and 9 yields Equation 10.

$$V_{OUT} = aV_{IN} - \frac{aZ_G V_{OUT}}{Z_G + Z_F} \quad (10)$$

Rearranging terms in Equation 10 yields Equation 11, which describes the transfer function of the circuit.

$$\frac{V_{OUT}}{V_{IN}} = \frac{a}{1 + \frac{aZ_G}{Z_G + Z_F}} \quad (11)$$

Equation 5 is repeated as Equation 12 to make a term by term comparison of the equations easy.

$$\frac{V_{OUT}}{V_{IN}} = \frac{A}{1 + A\beta} \quad (12)$$

By virtue of the comparison we get Equation 13, which is the loop-gain equation for the noninverting op amp. The loop-gain equation determines the stability of the circuit. The comparison also shows that the open-loop gain, A , is equal to the op-amp open-loop gain, a , for the noninverting circuit.

$$A\beta = \frac{aZ_G}{Z_G + Z_F} \quad (13)$$

Equation 13 is also derived with the aid of Figure 4, which shows the open-loop noninverting op amp.

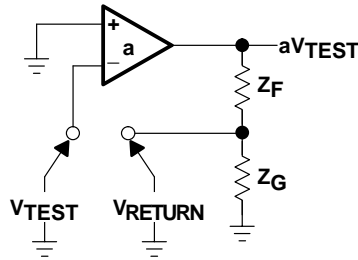


Figure 4. Open Loop Noninverting Op Amp

The test voltage, V_{TEST} , is multiplied by the op-amp open-loop gain to obtain the op-amp output voltage, aV_{TEST} . The voltage divider rule is used to calculate Equation 15, which is identical to Equation 14 after some algebraic manipulation.

$$V_{RETURN} = \frac{aV_{TEST} Z_G}{Z_F + Z_G} \quad (14)$$

$$\frac{V_{RETURN}}{V_{TEST}} = A\beta = \frac{aZ_G}{Z_F + Z_G} \quad (15)$$

4 Inverting Op Amps

The inverting op-amp circuit is shown in Figure 5. The dummy variable (V_A) is inserted to make the calculations easier, and a is the op amp open loop gain. See reference [3] (SLOA025) for more details.

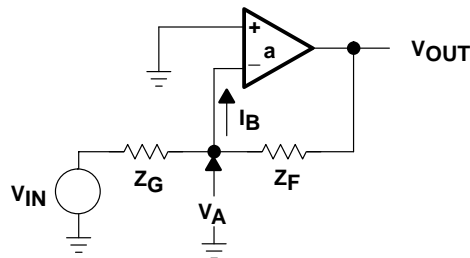


Figure 5. Inverting Op Amp

The transfer equation is:

$$V_{OUT} = -aV_A \quad (16)$$

The node voltage (Equation 17) is obtained with the aid of superposition and the voltage divider rule. Equation 18 is obtained by combining Equations 16 and 17.

$$V_A = \frac{V_{IN} Z_F}{Z_G + Z_F} + \frac{V_{OUT} Z_G}{Z_G + Z_F} \text{ for } I_B = 0 \quad (17)$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{\frac{-aZ_F}{Z_G + Z_F}}{1 + \frac{aZ_G}{Z_G + Z_F}} \quad (18)$$

Equation 16 is the transfer function of the inverting op amp. By virtue of the comparison between Equations 18 and 14, we get Equation 15 again, which is also the loop gain equation for the inverting op-amp circuit. The comparison also shows that the open-loop gain (A) is different from the op-amp open-loop gain (a) for the noninverting circuit.

The inverting op amp with the feedback loop broken is shown in Figure 6, and this circuit is used to calculate the loop gain given in Equation 19.

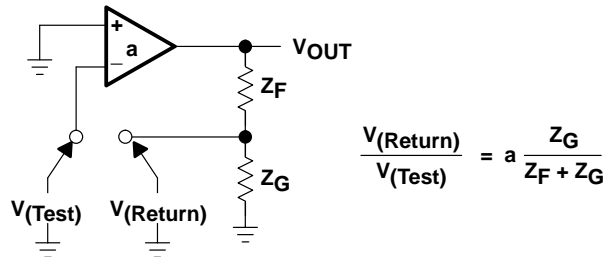


Figure 6. Inverting Op Amp: Feedback Loop Broken for Loop Gain Calculation

$$\frac{V_{\text{RETURN}}}{V_{\text{TEST}}} = \frac{aZ_G}{Z_G + Z_F} = A\beta \quad (19)$$

Several things must be mentioned at this point in the analysis. First, the transfer functions for the noninverting and inverting cases, Equations 13 and 18, are different. For a common set of Z_G and Z_F values, the magnitude and polarity of the gains are different. Second, the loop gain of both circuits, as given by Equations 15 and 19, is identical. Thus, the stability performance of both circuits is identical even though their transfer equations are different. This makes the important point that *stability is not dependent on the circuit inputs*. Third, the A -gain block shown in Figure 1 is different for each op-amp circuit. By comparing Equations 5, 11, and 18, we see that $A_{\text{NON-INV}} = a$ and $A_{\text{INV}} = aZ_F/(Z_G + Z_F)$.

5 Differential Op Amps

The differential-amplifier circuit is shown in Figure 7. The dummy variable, V_E , is inserted to make the calculations easier, and a is the open loop gain. See reference [3] (SLOA025) for more details.

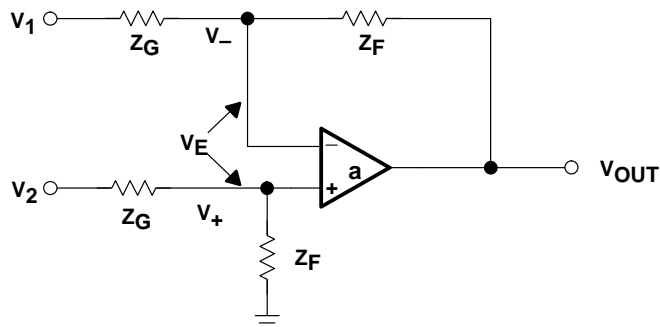


Figure 7. Differential Amplifier Circuit

Equation 20 is the circuit transfer equation.

$$V_{\text{OUT}} = a V_E = V_+ - V_- \quad (20)$$

The positive input voltage, V_+ , is written in Equations 21 and 22 with the aid of superposition and the voltage divider rule.

$$V_+ = V_2 \frac{Z_F}{Z_F + Z_G} \quad (21)$$

$$V_- = V_1 \frac{Z_F}{Z_F + Z_G} - V_{\text{OUT}} \frac{Z_G}{Z_F + Z_G} \quad (22)$$

Combining Equations 20, 21, and 22 yields Equation 23.

$$V_{\text{OUT}} = a \left[\frac{V_2 Z_F}{Z_F + Z_G} - \frac{V_1 Z_F}{Z_F + Z_G} - \frac{V_{\text{OUT}} Z_G}{Z_F + Z_G} \right] \quad (23)$$

After algebraic manipulation, Equation 23 reduces to Equation 24.

$$\frac{V_{\text{OUT}}}{V_2 - V_1} = \frac{\frac{aZ_F}{Z_F + Z_G}}{1 + \frac{aZ_G}{Z_F + Z_G}} \quad (24)$$

The comparison method reveals that the loop gain as shown in Equation 25 is identical to that shown in Equations 13 and 19.

$$A\beta = \frac{aZ_G}{Z_G + Z_F} \quad (25)$$

Again, the loop gain, which determines stability, is only a function of the closed loop, and independent of the inputs.

6 Conclusions

A set of nonideal op-amp equations has been developed to account for the frequency dependence of ac errors. These ac errors are caused by inherent inaccuracies in the op-amp internal components and are subject to drift. As the frequency increases, the op-amp loop gain decreases and the errors increase. Negative feedback is used to minimize or eliminate the errors of the op amp, and to make the transfer function dependent upon the *external* components instead. It is often much easier to find external components to meet tight specifications required by a design.

The op-amp circuit may become unstable once feedback is applied. The circuit designer has two types of op amp from which to choose: compensated and uncompensated. Compensated op amps are designed by the manufacturer to be stable for a specific set of operating conditions. Uncompensated op amps are unstable and require the addition of external components for stability, yet they allow the maximum flexibility for the circuit designer. Both types of op amp use external components to modify the transfer function and to render the circuit unconditionally stable.

The combination of the op-amp open-loop gain and negative feedback results in loop gain. Loop gain is the sole factor determining circuit stability—it is a function of the closed loop and not of the amplifier inputs. The larger the loop gain, the smaller the op-amp errors. The trade-off is stability, which increases as the loop gain increases.

7 References

1. *Stability Analysis of Voltage-Feedback Op Amps Including Compensation Techniques*, Application Note, Texas Instruments Literature Number SLOA020
2. *Feedback Amplifier Analysis Tools*, Application Note, Texas Instruments Literature Number SLOA017
3. *Understanding Basic Analog-Circuit Equations*, Application Note, Texas Instruments Literature Number SLOA025

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