Operational amplifier gain stability, Part 1: General system analysis

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Introduction

The goal of this three-part series of articles is to provide a more in-depth understanding of gain error and how it can be influenced by the actual parameters of an operational amplifier (op amp) in a typical closed-loop configuration. This first article explores general feedback control system analysis and synthesis as they apply to first-order transfer functions. This analysis technique is then used to calculate the transfer functions of both noninverting and inverting op amp circuits. The second article will focus on DC gain error, which is primarily caused by the finite open-loop gain of the op amp as well as its temperature dependency. The third and final article will discuss one of the most common mistakes in calculating AC closed-loop gain errors. Often, during circuit analysis, system designers have the tendency to use DC-gain calculation methods for AC-domain analysis, which provides worse results than the real performance of the circuit. With these three articles, the system designer will have the simple tools required to determine the overall closed-loop gain error for any specific op amp by using its data-sheet parameters.

Steady-state sinusoidal analysis and Bode plots

Before the main topic of this article is discussed, it is appropriate to briefly review the concepts of sinusoidal-frequency analysis and Bode plots. These two concepts will be used repeatedly throughout this series of articles.

It is often useful to characterize a circuit by measuring its response to sinusoidal input signals. Fourier analysis can be used to reconstruct any periodic signal by summing sinusoidal signals with various frequencies. Thus, the circuit designers can gather useful information about a circuit's response to various input signals by characterizing its response to sinusoidal excitations over a wide frequency range.

When a linear circuit is driven by a sinusoidal input signal of a specific frequency, the output signal is also a sinusoidal signal of the same frequency. The complex representation of a sinusoidal waveform can be used to represent the input signal as

$$v_1(t) = V_1 \times e^{j(\omega t+\varphi_1)},$$

and the output signal as

$$v_2(t) = V_2 \times e^{j(\omega t+\varphi_2)}.$$

V_1 and V_2 are the amplitudes of the input and output signals, respectively; and φ_1 and φ_2 are the phase of the input and output signals, respectively. The ratio of the output signals to the input signals is the transfer function, $H(j\omega)$.

In a sinusoidal steady-state analysis, the transfer function can be represented as

$$H(j\omega) = \left|H(j\omega)\right| \times e^{j\varphi(\omega)}, \tag{1}$$

where $\left|H(j\omega)\right|$ is the magnitude of the transfer function, and φ is the phase. Both are functions of frequency.

One way to describe how the magnitude and phase of a transfer function vary over frequency is to plot them graphically. Together, the magnitude and phase plots of the transfer function are known as a Bode plot. The magnitude part of a Bode diagram plots the expression given by Equation 2 on a linear scale:

$$\left|H(j\omega)\right|_{dB} = 20\log_{10}\left|H(j\omega)\right| \tag{2}$$

The phase part of a Bode diagram plots the expression given by Equation 3, also on a linear scale:

$$\varphi = \angle H(j\omega) \tag{3}$$

Both the magnitude and the phase plots are plotted against a logarithmic frequency axis.

The benefit of plotting the logarithmic value of the magnitude instead of the linear magnitude of the transfer function is the ability to use asymptotic lines to approximate the transfer function. These asymptotic lines can be drawn quickly without having to use Equation 2 to calculate the exact magnitude and can still represent the magnitude of the transfer function with reasonable accuracy.

As an example, consider a first-order (single-pole) transfer function,

$$H(j\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega_0}}, \tag{4}$$

where ω_0 is the angular cutoff frequency of the system. The magnitude, in decibels, of the transfer function from Equation 4 can be described by Equation 5:

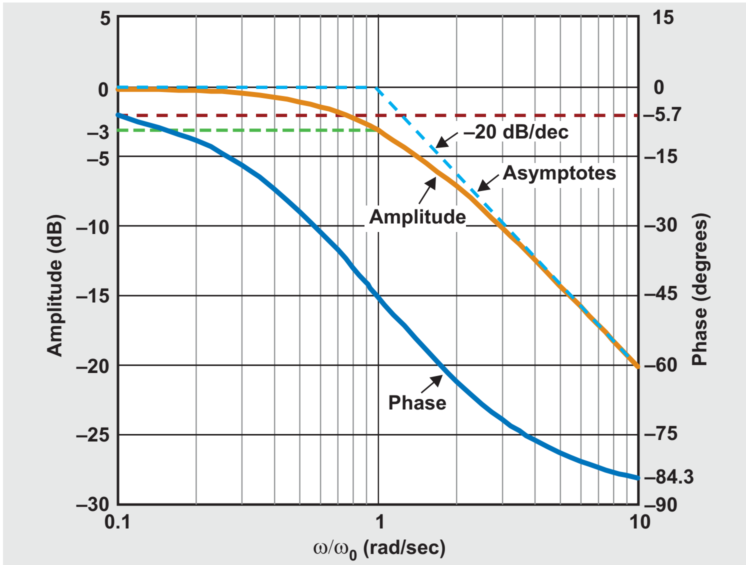
$$\left|H(j\omega)\right|_{dB} = 20\log\left|\frac{1}{1 + j\dfrac{\omega}{\omega_0}}\right| \tag{5}$$

The transfer function, $H(j\omega)$, is a complex function of the angular frequency, ω. To calculate the magnitude, both real and imaginary portions of the function need to be used:

$$\left|H(j\omega)\right|_{dB} = 20\log\frac{1}{\sqrt{1 + \dfrac{\omega^2}{\omega_0^2}}} \tag{6}$$

Equation 6 shows that at a frequency much lower than ω_0, the magnitude is near 1 V/V or 0 dB. At frequency

Figure 1. Bode plot of single-pole transfer function



$\omega = \omega_0$, the magnitude drops to $1/\sqrt{2} = 0.707$, or roughly –3 dB. Above this frequency, the magnitude rolls off at a rate of –20 dB/decade.

Both the real and imaginary parts of the transfer function can be used to calculate the phase response as

$$\varphi(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right). \tag{7}$$

Similarly, when the frequency is much lower than ω_0, the phase is 0°. At the frequency $\omega = \omega_0$, the phase is –45°. Finally, when the frequency is much higher than ω_0, the phase levels off at –90°.

Figure 1 shows the Bode plot of the first-order transfer function just described. Notice the use of the two asymptotic lines to simplify the magnitude plot of the transfer function. At the intersection of the two asymptotic lines, the simplified magnitude curve is off from the actual magnitude by about 3 dB. At frequencies much lower or much higher than ω_0, the error is negligible.

Deriving noninverting and inverting transfer functions

For simplicity, all the feedback networks in this article are shown as resistive networks. However, the analysis shown here will also be valid when these resistors are replaced with complex feedback networks.

Figure 2 depicts a typical noninverting op amp configuration. The closed-loop gain of the amplifier is set by two resistors: the feedback resistor, R_F, and the input resistor, R_I. The amount of output voltage, V_{OUT}, fed back to the feedback point is represented by the parameter β. The feedback point is the inverting input of the op amp. As stated, the β network is a simple resistive feedback network. From Figure 2, β is defined as

$$\beta = \frac{V_{FB}}{V_{OUT}} = \frac{R_I}{R_I + R_F}. \tag{8}$$

Figure 2. Typical noninverting op amp circuit with feedback

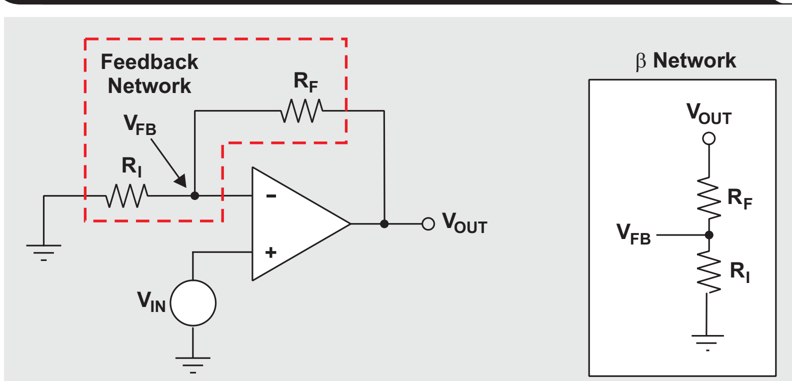


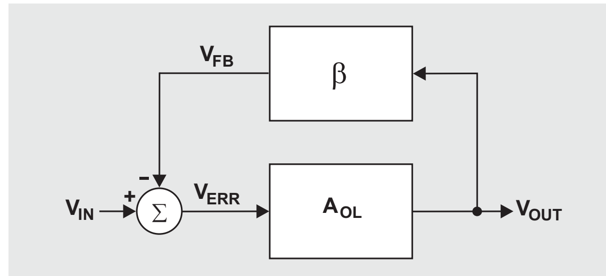
Figure 3 shows the control-loop model of the circuit in Figure 2. The parameter A_{OL} is the open-loop gain of the op amp and is always specified in any op amp data sheet. The control-loop model from Figure 3 can be used to express the closed-loop gain as

$$A_{CL} = \frac{V_{OUT}}{V_{IN}} = \frac{A_{OL}}{1+\beta\times A_{OL}}. \tag{9}$$

Assuming that this model is of a first-order system, the open-loop gain of an op amp as a function of angular frequency can be described as

$$A_{OL}(j\omega) = \frac{A_{OL\_DC}}{1+j\frac{\omega}{\omega_0}}. \tag{10}$$

Figure 3. Control-loop model of noninverting op amp circuit



The parameter A_{OL_DC} in Equation 10 is the open-loop gain of the op amp at a low frequency or at the DC level. The dominant pole of the op amp is given by the angular frequency, ω_0, or equivalently by $f_0 = \omega_0/2\pi$.

The Bode plot of the open-loop gain expression from Equation 10 is presented in Figure 4. Asymptotic curves are used in this figure to create a simplified version of the actual open-loop response. Now it is possible to express the closed-loop gain from Equation 9 in the frequency domain by replacing the parameter A_{OL} with Equation 10. After a few algebraic steps, the closed-loop transfer function can be written as

$$A_{CL}(j\omega) = \frac{\dfrac{A_{OL\_DC}}{1+\beta\times A_{OL\_DC}}}{1+j\dfrac{\omega}{\omega_0}\times\dfrac{1}{1+\beta\times A_{OL\_DC}}}. \tag{11}$$

$A_{CL}(j\omega)$ is a complex function of the angular frequency, ω. Also recall that to calculate the magnitude, both real and imaginary portions of the function need to be used in the same way that Equation 6 was obtained:

$$\left|A_{CL}(j\omega)\right|_{dB} = 20\log\frac{\dfrac{A_{OL\_DC}}{1+\beta\times A_{OL\_DC}}}{\sqrt{1+\dfrac{\omega^2}{\omega_0^2}\times\dfrac{1}{(1+\beta\times A_{OL\_DC})^2}}} \tag{12}$$

If the angular frequency, ω, is replaced with $2\pi f$, the closed-loop transfer function from Equation 12 can be rewritten as

$$\left|A_{CL}(jf)\right|_{dB} = 20\log\frac{\dfrac{A_{OL\_DC}}{1+\beta\times A_{OL\_DC}}}{\sqrt{1+\dfrac{f^2}{f_0^2}\times\dfrac{1}{(1+\beta\times A_{OL\_DC})^2}}}. \tag{13}$$

Figure 4. Open-loop gain vs. frequency of typical op amp

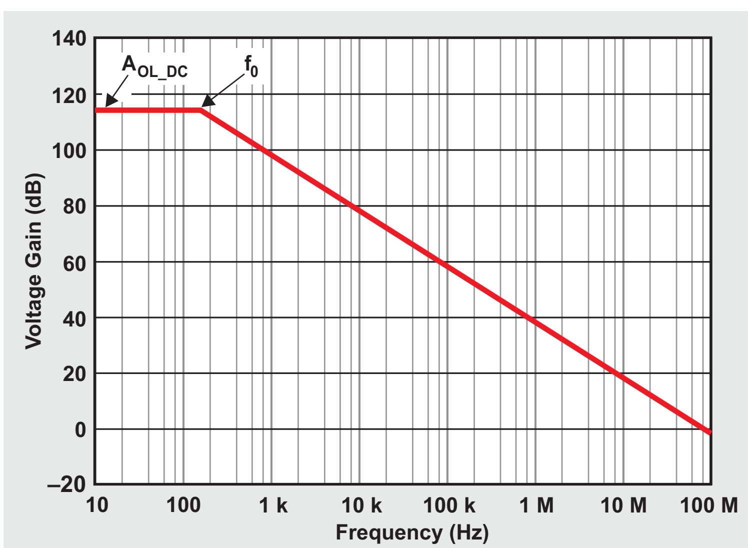
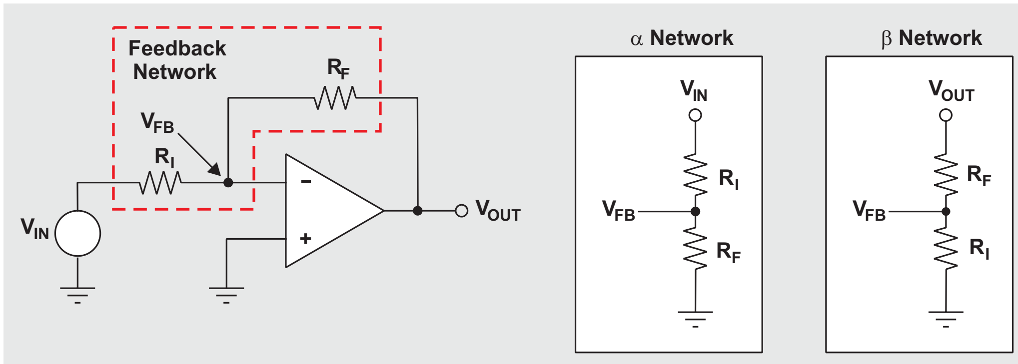


Figure 5 depicts a typical inverting op amp configuration. As with the analysis of the noninverting configuration, simple resistor networks are used that can be replaced with more complex functions. The closed-loop gain of the amplifier is again set by two resistors: the feedback resistor, R_F, and the input resistor, R_I. The amount of output voltage, V_{OUT}, fed back to the inverting input is again represented by β. In the inverting configuration, there is an additional signal arriving at the inverting node as a result of the input signal. The amount of this signal is represented by α.

Figure 5. Typical inverting op amp circuit with feedback



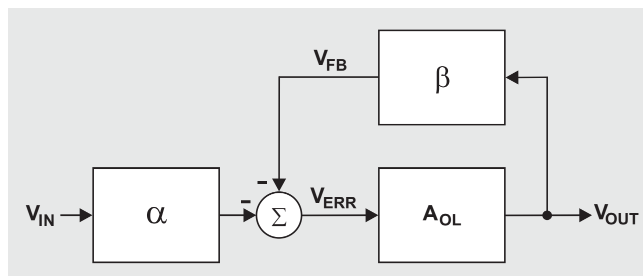
For the inverting op amp configuration, α is defined as

$$\alpha = \frac{V_{FB}}{V_{IN}} = \frac{R_F}{R_I + R_F}, \tag{14}$$

while β is defined by Equation 8.

Figure 6 shows the control-loop model of the circuit in Figure 5. This model can be used to express the closed-loop gain of the circuit as

Figure 6. Control-loop model of inverting op amp circuit



$$A_{CL} = \frac{V_{OUT}}{V_{IN}} = \frac{-\alpha \times A_{OL}}{1 + \beta \times A_{OL}}. \tag{15}$$

Substituting the A_{OL} term from Equation 10 into Equation 15 yields the closed-loop gain expression with its dependency on angular frequency:

$$A_{CL}(j\omega) = \frac{-\alpha \dfrac{A_{OL\_DC}}{1+\beta \times A_{OL\_DC}}}{1 + j\dfrac{\omega}{\omega_0} \times \dfrac{1}{1+\beta \times A_{OL\_DC}}} \tag{16}$$

As before, the transfer function, $A_{CL}(j\omega)$, is a complex function of the angular frequency, ω. To calculate the magnitude, both real and imaginary portions of the function must again be used:

$$\left|A_{CL}(j\omega)\right|_{dB} = 20\log \frac{\alpha \dfrac{A_{OL\_DC}}{1+\beta \times A_{OL\_DC}}}{\sqrt{1 + \dfrac{\omega^2}{\omega_0^2} \times \dfrac{1}{(1+\beta \times A_{OL\_DC})^2}}} \tag{17}$$

If the angular frequency, ω, is replaced with 2πf, the closed-loop transfer function from Equation 17 can be rewritten as

$$\left|A_{CL}(jf)\right|_{dB} = 20\log \frac{\alpha \dfrac{A_{OL\_DC}}{1+\beta \times A_{OL\_DC}}}{\sqrt{1 + \dfrac{f^2}{f_0^2} \times \dfrac{1}{(1+\beta \times A_{OL\_DC})^2}}} \tag{18}$$

Assuming that there is a first-order response from the op amp, the complete closed-loop equations for the noninverting and inverting gain amplifiers are respectively represented by Equation 13 and Equation 18.

Conclusion

This article has explored feedback control system analysis and synthesis as they apply to first-order transfer functions. The analysis technique was applied to both noninverting and inverting op amp circuits, resulting in a frequency-domain transfer function for each configuration. In Parts 2 and 3 of this series, these two transfer functions will be used to analyze the DC and AC gain error of closed-loop op amp circuits. Part 3 will help circuit designers avoid the mistake of using DC-gain calculations for AC-domain analysis.

References

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2. James W. Nilsson and Susan Riedel, *Electric Circuits* (Upper Saddle River, NJ: Prentice Hall, 2007).

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