AD-03 Effects of Aperture Time and Jitter in a Sampled Data

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Effects of Aperture Time and Jitter in a Sampled Data System

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The Nyquist sampling theorem states that band limited signals may be represented without error by sampled data. Among the effects which place additional restrictions on the signal, beyond those which are given by the Nyquist theorem, are aperture effects. Aperture effects such as aperture jitter, and aperture time, contribute to a signal degradation which is frequency dependent. In applications for high performance A/D converters, the aperture effects may be the dominant source of noise in the digitization process.

The purpose of this paper is to define the terms relating to aperture effects, to develop a mathematical framework to represent these effects, and to predict the errors that will be introduced into the sampled signal as a result of aperture effects.

Introduction and Terminology

The result of a sampling event is ideally the value of the input function at the sampling instant. This can be expressed as:

$$f_S(n) = \int_{-\infty}^{+\infty} f(t)\ \delta(t - n\tau)\ dt$$

where $\delta(t-n\tau)$ is a dirac delta distribution, f(t) is the input signal and $f_S(n)$ is the sampled function. In practice the sampling event is somewhat different, the instantaneous sampling is replaced by an integration over a small period of time, and there is uncertainty in the actual instant at which the sample is taken. These two non-idealities: the finite sampling time and the uncertainty of the sampling instant can be accounted for by replacing the delta distribution by another distribution $\Psi(t)$. $\Psi(t)$ will be an ordinary, continuous function of time which includes a random variable: ζ to denote the uncertainty in sampling instant. ζ will not be explicitly written but it is assumed that $\Psi(t)$ is $\Psi(t+\zeta)$. ζ represents the uncertainty in the sampling instant or aperture jitter and is a random variable with a mean value of 0. The RMS value of ζ is what will be specified as aperture jitter. The result of a sampling operation may now be written as:

$$f_S = \int_{-\infty}^{+\infty} f(t)\ \psi(t - n\tau)\ dt$$

A shorthand way of expressing this is $f_S = \langle f, \Psi \rangle$.

We are now in a position to evaluate the effects of aperture non-idealities in the sampling process on the sampled signal. One key is that since we are dealing with a random process, the result of a single sample is meaningless, we must consider the effects upon the signal, or the collection of the individual samples to have meaningful results.

The Effects of Sampling Instant Uncertainty (Aperture Jitter)

To consider Aperture Jitter we will assume that the sampling function Ψ is $\delta(t+\zeta)$, with ζ representing the uncertainty in sampling instant. The error generated in sampling is now $\langle f,\delta \rangle - \langle f, \delta(t+\zeta) \rangle$ or $f(t_0) - f(t_0+\zeta)$. If we consider the Taylor expansion of f(t) about t_0:

$$f(t) \approx f(t_0) + f'(t - t_0)$$

Then the error generated on each sample is approximately $f'(t) \bullet \zeta$. This is intuitively comfortable since it states that the error is proportional to the slew rate multiplied by the aperture jitter.

When a signal is sampled, the expected error that will be generated will be the RMS value of the slew rate multiplied by the RMS value of ζ (the RMS value of ζ is what is commonly specified as Aperture Jitter). Hence for a full scale sinusoidal input signal of 10MHz, the RMS slew rate is 0.02 full scale ranges/ns. If the Aperture Jitter is 10ps then the expected error would be 0.02 * 0.01ns or 213ppm. This corresponds to a signal to noise ratio of 73dB or approximately the same size error as the quantization error from an ideal 12 bit converter.

In Figure 1 the Signal-to-Noise Ratio is plotted vs. the Input frequency for various different values of aperture jitter.

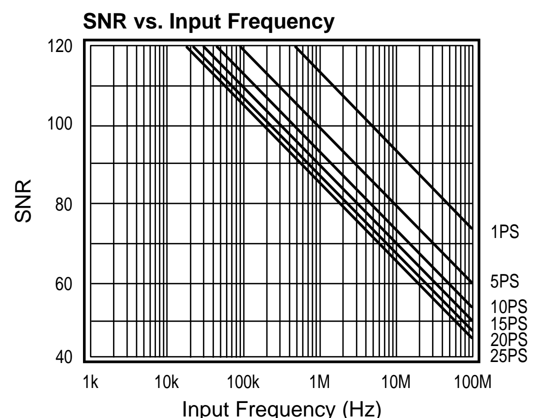

Figure 1: SNR vs. Input Frequency for Various Values of Aperture Jitter

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The Effects of Non-zero Aperture Time

For the analysis of aperture time we will assume ζ to be 0. This is the case of no aperture jitter. We will, however, consider the effects of $\Psi \neq \delta$. The error (as compared to an ideal sampling process) will then be $< f, \Psi > - < f, d >$ This is equivalent to $< f, (\Psi - \delta) >$. To progress we must now make some assumptions about the nature of Ψ. If we consider the physical process of sampling that takes place within a sample-and hold amplifier we can make an educated guess as to the shape of Ψ. Consider the sample-and hold circuit shown in Figure 2. When the switch is closed, V_{OUT} takes the form of the equation below:

$$V_{out}(t) = \int_{-\infty}^{t} V_{in}(\tau)\, e^{\frac{\tau - t}{RC}}\, d\tau$$

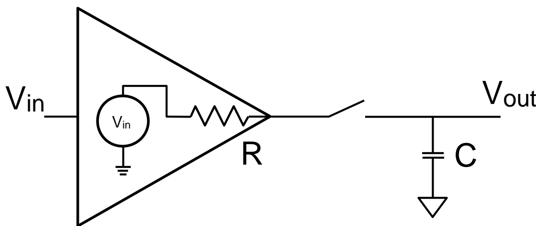

Figure 2: Simplified Sample-And-Hold Circuit

The RC time constant of this circuit will be proportional to the aperture time. This implies that Ψ has the form indicated below:

$$\psi = e^{\frac{t}{RC}}, t < 0 \qquad \psi = 0, t > 0$$

To get an idea of what effect this has on the sampling process we will expand f as a Taylor series expansion about the sampling instant: t. Once again we will consider the first two terms of the expansion:

$$f(t) \approx f(t_0) + f'(t - t_0)$$

Now we can examine the value of $< f, (\Psi - \delta) >$. $< f, \delta >$ is by definition $f(t_0)$ and the other terms of the expansion do not contribute to the result. Since Ψ has been normalized, $< f(t_0), \Psi >$ is $f(t_0)$: the ideal result of the sampling event. If we consider the next term: $f'(t_0)\ (t-t_0)$ this is a constant

$$f_s(t_0)\, error = f'(t_0) \int_{-\infty}^{0} t e^{\frac{t}{RC}} dt$$

slope going through 0 at the sampling point. This, expanded out gives us an approximation of the error generated by the non-zero sampling time. This implies that the error generated is proportional to the slew rate of the input, multiplied by the aperture time. This result is very similar to the result for aperture jitter.

Let us now look at the shape of Ψ that might be expected in a CCD or SAW device where the sample consists of the charge deposited in a bin as the bin passes beneath an input terminal. In this case, due to the symmetry of the sampling process, we can expect Ψ to be an even function:

$$f(t) \approx f(t_0) + f'(t_0)(t - t_0) + f''(t_0)\frac{(t - t_0)^2}{2}$$

such as a gaussian, or a rectangular pulse then the odd nature of $(t-t_0)$ will force $< f'(t_0)\ (t-t_0), \Psi > = 0$. Now in order to determine the error generated by the non-zero sampling time we must consider the next term in the Taylor series expansion.

Now the expected error is:

$$f_s(t_0)\, error \approx \frac{f'(t_0)}{2} < (t - t_0)^2, Y >$$

If the input function f(t) is sinusoidal in nature, all of it's derivatives have approximately equal values so the major difference between this case and the previous case comes in the comparison of $< t, \Psi >$ and $1/2 < t^2, \Psi >$.

How to Minimize Aperture Induced Errors

As we have seen in the preceding analysis the results of both aperture time and aperture jitter is an error signal which increases in amplitude as the slew rates at the input terminal of the A/D increase. One set of strategies that are used to reduce aperture errors therefore focus on minimizing this input slew rate. In fact, from an aperture error standpoint the only thing that is important is that the slew rate be small at the instant that the converter is sampling the input, rapid slewing between samples does not contribute to aperture error. Another tack that can be taken to minimize aperture related errors is one which takes advantage of the fact that the noise generated through aperture effects has a random characteristic and therefore lends itself to reduction through some standard signal processing techniques.

Reducing the Input Slew Rate

In some cases, the input slew rate is higher than it needs to be to recover the information content of the signal to be digitized. An example would be if an input signal were being digitally down converted through aliasing. As an example if a 100kHz signal, modulated on a 101MHz carrier is to be digitized, then there are several possible approaches:

1) Demodulate the signal and digitize the 100kHz base band signal at a rate of 1MHz. This scheme results in very low input slew rates and would be the preferred method from an aperture error standpoint.

2) Digitize the modulated signal at a 1MHz rate, allowing aliasing to perform the down conversion. In this method the usual mixers are eliminated and the digitized signal is identical to that obtained in method 1 above. The problem is that the input slew rates seen by the converter are over one thousand times greater than those seen in the above example and aperture related errors may dominate.

3) Down convert to an I.F. of 1MHz then digitize at a rate of 1MHz. In this method, some of the advantages of digital down conversion can be retained - multiple channels can be demodulated simultaneously - and complexity is reduced from a system that down converts to baseband. From an aperture standpoint we are still seeing input slew rates an order of magnitude greater than we did in example 1, but two orders of magnitude less than those seen in example 2.

4) Place a sample and hold circuit, sampling at a rate of 1MHz, in front of the A/D in scenario 2. This results in the A/D converter seeing a very low slew rate, but the aperture errors are shifted to the sample-and-hold circuit. In many cases a sample-and-hold has much lower aperture errors associated with it than a comparable speed A/D converter.

In all of the above scenarios, the digital output from the converter is at a 1 MHz rate and the digital output would be identical if the A/D converter were an ideal converter with no aperture jitter. With real world converters, there will be a vast difference in the signal to noise ratio in each of these three scenarios.

The solution involving placing a sample and hold circuit in front of an A/D converter is especially interesting with many high speed, high resolution, monolithic A/D converters. Many of these devices have particularly poor aperture performance and as a result even if the input frequencies are reduced to sub-nyquist rates, the aperture error may be the dominant error source. Use of a separate sample and hold will allow for the burden of sampling the signal to be shifted to the sample and hold, reducing the sensitivity to aperture jitter in the A/D converter.

Reducing Noise Through Signal Processing

A/D converter outputs contain noise that has as its origins quantization noise, noise that comes from aperture related effects and noise from other sources. Often it is desirable to lump all of these together and just treat them as noise and work towards reducing them. One technique that can be used for this is simply to over-sample the input then digitally low pass filter the output. If we take the example above, where we have a 100MHz carrier modulated with a 100kHz signal, if we sample at a rate of 2MHz instead of a rate of 1 MHz, the noise is distributed over a band that is twice as wide. If the digital signal is then filtered, and the half of the band that does not contain the signal of interest is thrown away, the result is a 3dB improvement in the signal to noise level. This could be carried on as far as the speed of the converter permits with the cost being carried mainly in the power and complexity of the digital filtering hardware. Imagine sampling the signal at 1GHz, then filtering out all but the lowest 500kHz band to obtain the equivalent of 1MHz sampling: we would be able to obtain a 30dB improvement in SNR over what we would have sampling at 1MHz.

Another signal conditioning method that can be used toreduce the noise both from aperture effects as well as other sources is implemented in SD A/D converters. A block diagram of a simple converter is shown in Figure 3.

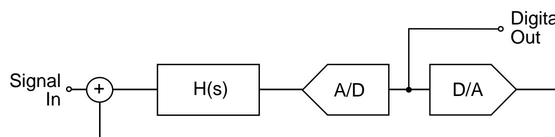

Figure 3

If we replace the SD A/D and D/A pair with a simple additive noise where this noise represents the noise contributed by the sampling process, both quantization noise and aperture jitter related noise, then the block diagram is modified to look like that in Figure 4. The transfer function for the signal in this system is given by:

$$STF = \frac{H(s)}{1 + H(S)}$$

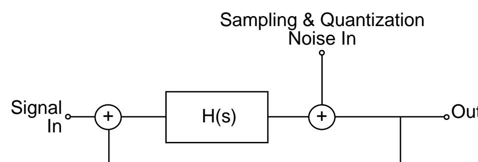

Figure 4

It can be seen from this that if H(s) is large for the frequency range in which the signal is, then the signal transfer function is near unity.

The noise however, sees a different transfer function:

$$NTF = \frac{1}{1 + H(S)}$$

If H(s) is large in the area of interest then the noise is attenuated in this same area. The result is that the signal to noise ratio is increased.

If an analysis is done of this (which is beyond the scope of this paper) it turns out that this is a much more effective method of reducing the noise than the simple over-sampling and low pass filtering that is outlined above. With a first order filter for H(s) then the SNR improvement that can be realized with ΣΔ techniques is 9dB per octave of oversampling as compared to the 3dB that we obtained above. As the order of the filter used increases the gains can be increased as well.

Conclusion

As digitizing systems increase in speed, aperture effects play a larger and larger role in the total error budget of the system. Techniques for analysis and prediction of the errors have been presented. Techniques for the reduction of aperture related errors have been presented.

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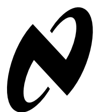

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