**TEXAS INSTRUMENTS**

*Application Report*

# OA-21 Component Pre-Distortion for Sallen Key Filters

## ABSTRACT

This application report shows a simple component pre-distortion method that works for many popular Sallen-Key (also called KRC or VCVS [voltage-controlled, voltage-source]) filter sections. This method compensates for voltage-feedback and current-feedback op amps. Several examples illustrate this method.

This revision obsoletes the previous revision of this application report, and covers additional material.

**Contents**

**List of Figures**

# 1    Introduction

KRC active filter sections use an op amp and two resistors to set a non-inverting gain of K. resistors and capacitors placed around this amplifier provide the desired transfer function. The op amp's finite bandwidth causes K to be a function of frequency. For this reason, KRC filters typically operate at frequencies well below the op amp's bandwidth ($f \ll f_{3dB}$).

"Pre-distortion" compensates for the op amp's finite bandwidth by modifying the nominal resistor and capacitor values. The pre-distortion method in this application report compensates for the op amp's group delay that is approximately constant when $f \ll f_{3dB}$.

One possible design sequence for KRC filters is:

- Design the filter assuming an ideal op amp (K is assumed constant over frequency)
  - Select components for low sensitivities
  - Do a worst case analysis
  - Do a temperature analysis
- Pre-distort the resistors and capacitors to compensate for the op amp's group delay
- Compensate for parasitic elements

# 2    Filter Component Pre-Distortion

This section outlines a simple pre-distortion method that works for many popular Sallen-Key filters using current-feedback or voltage-feedback op amps. Other more general pre-distortion methods are available (see Reference [4]) that requires more design effort.

To pre-distort your filter components:

1. Calculate the op amp's delay: $\tau_{oa} \approx -\frac{1}{f_c} \cdot \frac{\phi(f_c)}{360°}$ where, $\phi(f)$ is the op amp phase response in degrees, and $f_c$ is the cutoff frequency (passband edge frequency) of your filter.
   (a) Subtract the phase shift caused by your measurement jig from any measured value of $\phi(f_c)$
   (b) The group delay is specified at $f_c$ because it has the greatest impact on the filter response near the frequency.
   (c) Other less accurate estimates of the op amp delay at $f_c$ are:
      (i)  Step response propagation delay
      (ii) $1/(2\pi f_{3dB})$
2. The time delay around the filter feedback loop ("electrical loop delay") adds to the op amp delay. For this reason,
   (a) Make the filter feedback loop as physically short as possible.
   (b) If you need greater accuracy in the following calculation, use the electrical loop delay ($\tau_{eld}$) instead of the op amp delay ($\tau_{oa}$):$\tau_{eld} \leftarrow \tau_{oa}$. For information calculating $\tau_{eld}$, see Appendix B
3. Replace K in the filter transfer function with a simple approximation to the op amp's frequency response:
   (a) Start with a simple, single pole approximation:
   $K \leftarrow K/(1 + \tau_{oa}s)$ , s= jω                                                                      (1)
   (b) Alter the approximation to K and simplify:
   (c) *Do not create new terms* (a coefficient times a new power of s) in the transfer function *after simplifying*
   (d) Convert $(1 + \tau_{oa}s)$ to the exponential form (a pure time delay) when it multiplies, or divides, the entire transfer function
   (e) Do not change the gain at $\omega \approx \omega_p$ in allpass sections
   (f) The most useful alterations to K are:

$$\frac{K}{1 + \tau_{oa}s} \approx K \bullet \frac{1 - (\tau_{oa}/2)s}{1 + (\tau_{oa}/2)s}$$

$$\approx K(1 - \tau_{oa}s)$$

$$\approx Ke^{-\tau_{oa}s}$$

All of these approximations are valid when: $\omega \ll 1/\tau_{oa}$

4. Use an op amp with adequate bandwidth ($f_{3db}$) and slew rate (SR):

$f_{3db} \geq 10f_H$

$SR > 5f_H V_{peak}$

Where $f_H$ is the highest frequency in the passband of the filter, and $V_{peak}$ is the largest peak voltage. This increases the accuracy of the pre-distortion algorithm. It also reduces the filter's sensitivity to op amp performance changes over temperature and process. Make sure the op amp is stable at the gain of $A_v = K$.

Appendix A contains examples using transfer functions. Section 3 applies the results from Appendix A.

# 3    KRC Lowpass Biquad

The biquad shown in Figure 2 is a Sallen-Key lowpass biquad. $V_{in}$ needs to be a voltage source with low output impedance. $R_1$ and $R_2$ attenuate $V_{in}$ to keep the signal within the op amp's dynamic range. Using Section A.2 in Appendix A will show:

$$\frac{V_o}{V_{in}} \approx \frac{H_o}{1 + \left(1/\left(\omega_p Q_p\right)\right)s + \left(1/\omega_p^2\right)s^2} \bullet e^{-\tau_{oa}s}$$

$$\omega, \omega_p \ll 1/\tau_{oa}$$

where:

$$\alpha = R_2/(R_1 + R_2)$$

$$K = 1 + R_f/R_g$$

$$H_o = \alpha K$$

$$R_{12} = (R_1 \| R_2)$$

$$1/(\omega_p Q_p) = R_{12}C_5(1 - K) + R_3C_4 + R_{12}C_4$$

$$1/\omega_p^2 = R_{12}R_3C_4C_5 + K\tau_{oa}R_{12}C_5$$

(2)

After selecting $\alpha$ and $R_{12}$, calculate $R_1$ and $R_2$ as:

$$R_1 = R_{12}/\alpha$$

$$R_2 = R_{12}/(1 - \alpha)$$



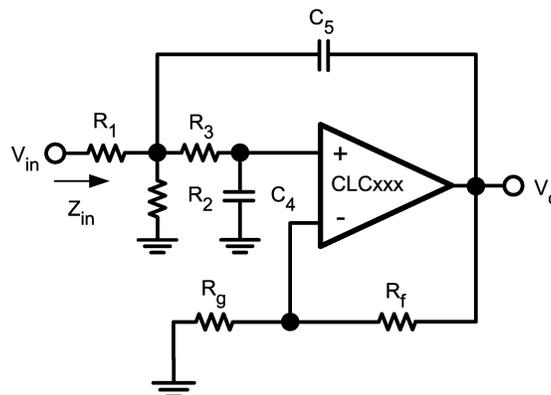**Figure 1. Lowpass Biquad**

**To pre-distort this filter:**

1. Design the filter assuming K constant ($\tau_{oa} = 0$). Use low values for K so that:

   (a) $\tau_{oa}$ will have less impact on the biquad's response.

   (b) For voltage-feedback op amps, $\tau_{oa}$ will be smaller ($\tau_{oa} \approx K$ divided by the gain-bandwidth products).

2.  Recalculate the resistors and capacitors using the pre-distorted values of $\omega_p$ and $Q_p$ ($\omega_{p(pd)}$ and $Q_{p(pd)}$) that compensates for $\tau_{oa}$:

$$1/\omega^2_{p(pd)} = 1/\omega^2_{p(nom)} - K\tau_{oa}R_{12}C_5$$
$$= R_{12}R_3C_4C_5$$

$$1/(\omega_{p(pd)}Q_{p(pd)}) = 1/(\omega_{p(nom)}Q_{p(nom)})$$
$$= R_{12}C_5(1 - K) + R_3C_4 + R_{12}C_4$$

(3)

where, $\omega_{p(nom)}$ and $Q_{p(nom)}$ are the nominal values of $\omega_p$ and $Q_p$

3.  Repeat step 2 until $\omega_p \approx \omega_{p(nom)}$ and $Q_p \approx Q_{p(nom)}$, where:

$$1/\omega^2_p = 1/\omega^2_{p(pd)} + K\tau_{oa}R_{12}C_5$$

$$1/(\omega_pQ_p) = 1/(\omega_{p(pd)}Q_{p(pd)})$$

(4)

# 4    Design Example

The circuit shown in is a third order Chebyshev lowpass filter. Section 4.2 is a buffered single pole section, and Section 4.3 is a lowpass biquad. Use a voltage source with low output impedance, such as the CLC111 buffer, for $V_{in}$.
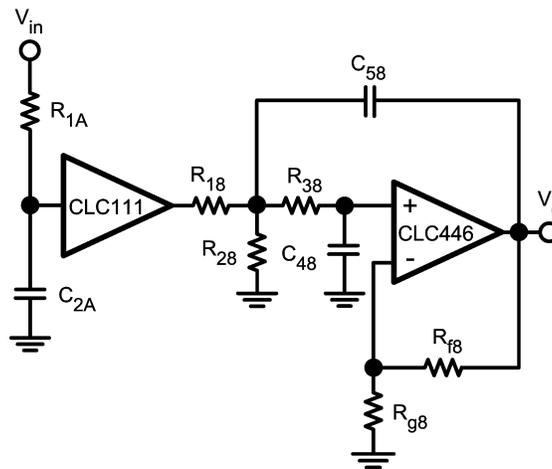


**Figure 2. Lowpass Filter**

The nominal filter specification are:

**$f_c$ = 50MHz—** (passband edge frequency)

**$f_s$ = 100MHz—** (stopband edge frequency)

**$A_p$ = 0.5dB—** (maximum passband ripple)

**$A_s$ = 19dB—** (minimum stopband attenuation)

**$H_o$ = 0dB—** (DC voltage gain)

The third order Chebyshev filter meets our specifications (see References [1] through [4]. The resulting -3dB frequency is 58.4MHz. The pole frequencies and quality factors are:

| Section | A | B |
|---|---|---|
| $\omega_p/2\pi$ [MHz] | 53.45 | 31.30 |
| $Q_p$ [ ] | 1.706 | – |

## 4.1   Overall Design

1. Use the CLC111 for Section 4.2. This is a closed loop buffer:
   (a) $f_{3dB}$ = 800MHz > $10f_c$ = 500MHz
   (b) SR = 3500V/μs, while a 50MHz, $2V_{pp}$ sinusoid requires more than 250V/μs
   (c) τoa ≈ 0.28ns at 50MHz
   (d) $C_{ni(111)}$ = 1.3pF (input capacitance)
2. Use the CLC446 for Section 4.3. This is a current feedback op amp:
   (a) $f_{3dB}$ = 400MHz ≈ $10f_c$ = 500MHz
   (b) SR = 2000V/μs > 250V/μs (see item #1)
   (c) $τ_{oa}$ ≈ 0.56ns at 50MHz
   (d) $C_{ni(446)}$ = 10pF (non-inverting capacitance)
3. Use 1% resistors (chip metal film, 1206 SMD, 25ppm/°C).
4. Use 1% capacitors (ceramic chip, 1206 SMD, 100ppm/°C).
5. Use standard resistor and capacitor values.
6. For the low-sensitivity design of this biquad, see Reference [6].

## 4.2   Section A Pre-Distortion

$R_{1A}$ was selected for noise, distortion and to properly isolate the CLC111's output and $C_{2A}$. The pole is then set by $C_{2A}$ . The pre-distorted value of $R_{1A}$, that also compensates for $C_{ni(111)}$, is (see Section A.1 in Appendix A):

$$R_{1A} = (1/\omega_p - \tau_{oa})/(C_{2A} + C_{ni(111)}) \tag{5}$$

The resulting components are in the table below:
- The Initial Value column shows the values before pre-distortion
- The Adjusted Value column shows the values after pre-distortion, and adjusting $C_{2A}$ for $C_{ni(111)}$
- The Standard Value column shows the nearest available standard 1% resistor and capacitor values

| Component | Value | | |
|---|---|---|---|
|  | **Initial** | **Adjusted** | **Standard** |
| $R_{1A}$ | 108Ω | 100Ω | 100Ω |
| $C_{2A}$ | 47pF | 47pF | 47pF |
| $C_{ni(111)}$ | – | 1.3pF | 1.3pF |

## 4.3   Section B Pre-Distortion

- The design started with these values:

$$\omega_{p(nom)} = 2\pi (53.45 MHz) \ Q_{p(nom)} = 1.706 \ K_B = 1.50 \ \alpha_B = 0.667 \ C_{4B} + C_{ni(446)} = 4.7pF \ C_{5B} = 47pF \tag{6}$$

- Iteration 0 shows the initial design results. Iterations 1-3 pre-distort $R_{12B}$ and $R_{3B}$ to compensate for the CLC446's group delay:

| Iteration | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\omega_{p(pd)}/2\pi$ [MHz] | 53.45 | 63.21 | 60.65 | 61.21 |
| $Q_{p(pd)}$ [ ] | 1.706 | 1.443 | 1.503 | 1.490 |
| $R_{12B}$ [Ω] | 64.00 | 50.17 | 53.32 | 52.63 |
| $R_{3B}$ [Ω] | 627.0 | 571.9 | 584.9 | 581.9 |
| $K\tau_{oa}R_{12B}C_{5B}$ [ns$^2$] | 2.527 | 1.981 | 2.105 | 2.078 |
| $\omega_p/2\pi$ [MHz] | 47.15 | 55.18 | 53.08 | 53.53 |
| $Q_p$ [ ] | 1.934 | 1.653 | 1.718 | 1.703 |

- The resulting components are:

| Component | Value | | |
|---|---|---|---|
| | Initial | Adjusted | Standard |
| $R_{1B}$ | 96.0Ω | 78.9Ω | 78.7Ω |
| $R_{2B}$ | 192Ω | 158Ω | 158Ω |
| $R_{3B}$ | 627Ω | 582Ω | 576Ω |
| $C_{4B}$ | 4.7pF | 3.7pF | 3.6pF |
| $C_{ni(446)}$ | – | 1.0pF | 1.0pF |
| $C_{5B}$ | 47pF | 47pF | 47pF |
| $R_{fB}$ | 348Ω | 348Ω | 348Ω |
| $R_{gB}$ | 696Ω | 696Ω | 698Ω |

Figure 3 and Figure 4 show simulated gains for the following conditions:

- Ideal (Initial Values, $\tau_{oa} = 0$)
- Without Pre-distortion (Initial Values, $\tau_{oa} \neq 0$)
- Without Pre-distortion (Standard Values, $\tau_{oa} \neq 0$)
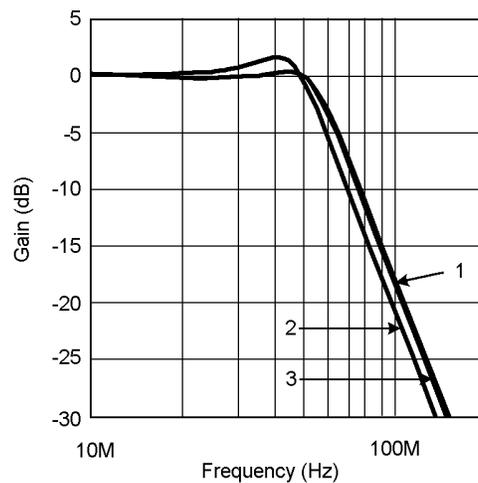


**Figure 3. Simulated Filter Magnitude Response**
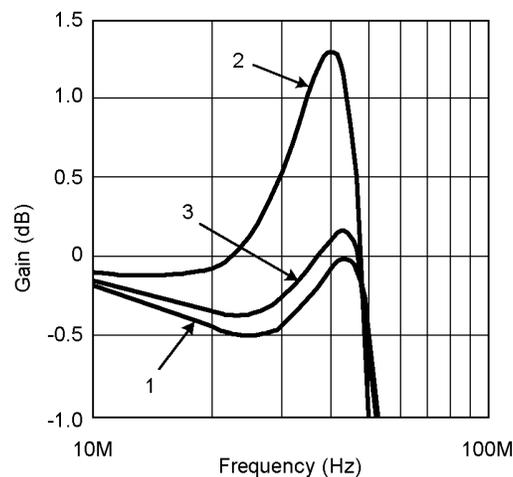


**Figure 4. Simulated Filter Magnitude Response**

## 5    SPICE Models

SPICE models are available for most of Comlinear's amplifiers. These models support nominal DC, AC, AC noise and transient simulations at room temperature.

We recommend simulating with Comlinear's SPICE models to:

*   Predict the op amp's influence on filter response
*   Support quicker design cycles

Include board and component parasitics to obtain a more accurate prediction of the filter's response, and to further improve your design.

To verify your simulations, we recommend bread-boarding your circuit.

## 6    Summary

This application report demonstrates a component pre-distortion method that:

*   Works for popular Sallen-Key filter sections
*   Is quick and simple to use
*   Shows the op amp's effect on the filter response
*   Gives reasonable op amp selection criteria

Appendix A and Section 4 contain illustrations of this method.

## 7    References

1.  R. Schaumann, M. Ghausi and K. Laker, *Design of Analog Filters: Passive, Active RC, and Switched Capacitor.* New Jersey: Prentice Hall, 1990.
2.  A. Zverev, *Handbook of FILTER SYNTHESIS.* John Wiley & Sons, 1967.
3.  A. Williams and F. Taylor, *Electronic Filter Design Handbook.* McGraw Hill, 1995.
4.  S. Natarajan, *Theory and Design of Linear Active Networks.* Macmillan, 1987.
5.  *OA-21 Component Pre-Distortion for Sallen Key Filters* (SNOA369).
6.  *OA-27 Low-Sensitivity, Lowpass Filter Design* (SNOA372)
7.  *CLC to LMH Conversion Table* (SNOA428)

> **NOTE:**    The circuits included in this application report have been tested with Texas Instruments parts that may have been obsoleted and/or replaced with newer products. To find the appropriate replacement part for the obsolete device, see the *CLC to LMH Conversion Table* (SNOA428).

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# Appendix A  Transfer Function Examples

## A.1    Single Pole Section, K in the Numerator

$$\frac{V_o}{V_{in}} \approx \frac{K}{1 + (1/\omega_p)s}$$

$$1/\omega_p = \tau_1$$

(7)

where $\tau_1$ is a time constant set by resistors and capacitors.

To include the op amp's group delay, substitute for K and simplify:

$$\frac{V_o}{V_{in}} \approx \frac{1}{1 + (\tau_1)s} \bullet \frac{K}{1 + (\tau_{oa})s}$$

$$\approx \frac{K}{1 + (1/\omega_p)s} \; ; \quad \omega, \omega_p << 1/\tau_{oa}$$

$$1/\omega_p = \tau_1 + \tau_{oa}$$

(8)

Notice that:

- There are no new powers of s in the transfer function
- Changing the resistor and capacitor values can compensate for $\tau_{oa}$
- The approximation is reasonably accurate when $f << f_{3dB}$
  - To pre-this filter section, recalculate the resistors and capacitors using Equation 9:

$$\tau_1 = 1/\omega_p - \tau_{oa}$$

(9)

## A.2    Single Pole Allpass Section, K Times the Numerator

$$\frac{V_o}{V_{in}} \approx \frac{1 - (1/\omega_z)s}{1 + (1/\omega_p)s} \bullet K$$

$$1/\omega_p = \tau_1$$

$$1/\omega_z = \tau_2$$

(10)

where $\tau_1$ and $\tau_2$ are time constants set by resistors and capacitors. This section operates as an allpass filter when:

$$\tau_1 = \tau_2$$

(11)

To include the op amp's group delay, substitute for K and simplify. Since this is an allpass transfer function, the approximation to K does not change gain at $\omega = \omega_p$:

$$\frac{V_o}{V_{in}} \approx \frac{1 - (\tau_2)s}{1 + (\tau_1)s} \bullet \frac{1 - (\tau_{oa}/2)s}{1 + (\tau_{oa}/2)s} \bullet K$$

$$\approx \frac{1 - (1/\omega_z)s}{1 + (1/\omega_p)s} \bullet K$$

$$\omega, \omega_p, \omega_z << 1/\tau_{oa}$$

$$1/\omega_z = \tau_2 + \tau_{oa}/2$$

$$1/\omega_p = \tau_1 + \tau_{oa}/2$$

(12)

Notice that:

- There are no new powers of s in the transfer function
- The gain at $\omega_p$ does not change (this is an allpass section)
- Changing the resistor and capacitor values can compensate for $\tau_{oa}$
- The approximation is reasonably accurate when $f << f_{3dB}$

To pre-distort this filter, recalculate the resistor and capacitors using Equation 13:

$$\tau_2 = /\omega_z - \tau_{oa}/2 \quad \tau_1 = /\omega_p - \tau_{oa}/2$$

(13)

## A.3   Biquad Section, s Term in the Denominator That Includes K

$$\frac{V_o}{V_{in}} \approx \frac{1}{1 + (1/(\omega_p Q_p))s + (1/\omega_p^2)s^2}$$

$$1/(\omega_p Q_p) = \tau_1 + K\tau_2$$

$$1/\omega_p^2 = \tau_3^2$$

(14)

where $\tau_1$, $\tau_2$ and $\tau_3$ are time constants set by resistors and capacitors.

To include the op amp's group delay, substitute for K and simplify:

$$\frac{V_o}{V_{in}} \approx \frac{1}{1 + (\tau_1 + K(1 - \tau_{oa}s) \bullet \tau_2)s + (\tau_3^2)s^2}$$

$$\approx \frac{1}{1 + (1/(\omega_p Q_p))s + (1/\omega_p^2)s^2}$$

$$\omega, \omega_p, << 1/\tau_{oa}$$

$$1/(\omega_p Q_p) = \tau_1 + K\tau_2$$

$$1/\omega_p^2 = \tau_3^2 - K\tau_2\tau_{oa}$$

(15)

Notice that:

- There are no new powers of s in the transfer function
- Changing the resistor and capacitor values can compensate for $\tau_{oa}$
- The approximation is reasonably accurate when $f << f_{3dB}$

To pre-distort this filter:

1. Design the filter assuming K constant ($\tau_{oa} = 0$).
2. Recalculate the resistors and capacitors using the pre-distorted values of $\omega_p$ and $Q_p$ ($\omega_{p(pd)}$ and $Q_{p(pd)}$)

$$1/\omega_{p(pd)}^2 = 1/\omega_{p(nom)}^2 + K\tau_2\tau_{oa}$$
$$= \tau_3^2$$
$$1/(\omega_{p(pd)}Q_{p(pd)}) = 1/(\omega_{p(nom)}Q_{p(nom)})$$
$$= \tau_1 + K\tau_2$$

that will compensate for $\tau_{oa}$:                        where $\omega_{p(nom)}$ and $Q_{p(nom)}$ are the nominal values of $\omega_p$ and $Q_p$

3. Repeat step 2 until $\omega_p \approx \omega_{p(nom)}$ and
   - $Q_p \approx Q_{p(nom)}$, where:

$$1/\omega_p^2 = 1/\omega_{p(pd)}^2 - K\tau_2\tau_{oa}$$

$$1/(\omega_p Q_p) = 1/(\omega_{p(pd)}Q_{p(pd)})$$

(16)

## A.4    Biquad Section, $s^2$ Term in the Denominator Multiplied by K

$$\frac{V_o}{V_{in}} \approx \frac{1}{1 + (1/(\omega_p Q_p))s + (1/\omega_p^2)s^2}$$

$$1/(\omega_p Q_p) = \tau_1$$

$$1/\omega_p^2 = K\tau_2^2 \tag{17}$$

where, $\tau_1$ and $\tau_2$ are time constants set by resistors and capacitors.

To include the op amp's group delay, substitute for K and simplify:

$$\frac{V_o}{V_{in}} \approx \frac{1}{1 + (\tau_1)s + (\tau_2^2 \bullet K/(1 + \tau_{oa}s))s^2}$$

$$\approx \frac{e^{\tau_{oa}s}}{1 + (1/\omega_p Q_p))s + (1/\omega_p^2)s^2} \tag{18}$$

$$\omega, \omega_p << 1/\tau_{oa}$$

$$1/(\omega_p Q_p) = \tau_1 + \tau_{oa}$$

$$1/\omega_p^2 = K\tau_2^2 + \tau_1\tau_{oa} \tag{19}$$

Notice that:

- The $(1 + \tau_{oa}s)$ factor in the numerator was converted to the exponential form, which represents a constant group delay
- There are no new powers of s in the transfer function
- Changing the resistor and capacitor values can compensate for $\tau_{oa}$
- The approximation is reasonably accurate when f << $f_{3dB}$

To pre-distort this filter:

1. Design the filter assuming K constant ($\tau_{oa} = 0$).
2. Recalculate the resistors and capacitors using the pre-distorted values of $\omega_p$ and $Q_p$ $\omega_{p(pd)}$ and $Q_{p(pd)}$)

$$1/\omega_{p(pd)}^2 = 1/\omega_{p(nom)}^2 - \tau_1\tau_{oa}$$
$$= K\tau_2^2$$
$$1/(\omega_{p(pd)} Q_{p(pd)}) = 1/(\omega_{p(nom)}Q_{p(nom)}) -$$
$$\tau_{oa}$$

that will compensate for $\tau_{oa}$:                    $= \tau_1$                    where $\omega_{p(nom)}$ and $Q_{p(nom)}$ are the nominal values of $\omega_p$ and $Q_p$

$$1/\omega_p^2 = 1/\omega_{p(pd)}^2 + \tau_1\tau_{oa}$$

3. Repeat step 2 until $\omega_p \approx \omega_{p(nom)}$ and $Q_p \approx Q_{p(nom)}$, where: $1/(\omega_p Q_p) = 1/(\omega_{p(pd)}Q_{p(pd)}) + \tau_{oa}$

*OBSOLETE*

## Appendix B  Electrical Loop Delay

$\tau_{eld}$ can be calculated as:

$$\tau_{eld} = x \bullet \sqrt{\varepsilon_r \mu_r} / c + \tau_{oa} \qquad (20)$$

where,

- x is the distance around the filter feedback loop, excluding the op amp

- $\varepsilon_r$ is the equivalent relative permittivity of the PCB trace
- $\mu_r$ is the equivalent relative permeability of the PCB trace
- **c** is the speed of light in free space ($3.00 \times 10^8$ m/s)
- $\tau_{oa}$ is the op amp group delay at $f_c$

For a typical printed circuit board, $\sqrt{\varepsilon_r \mu_r} \approx 2.0$ . This gives:

$$\tau_{eld} \approx x \cdot (0.067\text{ns/cm}) + \tau_{oa} \qquad (21)$$

where, x is in centimeters, and $\tau_{oa}$ is in nanoseconds.

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