OBSOLETE

Texas Instruments

Application Report

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# OA-27 Low-Sensitivity, Lowpass Filter Design


## ABSTRACT

This application report covers the design of a Sallen-Key (also called KRC or VCVS [voltage-controlled, voltage-source]) lowpass biquad with low component and op amp sensitivities. This method is valid for either voltage-feedback or current-feedback op amps. Basic techniques for evaluating filter sensitivity performance are included. A filter design example illustrates the method.


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# 1 Introduction

Changes in component values over process, environment and time affect the performance of a filter. To achieve a greater production yield, we need to make the filter insensitive to these changes. This application report presents a design algorithm that results in low sensitivity to component variation.

Lowpass biquad filter sections have the transfer function:

$$\frac{V_O}{V_{IN}} \approx \frac{H_O}{1 + \left(1/(\omega_p Q_p)\right) s + (1/\omega_p^2) s^2} \quad (1)$$

where, s=jω, $H_o$ is the DC gain, $\omega_p$ is the pole frequency, and $Q_p$ is the pole quality factor. Both $\omega_p$ and $Q_p$ affect the filter phase response, $\omega_p$ the filter cutoff frequency, $Q_p$ the peaking, and $H_o$ the gain. For these reasons, we will minimize the sensitivities of $H_o$, $\omega_p$ and $Q_p$ to all of the components (see Appendix A).

To achieve the best production yield, the nominal filter design must also compensate for component and board parasitics. For information on filter component pre-distortion, see [5]. SPICE simulations, with good component and board models, help adjust the nominal design point to compensate for parasitics.

For an overview of sensitivity analysis, with applications to filter design, see Appendix A. For useful sensitivity properties and formulas, see Appendix B. For a more complete discussion of sensitivity functions, their applications, and other approaches to improving the manufacturing yield of your filter, see the references listed in Section 6.

# 2 KRC Lowpass Biquad

The biquad shown in Figure 1 is a Sallen-Key lowpass biquad. $V_{IN}$ needs to be a voltage source with low output impedance. $R_1$ and $R_2$ attenuate $V_{IN}$ to keep the signal within the op amp's dynamic range. The Thevenin equivalent of $V_{IN}$, $R_1$ and $R_2$ is a voltage source $\alpha V_{IN}$, with an output impedance of $R_{12}$, where:

$\alpha = R_2/(R_1 + R_2)$

$R_{12} = (R_1 \parallel R_2)$

The input impedance in the passband is:

$Z_{IN} = R_1 + R_2, \quad \omega << \omega_p$

The transfer function is:

$$\frac{V_O}{V_{IN}} \approx \frac{H_O}{1 + \left(1/(\omega_p Q_p)\right) s + (1/\omega_p^2) s^2}$$

where:

$$\begin{aligned} K &= 1 + R_f/R_g \\ H_O &= \alpha K \\ 1/(\omega_p Q_p) &= R_{12} C_5 (1 - K) + R_3 C_4 + R_{12} C_4 \\ 1/\omega_p^2 &= R_{12} R_3 C_4 C_5 \end{aligned} \quad (2)$$

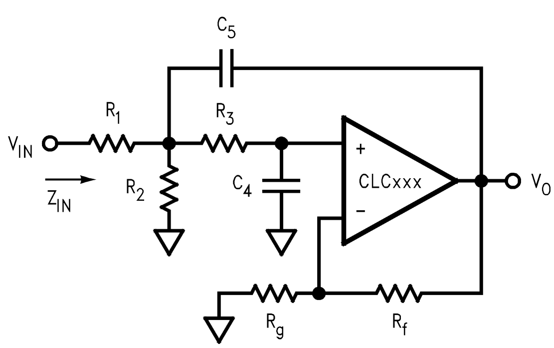


**Figure 1. Lowpass Biquad**

To achieve low sensitivities, use this design algorithm:

1. Partition the gain for good $Q_p$ sensitivity and dynamic range performance:
   (a) Use a low noise amplifier before this biquad if you need a large gain.
   (b) Select K for good sensitivity with this empirical formula:

$$K = \begin{cases} 1 & , 0.1 \le Q_p \le 1.1 \\ \dfrac{2.2\,Q_p - 0.9}{Q_p + 0.2} & , 1.1 < Q_p < 5 \end{cases} \tag{3}$$

   (c) These values also reduce the op amp bandwidth's impact on the filter response, and increase the bandwidth for voltage-feedback op amps. When $Q_p \geq 5$, the sensitivities of this biquad are very high
   (d) Set α as close to 1 as possible while keeping the signal within the op amp's dynamic range
2. Select an op amp with adequate bandwidth ($f_{3\ dB}$) and slew rate: (SR):
   (a) $f_{3\ dB} \geq 10 f_c$
   (b) $SR > 5 f_c\ V_{peak}$
   (c) where $f_c$ is the corner frequency of the filter, and $V_{peak}$ is the largest peak voltage. Make sure the op amp is stable at a gain of $A_v = K$.
3. Select $R_f$ and $R_g$ so that:
   (a) $K = 1 + R_f/R_g$
   (b) For current-feedback op amps, use the recommended value of $R_f$ for a gain of $A_v = K$. For voltage-feedback op amps, select $R_f$ for noise and distortion performance.
4. Initialize the resistance level $\left(R = \sqrt{R_{12} R_3}\right)$. This value is a compromise between noise performance, distortion performance, and adequate isolation between the op amp outputs and the capacitors.
5. Initialize the capacitance level $\left(C = \sqrt{C_4 C_5}\right)$, the resistor ratio ($r^2 = R_{12} / R_3$), the capacitor ratio ($c^2 = C_4/C_5$) and the capacitors:
   (a) $C = 1/(R\omega_p)$
   (b) $r^2 = 0.10$

$$c^2 = \max\left(\left(\frac{1 + \sqrt{1 + 4Q_p^2(1 + r^2)(K - 1)}}{2 \cdot Q_p \cdot (1 + r^2)/r}\right)^2, 0.10\right) \tag{4}$$

   (c) $C_4 = cC$
   (d) $C_5 = C/c$
6. Set the capacitors $C_4$ and $C_5$ to the nearest standard values.
7. Recalculate C, $c^2$, R and $r^2$:
   (a) $C = \sqrt{C_4 C_5}$
   (b) $c^2 = C_4/C_5$
   (c) $R = 1/(C\omega_p)$

$$r^2 = \left(\frac{2 \cdot cQ_p}{1 + \sqrt{1 + 4Q_p^2(K - 1 - c^2)}}\right)^2 \tag{5}$$

8. Calculate $R_{12}$ and the resistors
   (a) $R_{12} = rR$
   (b) $R_1 = R_{12}/\alpha$
   (c) $R_2 = R_{12}/(1–\alpha)$
   (d) $R_3 = R/r$

$V_{IN}$ can represent a source driving a transmission line, with $R_1$ and $R_2$ the source and terminating resistances. For this type of application, make these modifications to the design algorithm:

- Select $R_1$ and $R_2$ to properly terminate the transmission line ($R_1$ includes the source resistance)
- Calculate α and $R_{12}$

- Adjust C and R so that $R_{12}$ = rR

To evaluate the sensitivity performance of this design, follow these steps:

1. Calculate the resulting sensitivities:

| $\alpha_i$ | $S_{\alpha_i}^{H_o}$ | $S_{\alpha_i}^{\omega_p}$ | $S_{\alpha_i}^{Q_p}$ |
|---|---|---|---|
| K | 1 | 0 | $\left(K \cdot Q_p \cdot \frac{r}{c}\right)$ |
| $R_1$ | $-(1-\alpha)$ | $-\frac{\alpha}{2}$ | $(\alpha) \cdot \left(Q_p \cdot \frac{c}{r} - \frac{1}{2}\right)$ |
| $R_2$ | $(1-\alpha)$ | $-\frac{1-\alpha}{2}$ | $(1-\alpha) \cdot \left(Q_p \cdot \frac{c}{r} - \frac{1}{2}\right)$ |
| $R_3$ | 0 | $-\frac{1}{2}$ | $-\left(Q_p \cdot \frac{c}{r} - \frac{1}{2}\right)$ |
| $R_f$ | $\frac{K-1}{K}$ | 0 | $\left((K-1) \cdot Q_p \cdot \frac{r}{c}\right)$ |
| $R_g$ | $-\frac{K-1}{K}$ | 0 | $-\left((K-1) \cdot Q_p \cdot \frac{r}{c}\right)$ |
| $C_4$ | 0 | $-\frac{1}{2}$ | $-\left((K-1) \cdot Q_p \cdot \frac{r}{c} + \frac{1}{2}\right)$ |
| $C_5$ | 0 | $-\frac{1}{2}$ | $\left((K-1) \cdot Q_p \cdot \frac{r}{c} + \frac{1}{2}\right)$ |

   Reducing $\left|S_K^{Q_p}\right|$ lowers the biquad's sensitivity to the op amp bandwidth.

2. Calculate the relative standard deviations of $H_o$, $\omega_p$ and $Q_p$: $\left(\frac{\sigma_X}{X}\right)^2 \approx \sum_i \left(\left|S_{\alpha_i}^X\right| \cdot \frac{\sigma_{\alpha_i}}{\alpha_i}\right)^2$ In this formula, use:
   (a) The nominal values of $H_o$, $\omega_p$ and $Q_p$ for X
   (b) The nominal values of $R_1$, $R_2$, $R_3$, $R_f$, $R_g$, $C_4$ and $C_5$ for $\alpha_i$ (*do not use* K since it is not a component)
   (c) The capacitor and resistor standard deviations for $\sigma_{\alpha_i}$. For parts with a uniform probability distribution,

$$\sigma_{\alpha_i} = \frac{\max(\alpha_i) - \min(\alpha_i)}{\sqrt{12}} \tag{6}$$

3. If temperature performance is a concern, then estimate the change in nominal values of $H_o$, $\omega_p$ and $Q_p$ over the design temperature range:

$$X(T) \approx X\left(1 + \sum_i \left(S_{\alpha_i}^X \cdot \frac{\alpha_i(T) - \alpha_i}{\alpha_i}\right)\right)$$

   In this formula, use:
   (a) The nominal values, at room temperature, of $H_o$, $\omega_p$ and $Q_p$ for X
   (b) The nominal values, at room temperature, of $R_1$, $R_2$, $R_3$, $R_f$, $R_g$, $C_4$ and $C_5$ for $\alpha_i$ (*do not use* K since it is not a component)
   (c) The nominal resistor and capacitor values at temperature T for $\alpha_i(T)$

4. Estimate the probable ranges of values for $H_o$, $\omega_p$ and $Q_p$:
   (a) $X \geq (1-3 \cdot \sigma_x/X) \cdot \min(X(T))$
   (b) $X \leq 1+3 \cdot \sigma_x/X) \cdot \max(X(T))$
   (c) where X is $H_o$, $\omega_p$ and $Q_p$

# 3 Design Example

The circuit shown in Figure 2 is a third order Chebyshev lowpass filter. Section 3.2 is a buffered single pole section, and Section 3.3 is a lowpass biquad. Use a voltage source with low output impedance, such as the CLC111 buffer, for $V_{IN}$.

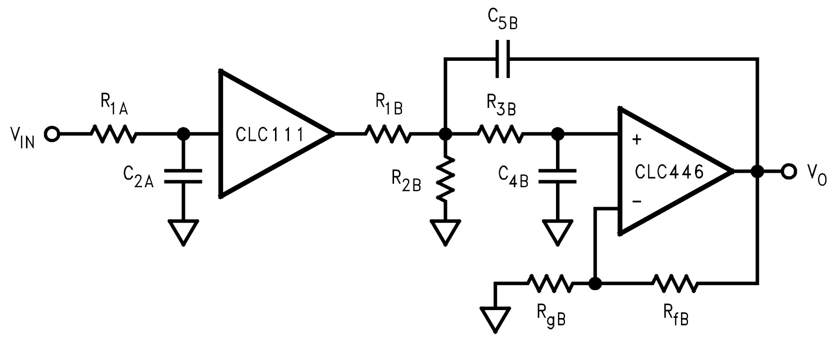


**Figure 2. Lowpass Filter**

The nominal filter specifications are:

$f_c$ = 500 MHz (passband edge frequency)

$f_s$ = 100 MHz (stopband edge frequency)

$A_p$ = 0.5 dB (maximum passband ripple)

$A_s$ = 19 dB (minimum stopband attenuation)

$H_o$ = 0 dB (DC voltage gain)

The third order Chebyshev filter meets our specifications (see References [1] through [4]). The resulting −3 dB frequency is 58.4 MHz. The pole frequencies and quality factors are:

| Section | A | B |
|---|---|---|
| $\omega_p/2\pi$ [MHz] | 53:45 | 31:30 |
| $Q_p$ [ ] | 1.706 | — |

## 3.1 Overall Design

1. Restrict the resistor and capacitor ratios to:
   (a) $0.1 \le r^2 \le 10$
   (b) $0.1 \le c^2 \le 10$
2. Use 1% resistors (chip metal film, 1206 SMD, 25 ppm/°C)
3. Use 1% capacitors (ceramic chip, 1206 SMD, 100 ppm/°C)
4. Use standard resistor and capacitor values
5. The temperature range is −40°C to 85°C, and room temperature is 25°C

### 3.2 Section A Design

1. Use the CLC111. This is a closed-loop buffer.
   (a) $f_{3\ db}$ = 800 MHz > 10 $f_c$ = 500 MHz
   (b) SR = 3500V/µs, while a 50 MHz, 2$V_{pp}$ sinusoid requires more than 250V/µs
   (c) $C_{ni(111)}$ = 1.3 pF (input capacitance)
2. We selected $R_{1A}$ for noise, distortion and to properly isolate the CLC111's output and $C_{2A}$. The capacitor $C_{2A}$ then sets the pole frequency:$1/\omega_p = R_{1A}C_{2A}$. The results are in the table below:
   (a) The Initial Value column shows values from the calculations above
   (b) The Adjusted Value column shows the component values that compensate for $C_{ni(111)}$ and for the CLC111's finite bandwidth (see Comlinear's application report on filter component pre-distortion [5])
   (c) The Standard Value column shows the nearest available standard 1% resistors and capacitors

| Component | Value | | |
|---|---|---|---|
| | **Initial** | **Adjusted** | **Standard** |
| $R_{1A}$ | 108Ω | 100Ω | 100Ω |
| $C_{2A}$ | 47 pF | 47 pF | 47 pF |
| $C_{ni(111)}$ | — | 1.3 pF | 1.3 pF |

### 3.3 Section B Design

1. The recommended value of $K_B$ for $Q_p$ = 1.706 is: $K_B = \frac{2.2(1.706) - 0.9}{(1.706) + 0.2} = 1.50$ Set $\alpha_B = H_o/K_B$ = 0.667.
2. Use the CLC446. This is a current-feedback op amp
   (a) $f_{3\ dB}$ = 400 MHz ≈ 10 $f_c$ = 500 MHz
   (b) SR = 2000V/µs > 250V/µs (see Item #1 in "Section A Design")
   (c) $C_{ni(446)}$ = 1.0 pF (non-inverting input capacitance)
3. Set $R_{fB}$ to the CLC446's recommended $R_f$ at $A_V$ = +15:
   (a) $R_{fB}$ = 348Ω
   (b) Then set $R_{gB}$ = 696Ω so that $K_B$ = 1.50.
4. Initialize the resistor level for noise and distortion performance:
   (a) R ≈ 200Ω
5. Initialize the capacitor level, resistor and capacitor ratios, and the capacitors: $C \approx \frac{1}{(200\Omega)\cdot(2\pi(53.45\ \text{MHz}))} = 15\ \text{pF}$
   (a) $r^2 \approx 0.10$
   (b) $c^2 \approx$ max (0.0983, 0.10) = 0.1000
   (c) $C_{4B}$ ≈ 4.7 pF
   (d) $C_{5B}$ ≈ 4.7 pF
6. Set the capacitors to the nearest standard values:
   (a) $C_{4B}$ = 4.7 pF
   (b) $C_{5B}$ = 4.7 pF
7. Recalculate the capacitor level and ratio, and the resistor level and ratio:

$$C = \sqrt{(4.7\ \text{pF})\cdot(47\ \text{pF})} = 14.86\ \text{pF}$$
$$c^2 = (4.7\ \text{pF})/(47\ \text{pF}) = 0.1000$$
$$R = \frac{1}{(14.86\ \text{pF})\cdot(2\pi(53.45\ \text{MHz}))} = 200.4\Omega$$
$$r^2 = 0.1020$$

8. Calculate $R_{12B}$ and the resistor values:
   (a) $R_{12B} = 64.0\Omega$
   (b) $R_{1B} = 96.0\Omega$
   (c) $R_{2B} = 192\Omega$
   (d) $R_{3B} = 627\Omega$
   (e) The resulting component values are:

| Component | Value | | |
|---|---|---|---|
| | Initial | Adjusted | Standard |
| $R_{1B}$ | 96.0Ω | 78.9Ω | 78.7Ω |
| $R_{2B}$ | 192Ω | 158Ω | 158Ω |
| $R_{3B}$ | 627Ω | 582Ω | 576Ω |
| $C_{4B}$ | 4.7 pF | 3.7 pF | 3.6 pF |
| $C_{ni(446)}$ | — | 1.0 pF | 1.0 pF |
| $C_{5B}$ | 47 pF | 47 pF | 47 pF |
| $R_{fB}$ | 348Ω | 348Ω | 348Ω |
| $R_{gB}$ | 696Ω | 696Ω | 698Ω |

9. The sensitivities for this design are:

| $\alpha_i$ | $S_{\alpha i}^{H_o}$ | $S_{\alpha i}^{\omega_p}$ | $S_{\alpha i}^{Q_p}$ |
|---|---|---|---|
| K | 1.00 | 0.00 | 2.58 |
| $R_{1B}$ | −0.33 | −0.33 | 0.79 |
| $R_{2B}$ | 0.33 | −0.17 | 0.40 |
| $R_{3B}$ | 0.00 | −0.50 | −1.19 |
| $R_{fB}$ | 0.33 | 0.00 | 0.86 |
| $R_{gB}$ | −0.33 | 0.00 | −0.86 |
| $C_{4B}$ | 0.00 | −0.50 | −1.36 |
| $C_{5B}$ | 0.00 | −0.50 | 1.36 |

10. The relative standard deviations of $H_o$, $\omega_p$ and $Q_p$ are:
   (a) $\sigma_{H_o}/H_o \approx 0.38\%$
   (b) $\sigma_{\omega_p}/\omega_p \approx 0.55\%$
   (c) $\sigma_{Q_p}/Q_p \approx 1.58\%$
   These standard deviations are based on a uniform distribution, with all resistors and capacitor values being independent:

$$\frac{\sigma_R}{R} \approx \frac{\sigma_C}{C} \approx \frac{1.00\% - (-1.00\%)}{\sqrt{12}} \approx 0.58\% \quad (7)$$

11. The nominal values of $H_o$, $\omega_p$ and $Q_p$ over the design temperature range are:

| | | | |
|---|---|---|---|
| T [°C] | −40 | 25 | 85 |
| $H_o$ [V/V] | 1.000 | 1.000 | 1.000 |
| $\omega_p/2\pi$ [MHz] | 53.88 | 53.45 | 53.00 |
| $Q_p$ [ ] | 1.706 | 1.706 | 1.706 |

12. The probable ranges of values for $H_o$, $\omega_p$ and $Q_p$, over the design temperature range, are:
   (a) $0.99 \le H_o \le 1.01$
   (b) $52.1\text{ MHz} \le (\omega_p/2\pi) \le 54.8\text{ MHz}$
   (c) $1.63 \le Q_p \le 1.79$

13. Based on the results in #10 and #12, we can conclude that:
    (a) The DC gain and cutoff frequency change little with component value and temperature changes
    (b) $Q_p$ has the greatest sensitivity to fabrication changes
    (c) The greatest filter response variation is in the peaking near the cutoff frequency

Figure 3 shows the results of a Monte-Carlo simulation at room temperature, with 100 cases simulated. These simulations used the "Standard Values" of the components. The gain curves are:

- Lower 3-sigma limit (mean minus 3 times the standard deviation)
- Mean value
- Upper 3-sigma limit (mean plus 3 times the standard deviation)

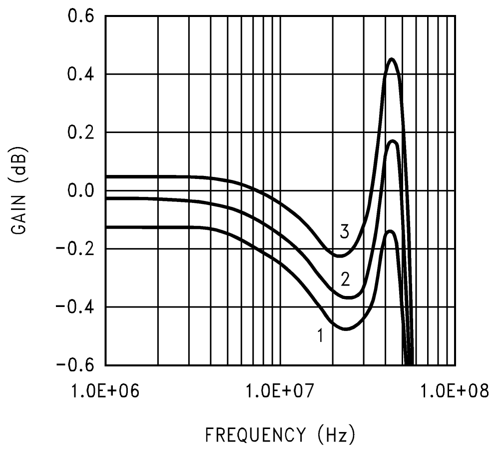


**Figure 3. Monte-Carlo Simulation Results**

## 4 SPICE Models

SPICE models are available for most of Comlinear's amplifiers. These models support nominal DC, AC, AC noise and transient simulations at room temperature.

It is recommended simulating with Comlinear's SPICE models to:

- Predict the op amp's influence on filter response
- Support quicker design cycles

Include board and component parasitic models to obtain a more accurate prediction of the filter's response.

To verify your simulations, we recommend bread-boarding your circuit.

## 5 Summary

This application report contains an easy to use design algorithm for a low sensitivity, Sallen-Key lowpass biquad, which works for $Q_p < 5$. It also shows the basics of evaluating filter sensitivity performance.

Designing for low $\omega_p$ and $Q_p$ sensitivities gives:

- Reduced filter variation over process, temperature and time
- Higher manufacturing yield
- Lower component cost

A low sensitivity design is not enough to produce high manufacturing yields. The nominal design must also compensate for any component parasitics, board parasitics, and op amp bandwidth (see Comlinear's application report on filter component pre-distortion [5]). The components must also have low enough tolerance and temperature coefficients.

## 6 References

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8. A. Lokanathan and J. Brockman, “Efficient Worst Case Analysis of Integrated Circuits,” *IEEE 1995 Custom Integrated Circuits Conf.*, pp. 11.4.1–4, 1995.

# Appendix A Transfer Function Examples

## A.1 Sensitivity Analysis Overview

The classic logarithmic sensitivity function is:

$$S^{X}_{\alpha_i} = \frac{\partial(\ln X)}{\partial(\ln \alpha_i)}\;;\quad \alpha_i, X \neq 0$$
$$= \frac{\alpha_i}{X} \cdot \frac{\partial X}{\partial \alpha_i}$$
$$\approx \frac{\Delta X / X}{\Delta \alpha_i / \alpha_i} \tag{8}$$

where, $\alpha_i$ is a component value, and X is a filter performance measure (in the most general case, this is a complex-value function or frequency). The sensitivity function is a dimensionless figure of merit used in filter design.

You can approximate the relative change in X caused by the relative changes in the components $\alpha_i$ as:

$$\frac{\Delta X}{X} \approx \sum_i S^{X}_{\alpha_i} \cdot \frac{\Delta \alpha_i}{\alpha_i} \tag{9}$$

where,

$$\alpha_i, X \neq 0$$
$$\left|\frac{\Delta \alpha_i}{\alpha_i}\right|, \left|\frac{\Delta X}{X}\right| << 1 \tag{10}$$

The relative standard deviation of X is calculated using:

$$\left(\frac{\sigma_X}{X}\right)^2 \approx \sum_i \left(\left|S^{X}_{\alpha_i}\right| \cdot \frac{\sigma_{\alpha_i}}{\alpha_i}\right)^2 \tag{11}$$

where,

- The summation is over all component values ($\alpha_i$) that affect X
- All component values ($\alpha_i$) are physically independent (no statistical correlation)

The nominal value of X is a function of temperature:

$$X(T) = X\left(1 + \frac{X(T) - X}{X}\right)$$
$$\approx X\left(1 + \sum_i \left(S^{X}_{\alpha_i} \cdot \frac{\alpha_i(T) - \alpha_i}{\alpha_i}\right)\right) \tag{12}$$

where,

- X is the nominal value of X at room temperature
- $\alpha_i$ (T) is the nominal value of $\alpha_i$ at temperature T
- X(T) is the nominal value of X at temperature T

To help reduce variation in filter performance:

- Reduce the sensitivity function magnitudes ($\left|S^{X}_{\alpha_i}\right|$), where X is $H_o$, $\omega_p$ and $Q_p$, and $\alpha_i$ is any of the component values, the gain K, or operating conditions (such as temperature or supply voltage)
- Use components with smaller tolerances
- Use components with lower temperature coefficients

# Appendix B Biquad Section, s Term in the Denominator That Includes K

## B.1 Handy Sensitivity Formulas

Notation:

- k, m, n = constants
- α, β = [non-zero] component parameters
- X, Y = [non-zero] performance measures

Formulas:

$$
\begin{aligned}
&1.\quad S_{\alpha}^{k\alpha^{n}} = n \\
&2.\quad S_{\alpha}^{kX} = S_{k\alpha}^{X} = S_{\alpha}^{X} \\
&3.\quad S_{\alpha^{k}}^{X^{m}Y^{n}} = \frac{m}{k} \cdot S_{\alpha}^{X} + \frac{n}{k} \cdot S_{\alpha}^{Y} \\
&4.\quad S_{\alpha}^{X} = 1/S_{X}^{\alpha} \\
&5.\quad S_{\alpha}^{Y(X(\alpha))} = S_{X}^{Y} S_{\alpha}^{X} \\
&6.\quad S_{\gamma}^{X(\alpha,\beta)} = S_{\alpha}^{X} S_{\gamma}^{\alpha} + S_{\beta}^{X} S_{\gamma}^{\beta} \\
&7.\quad S_{\alpha}^{X} = \mathrm{Re}\left(S_{\alpha}^{X}\right) + j\,\mathrm{Im}\left(S_{\alpha}^{X}\right) \\
&\quad\text{where:} \\
&\quad\quad \mathrm{Re}\left(S_{\alpha}^{X}\right) = S_{\alpha}^{|X|} \\
&\quad\quad \mathrm{Im}\left(S_{\alpha}^{X}\right) = \arg(X) \cdot S_{\alpha}^{\arg(X)} = \alpha \cdot \frac{\partial(\arg(X))}{\partial(\alpha)}
\end{aligned}
\tag{13}
$$

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