ABSTRACT
This application note presents a detail modeling and design of current mode control boost converters operating in the continuous conduction mode (CCM). Based on the derived small signal models, the design of a lag compensator for current mode control boost converters will be detailed. The LM3478 boost controller will be used in the example. Simulation and hardware measurement of frequency responses will be shown.

Contents
1 Basic Operation of a Boost Converter ............................................................................................................ 2
2 Modeling of an Open Loop Boost Converter .................................................................................................... 2
3 Modeling of a Current Mode Control Boost Converter ...................................................................................... 4
4 Principle of a Lag Compensator ...................................................................................................................... 6
5 Illustrative Example ............................................................................................................................................ 7
6 Conclusion .......................................................................................................................................................... 9

List of Figures
1 An Open Loop Boost Converter ....................................................................................................................... 2
2 Inductor Current Waveform at the On Period ....................................................................................................... 4
3 Frequency Response of a Lag Compensator ....................................................................................................... 6
4 A Lag Compensator Implemented by a Transconductance Amplifier Circuit .................................................. 6
5 Frequency Response of the Un-Compensated System ..................................................................................... 8
6 Frequency Response of the Compensated System with 90° Phase Margin .................................................... 9

List of Tables
1 Major Parameters of the Example Boost Converter .......................................................................................... 7
2 Parameters of the LM3478 ............................................................................................................................... 7
1 Basic Operation of a Boost Converter

Figure 1. An Open Loop Boost Converter

Figure 1 shows an open loop boost converter with an inductor \( L_1 \), a diode \( D_1 \), an output capacitor \( C_{OUT} \) with an equivalent series resistance \( R_{OUT} \). It is assumed that the load is a resistor \( R_{OUT} \), and the switch \( Q_1 \) is ideal. Let \( v_{IN} \), \( v_{OUT} \), and \( v_{COUT} \) be the input voltage, output voltage, and the voltage across \( C_{OUT} \); \( i_{L1} \) be the current through \( L_1 \); and \( V_{D1} \) be the forward voltage drop of \( D_1 \) when \( D_1 \) is turned on. Under the CCM, when \( Q_1 \) is turned on, the state equations are

\[
\begin{align*}
v_{IN} &= L_1 \frac{d}{dt} i_{L1}, \\
v_{OUT} &= v_{COUT} - R_{OUT} \frac{d}{dt} v_{COUT}
\end{align*}
\]

(1) (2)

Also, the output equation is

\[
v_{OUT} = v_{COUT} + C_{OUT} \frac{d}{dt} v_{COUT} R_{OUT}\n\]

(3)

Similarly, when \( Q_1 \) is turned off, the output equation of (3) still holds, while the state equations become

\[
\begin{align*}
v_{IN} &= L_1 \frac{d}{dt} i_{L1} + V_{D1} + v_{OUT}, \\
v_{OUT} &= i_{L1} \cdot C_{OUT} \frac{d}{dt} v_{COUT}
\end{align*}
\]

(4) (5)

2 Modeling of an Open Loop Boost Converter

To obtain a small signal model of boost converters, it is required to apply the averaging technique, perturbation, and the linearization technique. First, by applying the averaging technique, the averaged state equations and output equation are

\[
\begin{align*}
a_{IN} &= L_1 \frac{d}{dt} \bar{i}_{L1} + (1 - d) V_{D1} + (1 - d) a_{OUT}, \\
a_{OUT} &= (1 - d) i_{L1} \cdot C_{OUT} \frac{d}{dt} a_{COUT}. \\
a_{OUT} &= a_{COUT} + R_{OUT} C_{OUT} \frac{d}{dt} a_{COUT}
\end{align*}
\]

(6) (7) (8)

where \( d \) is the duty cycle, \( \bar{x} \) is the averaged variable for the variable \( x \) (which can represent \( v_{IN} \), \( v_{OUT} \), \( v_{COUT} \), and \( i_{L1} \)).

Second, by applying small signal perturbations to (6) to (8), i.e. let

\[
x = \bar{x} + \tilde{x},
\]

where \( \bar{x} \) and \( \tilde{x} \) are nominal (DC) and perturbed (AC) variables,

\[
\begin{align*}
\bar{v}_{IN} + \tilde{v}_{IN} &= L_1 \frac{d}{dt} (\bar{i}_{L1} + \bar{i}_{L1}) + (1 - d) V_{D1} + (1 - d)\bar{v}_{OUT} + \tilde{v}_{OUT}.
\end{align*}
\]

(9)
Third, by applying the linearization technique (assume that the high order non-linear terms are small and negligible), a set of DC and AC equations can be obtained as follows:

DC equations:

From (9)
\[
\overline{V}_\text{IN} = (1 - D)\overline{V}_\text{DIN} + (1 - D)\overline{V}_\text{OUT}
\]
\[
\overline{D} = \frac{\overline{V}_\text{OUT} - \overline{V}_\text{IN} + \overline{V}_\text{DIN}}{\overline{V}_\text{OUT} + \overline{V}_\text{DIN}}
\] (12)

Also
\[
\overline{V}_\text{OUT} = \frac{\overline{V}_\text{IN} - (1 - D)\overline{V}_\text{DIN}}{(1 - D)}
\]

For simplicity, consider that \((1 - D)\overline{V}_\text{DIN}\) is small compared with \(\overline{V}_\text{IN}\),
\[
\overline{V}_\text{OUT} = \frac{\overline{V}_\text{IN}}{(1 - D)}
\] (13)

From (10),
\[
I_{L1} = \frac{\overline{V}_\text{OUT}}{(1 - D)R_{\text{OUT}}}
\] (14)

AC equations:

From (9),
\[
\tilde{V}_\text{IN} = sL_{\text{I}L1} + (1 - D)\tilde{V}_\text{OUT} - d\overline{V}_\text{OUT}
\]
\[
sL_{\text{I}L1} = \tilde{V}_\text{IN} - (1 - D)\tilde{V}_\text{OUT} + d\overline{V}_\text{OUT}
\] (15)

From (10),
\[
\tilde{V}_\text{OUT} = (1 - D)\tilde{I}_{L1} - sC_{\text{OUT}}\overline{V}_{\text{COUT}}
\] (16)

From (11),
\[
\overline{V}_\text{OUT} = \overline{V}_{\text{COUT}} + sR_{\text{COUT}}C_{\text{OUT}}\overline{V}_{\text{COUT}}
\]
\[
\overline{V}_{\text{COUT}} = \frac{\overline{V}_\text{OUT}}{1 + sR_{\text{COUT}}C_{\text{OUT}}}
\] (17)

Substitute (13), (14), (15), and (17) into (16),
\[
\tilde{V}_\text{OUT} = \left(\frac{(1 + sR_{\text{COUT}}C_{\text{OUT}})\overline{V}_\text{IN}}{(1 - D)}\right) + \left(\frac{(1 + sR_{\text{COUT}}C_{\text{OUT}})\overline{V}_\text{IN}}{(1 - D)^2}\right) \frac{sL_{\text{I}L1}}{R_{\text{OUT}}(1 - D)^2} - \left(\frac{s^2L_{\text{I}L1}R_{\text{OUT}} + s^2L_{\text{I}L1}R_{\text{COUT}}}{R_{\text{OUT}}(1 - D)^2}\right)
\] (18)
3 Modeling of a Current Mode Control Boost Converter

Under current mode control, \( i_{L1} \) is fed back when \( Q_1 \) is turned on to determine the on-time of \( Q_1 \) by comparing it to a current control signal \( i_C \). A compensation ramp of slope \(-m_C\) is normally added to avoid sub-harmonic oscillation. Figure 2 shows the current waveform of an on period. The averaged state equation is as follows:

\[
\bar{i}_{L1} = i_C - m_C \bar{d} T_{SW} - \frac{m_1}{2} \bar{d} T_{SW},
\]

(19)

where

\[
m_1 = \frac{V_{IN}}{L_1}
\]

(20)

is the slope of \( i_{L1} \) during the on period. By adding small signal perturbations to the above equation,

\[
i_{L1} + \tilde{i}_{L1} = i_C + \tilde{i}_C - m_C(D + \tilde{d})T_{SW} - \frac{T_{SW}}{2}(\bar{M}_1 + \bar{m}_1)(D + \tilde{d}).
\]

(21)

By applying the linearization technique (assume that the high order non-linear terms are small and negligible), a set of DC and AC equations can be obtained as follows:

**DC equation:**

\[
i_{L1} = i_C - m_C D T_{SW} - \frac{T_{SW}}{2} \bar{M}_1 D
\]

(22)

where

\[
\bar{M}_1 = \frac{V_{IN}}{L_1}
\]

**AC equation:**

\[
i_{L1} = i_C - m_C T_{SW} \bar{d} - \frac{T_{SW}}{2} \bar{m}_1 \bar{d} - \frac{T_{SW}}{2} D \bar{m}_1
\]

(23)

Define

\[
T_2 = \frac{T_{SW}}{2}
\]

(24)

\[
T_M = \frac{T_{SW}}{2}(2m_C + \bar{M}_1)
\]

(25)

From (20), (23), (24), (25),

\[
i_{L1} = i_C - T_0 \bar{d} - D T_2 \bar{V}_{IN} L_1.
\]

(26)

Substitute (13) into (15),

\[
sL \bar{i}_{L1} = \bar{V}_{IN} - (1 - D)\bar{V}_{OUT} + \bar{d} \frac{V_{IN}}{(1 - D)}
\]

(27)
Substitute (27) into (26),
\[
\tilde{d} = \frac{sL_i C + (1 - D)\tilde{V}_{OUT} - \left(1 + \frac{sL_i D T_2}{L_1}\right)\tilde{V}_{IN}}{\tilde{v}_{IN} + sL_i T M\
\]
(28)

By substituting (28) into (18), the small signal model of the current mode control boost converter can be formulated as follows:
\[
\tilde{v}_{OUT} = \frac{G_{NC}(s)\tilde{v}_{IN} + G_{IC}(s)i_C}{\Delta(s)},
\]
(29)

where
\[
G_{NC}(s) = R_{OUT}(1 - D)(1 + sR_{COUT}C_{OUT})\
\left\{\frac{(1 - D)T M}{\tilde{v}_{IN}} + \frac{1}{R_{OUT}(1 - D)^2} - \frac{DT_2}{L_1} + \frac{sDT_2}{R_{OUT}(1 - D)^2}\right\}
\]
\[
G_{IC}(s) = R_{OUT}(1 - D)(1 + sR_{COUT}C_{OUT})\left[1 + sL_1\frac{R_{COUT}}{R_{OUT}(1 - D)^2}\right]
\]
\[
\Delta(s) = 2 + R_{OUT}(1 - D)^2 T M\left(\frac{T M}{\tilde{v}_{IN}} + (1 - D) + (R_{OUT} + R_{COUT})C_{OUT}\right)\
+ s\left\{L_1 C_{OUT} (R_{OUT} + R_{COUT}) T M\left(\frac{1 - D}{\tilde{v}_{IN}}\right)\right\}
\]

If the current \(i_{L1}\) is sensed by a resistor \(R_{SN}\) connecting between Q1 and the ground, the current control signal \(i_C\) can be converted to a voltage control signal \(v_C\). The relationship between the output voltage and the voltage control signal can be formulated as follows:
\[
\tilde{v}_{OUT} = \frac{G_{IC}(s)}{\Delta(s)R_{SN}} \tilde{v}_{C}.
\]
(30)
4 Principle of a Lag Compensator

A lag compensator consists of a pair of pole and zero at the frequency \( f_{PC} \) and \( f_{ZC} \), with \( f_{PC} < f_{ZC} \). It also provides a dc gain \( A_C \). As shown in Figure 3, the lag compensator provides an attenuation in magnitude at the high frequency. The degree of attenuation is determined by the distance between \( f_{PC} \) and \( f_{ZC} \). It is because the magnitude is decreased at a slope of 20dB/decade between \( f_{PC} \) and \( f_{ZC} \). The lag compensator also provides a phase lag. However, \( f_{PC} \) and \( f_{ZC} \) can be placed at a low frequency (much lower than the frequency of interest, for example, the cross over frequency \( f_C \)) such that the lag compensator nearly does not affect the phase at the high frequency.

The aim of designing a lag compensator is to provide a desired phase margin for the compensated system. Starting from a bode plot of an un-compensated system, and a requirement of phase margin of \( \Phi_m \), a new \( f_C \) can be selected at the frequency corresponding to \( 180^\circ - \Phi_m \) of the un-compensated system. Then the magnitude of the un-compensated system at \( f_C \) is found. The magnitude at \( f_C \) can be attenuated to 0dB by the lag compensator through proper design of \( f_{PC} \) and \( f_{ZC} \). As a result, the compensated system will have a phase margin of \( \Phi_m \), and the cross over frequency will be \( f_C \). An illustrative example (see Section 5) will be presented to show the design steps.

![Figure 3. Frequency Response of a Lag Compensator](image)

By adding small signal perturbations, the AC equation can be obtained as follows:

\[
\dot{v}_C = \frac{R_{F2}}{R_{F1} + R_{F2}} g_m R_0 \frac{1 + sR_C C_1}{1 + s(R_C + R_0)C_1} \dot{v}_{OUT}. \tag{32}
\]

Hence,
Illustrative Example

The design of a current mode control Boost converter with a nominal input voltage of 5V and an output voltage of 12V and an output current of 0.5A will be shown. The major components are listed in Table 1. A current mode controller LM3478 will be used. The parameters of the LM3478, which can be derived from the data sheet, are also listed in Table 2.

### Table 1. Major Parameters of the Example Boost Converter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>V&lt;sub&gt;IN&lt;/sub&gt;</td>
<td>5V</td>
</tr>
<tr>
<td>V&lt;sub&gt;OUT&lt;/sub&gt;</td>
<td>12V</td>
</tr>
<tr>
<td>R&lt;sub&gt;OUT&lt;/sub&gt;</td>
<td>24Ω</td>
</tr>
<tr>
<td>L&lt;sub&gt;1&lt;/sub&gt;</td>
<td>10 µH</td>
</tr>
<tr>
<td>C&lt;sub&gt;OUT&lt;/sub&gt;</td>
<td>150 µF</td>
</tr>
<tr>
<td>R&lt;sub&gt;COUT&lt;/sub&gt;</td>
<td>0.05Ω</td>
</tr>
<tr>
<td>f&lt;sub&gt;SW&lt;/sub&gt;</td>
<td>400 kHz</td>
</tr>
<tr>
<td>R&lt;sub&gt;SN&lt;/sub&gt;</td>
<td>0.05Ω</td>
</tr>
<tr>
<td>R&lt;sub&gt;SL&lt;/sub&gt;</td>
<td>604Ω</td>
</tr>
</tbody>
</table>

### Table 2. Parameters of the LM3478

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>V&lt;sub&gt;REF&lt;/sub&gt;</td>
<td>1.26V</td>
</tr>
<tr>
<td>g&lt;sub&gt;m&lt;/sub&gt;</td>
<td>800 µΩ&lt;sup&gt;-1&lt;/sup&gt;</td>
</tr>
<tr>
<td>R&lt;sub&gt;0&lt;/sub&gt;</td>
<td>$A_V/g_m = 38/800$ µΩ&lt;sup&gt;-1&lt;/sup&gt; = 47.5 kΩ</td>
</tr>
<tr>
<td>V&lt;sub&gt;SL&lt;/sub&gt;</td>
<td>92 mV</td>
</tr>
</tbody>
</table>

Other parameters of (30) are calculated below.

From (13),
\[ D = 0.5833 \]

From (24),
\[ T_2 = \frac{T_{SW}}{2} = \frac{1}{2f_{SW}} \]
\[ T_2 = 1.25 \mu s \]  \hspace{1cm} (34)

The parameter \( m_c \) is determined by an internal compensation ramp \( V_{SL} \) and an external compensation ramp determined by an internal current of 40 µA passing through an external resistor \( R_{SL} \). It can be calculated by the following equation:
\[ m_c = (V_{SL} + 40 \mu A \times R_{SL})f_{SW}R_{SN} = 929280 \text{As}^{-1} \]  \hspace{1cm} (36)

From (25),
\[ T_M = \frac{T_{SW}}{2} \left( 2m_c + \frac{V_{IN}}{L_1} \right) \]
\[ = 2.9482A \]  \hspace{1cm} (37)
Hence, all parameters of (30) are obtained. A bode plot of (30) with the above parameters is shown in Figure 5. The DC gain, zeros, and poles are

DC gain: 36.39 dB
Zeros: 53 kHz, 66 kHz (right half plane zero)
Poles: 133 Hz, 65 kHz

![Bode Plot](image)

**Figure 5. Frequency Response of the Un-Compensated System**

The design of the lag compensator can be achieved by following (32). Since $V_{OUT}$ and $V_{REF}$ are 12V and 1.26V respectively, we can design that

\[ R_{F1} = 84.5 \, \text{k}\Omega \]  \hspace{1cm} (41)  \\
\[ R_{F2} = 10 \, \text{k}\Omega \]  \hspace{1cm} (42)

From (32),

\[ A_C = \frac{R_{F2}}{R_{F1} + R_{F2}} \cdot g_m \cdot R_0 \]
\[ = 4.02 \]
\[ = 12.09 \, \text{dB} \]  \hspace{1cm} (43)

In this example, a compensated system with a phase margin of around 90° is desired. From Figure 5, $f_C$ can be selected as 3.5kHz (the frequency at which the phase is around $180° - 90° = 90°$). The attenuation required is $7\, \text{dB} + A_C = 19.09\, \text{dB}$, which implies that the distance between $f_{PC}$ and $f_{ZC}$ should be 0.96 decade. Select $f_{ZC}$ to be 350 Hz, i.e. one decade lower than $f_C$, then $f_{PC}$ should be 38.3 Hz.

\[ \frac{1}{R_{C1} C_{C1}} = 2\pi \times 350 \, \text{Hz} \]  \hspace{1cm} (44)
\[ \frac{1}{(R_{C1} + R_0) C_{C1}} = \frac{1}{2\pi \times 38.3 \, \text{Hz}} \cdot R_1 C_{C1} \]
\[ R_0 C_{C1} = \frac{1}{2\pi \times 38.3 \, \text{Hz}} \cdot R_1 C_{C1} \]
\[ C_{C1} = 78 \, \text{nF} \]
\[ R_{C1} = 5.85 \, \text{k}\Omega \]  \hspace{1cm} (45)
Finally, select $R_{C1} = 5.9 \, \text{k}\Omega$ and $C_{C1} = 100 \, \text{nF}$. The frequency response of the compensated system is shown in Figure 6. It can be found that the 0dB point is at around 4 kHz, and the phase margin is around 95°.

6 Conclusion

This application note details the modeling of an open loop and a current mode control boost converter under the continuous conduction mode. The principle and design of a lag compensator have also been addressed. An example has been presented to illustrate the design. A lag compensator has been designed for a compensated system with around 90° phase margin.

The selection of a desired phase margin affects the transient response of the output voltage. Moreover, some practical systems are suffered from noise, and transient responses are not a major concern. A lower cross over frequency may be required. In this case, the application of the dominant pole compensation method may be more appropriate. The lag compensator can be easily changed to a compensator with a dominant pole by setting $R_{C1}$ to zero, that is, to eliminate the zero. Application engineers are suggested to design properly based on practical situations.
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