AN-104 Noise Specs Confusing?

ABSTRACT

It's really all very simple—once you understand it. Then, here's the inside story on noise for those of us who haven't been designing low noise amplifiers for ten years.

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1   Introduction

You hear all sorts of terms like signal-to-noise ratio, noise figure, noise factor, noise voltage, noise current, noise power, noise spectral density, noise per root Hertz, broadband noise, spot noise, shot noise, flicker noise, excess noise, I/F noise, fluctuation noise, thermal noise, white noise, pink noise, popcorn noise, bipolar spike noise, low noise, no noise, and loud noise. No wonder not everyone understands noise specifications.

In a case like noise, it is probably best to sort it all out from the beginning. So, in the beginning, there was noise; and then there was signal. The whole idea is to have the noise very small compared to the signal; or, conversely, we desire a high signal-to-noise ratio S/N. Now it happens that S/N is related to noise figure NF, noise factor F, noise power, noise voltage $\bar{e}_n$, and noise current $\bar{i}_n$. To simplify matters, it also happens that any noisy channel or amplifier can be completely specified for noise in terms of two noise generators $\bar{e}_n$ and $\bar{i}_n$ as shown in Figure 1.
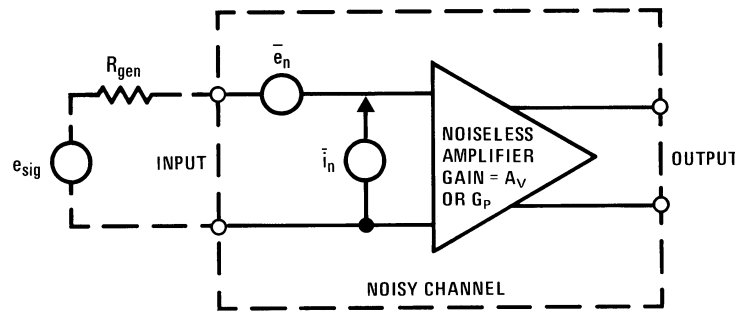
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Figure 1. Noise Characterization of Amplifier

All we really need to understand are NF, $\bar{e}_n$, and $\bar{i}_n$. So here is a rundown on these three.

NOISE VOLTAGE, $\bar{e}_n$, or more properly, EQUIVALENT SHORT-CIRCUIT INPUT RMS NOISE VOLTAGE is simply that noise voltage which would appear to originate at the input of the noiseless amplifier if the input terminals were shorted. It is expressed in nanovolts per root Hertz $nV/\sqrt{Hz}$ at a specified frequency, or in microvolts in a given frequency band. It is determined or measured by shorting the input terminals, measuring the output rms noise, dividing by amplifier gain, and referencing to the input. Hence the term, equivalent noise voltage. An output bandpass filter of known characteristic is used in measurements, and the measured value is divided by the square root of the bandwidth $\sqrt{B}$ if data is to be expressed per unit bandwidth or per root Hertz. The level of $\bar{e}_n$ is not constant over the frequency band; typically it increases at lower frequencies as shown in Figure 2. This increase is 1/f NOISE.
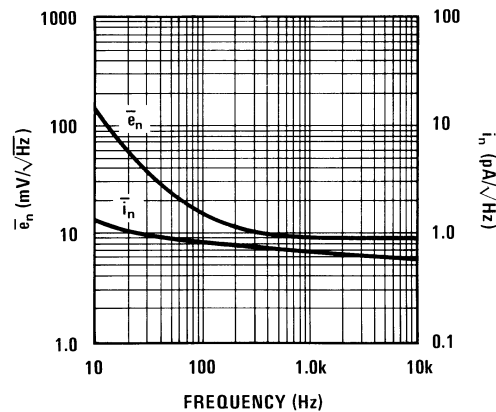

Figure 2. Noise Voltage and Current for an Op Amp

NOISE CURRENT, $\bar{i}_n$, or more properly, EQUIVALENT OPEN-CIRCUIT RMS NOISE CURRENT is that noise which occurs apparently at the input of the noiseless amplifier due only to noise currents. It is expressed in picoamps per root Hertz $pA/\sqrt{Hz}$ at a specified frequency or in nanoamps in a given frequency band. It is measured by shunting a capacitor or resistor across the input terminals such that the noise current will give rise to an additional noise voltage which is $\bar{i}_n \times R_{in}$ (or X_{cin}). The output is measured, divided by amplifier gain, referenced to input, and that contribution known to be due to $\bar{e}_n$ and resistor noise is appropriately subtracted from the total measured noise. If a capacitor is used at the input, there is only $\bar{e}_n$ and $\bar{i}_n X_{cin}$. The $\bar{i}_n$ is measured with a bandpass filter and converted to

$pA\sqrt{Hz}$

if appropriate; typically it increases at lower frequencies for op amps and bipolar transistors, but increases at higher frequencies for field-effect transistors.

NOISE FIGURE, NF is the logarithm of the ratio of input signal-to-noise and output signal-to-noise.

$$NF = 10 \, Log \frac{(S/N)_{in}}{(S/N)_{out}}$$

(1)

where: S and N are power or (voltage)^2 levels

This is measured by determining the S/N at the input with no amplifier present, and then dividing by the measured S/N at the output with signal source present.

The values of R_{gen} and any X_{gen} as well as frequency must be known to properly express NF in meaningful terms. This is because the amplifier $\bar{i}_n$ x Z_{gen} as well as R_{gen} itself produces input noise. The signal source in Figure 1 contains some noise. However e_{sig} is generally considered to be noise free and input noise is present as the THERMAL NOISE of the resistive component of the signal generator impedance R_{gen}. This thermal noise is WHITE in nature as it contains constant NOISE POWER DENSITY per unit bandwidth. It is easily seen from Equation 2 that the $\bar{e}_n^2$ has the units V^2/Hz and that $(\bar{e}_n)$ has the units $V/\sqrt{Hz}$

$$ne_R{}^2 = 4kTRB \tag{2}$$

where:— T is the temperature in °K
 R is resistor value in Ω
 B is bandwidth in Hz
 k is Boltzman's constant

1.1 Relation Between $\bar{e}_n$, $\bar{i}_n$, NF

Now we can examine the relationship between $\bar{e}_n$ and $\bar{i}_n$ at the amplifier input. When the signal source is connected, the $\bar{e}_n$ appears in series with the e_{sig} and $\bar{e}_R$. The $\bar{i}_n$ flows through R_{gen} thus producing another noise voltage of value $\bar{i}_n$ x R_{gen}. This noise voltage is clearly dependent upon the value of R_{gen}. All of these noise voltages add at the input in rms fashion; that is, as the square root of the sum of the squares. Thus, neglecting possible correlation between $\bar{e}_n$ and $\bar{i}_n$, the total input noise is

$$ne_N{}^2 = ne_n{}^2 + ne_R{}^2 + + ni_n{}^2\, R_{gen}{}^2 \tag{3}$$

Further examination of the NF equation shows the relationship of $\bar{e}_N$, $\bar{i}_n$, and NF.

$$NF = 10 \log \frac{S_{in} \times N_{out}}{S_{out} \times N_{in}}$$

$$= 10 \log \frac{S_{in}\, G_p\, \overline{e_N{}^2}}{S_{in}\, G_p\, \overline{e_R{}^2}}$$

where: G_p = power gain

$$= 10 \log \frac{\overline{e_N{}^2}}{\overline{e_R{}^2}}$$

$$= 10 \log \frac{\overline{e_n{}^2} + \overline{e_R{}^2} + \overline{i_n{}^2}\, R_{gen}{}^2}{\overline{e_R{}^2}}$$

$$NF = 10 \log \left(1 + \frac{\overline{e_n{}^2} + \overline{i_n{}^2}\, R_{gen}{}^2}{\overline{e_R{}^2}} \right) \tag{4}$$

Thus, for small R_{gen}, noise voltage dominates; and for large R_{gen}, noise current becomes important. A clear advantage accrues to FET input amplifiers, especially at high values of R_{gen}, as the FET has essentially zero $\bar{i}_n$. Note, that for an NF value to have meaning, it must be accompanied by a value for R_{gen} as well as frequency.

1.2 Calculating Total Noise, $\bar{e}_N$

We can generate a plot of $\bar{e}_N$ for various values of R_{gen} if noise voltage and current are known vs frequency. Such a graph is shown in Figure 3 drawn from Figure 2. To make this plot, the thermal noise $\bar{e}_R$ of the input resistance must be calculated from Equation 2 or taken from the graph of Figure 4. Remember that each term in Equation 3 must be squared prior to addition, so the data from Figure 4 and from Figure 2 is squared. A sample of this calculation follows:
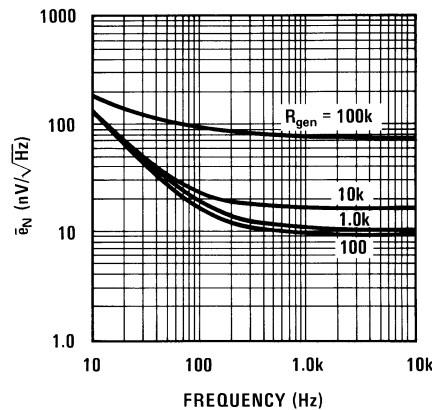
Figure 3. Total Noise for the Op Amp of Figure 2
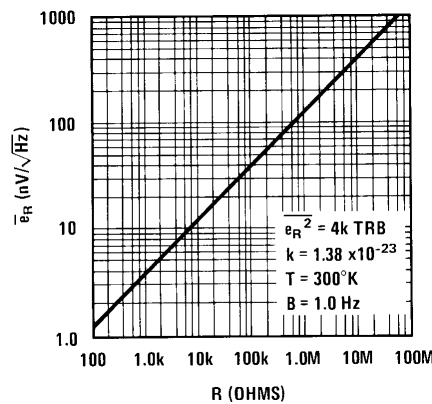
Figure 4. Thermal Noise of Resistor

Example 1: Determine total equivalent input noise per unit bandwidth for an amplifier operating at 1 kHz from a source resistance of 10 kΩ. Use the data from Figure 2 and Figure 4.

1. Read $\bar{e}_R$ from Figure 4 at 10 kΩ; the value is

    $12.7 \, nV/\sqrt{Hz}$.

2. Read $\bar{e}_n$ from Figure 2 at 1 kHz; the value is

    $9.5 \, nV/\sqrt{Hz}$.

3. Read $\bar{i}_n$ from Figure 2 at 1 kHz; the value is
   Multiply by 10 kΩ to obtain

    $0.68 \, pA/\sqrt{Hz}$.

    $6.8 \, nV/\sqrt{Hz}$.

4. Square each term individually, and enter into Equation 3.

$$\bar{e}_N = \sqrt{\overline{e_n^2} + \overline{e_R^2} + \overline{i_n^2}\,R_{gen}^2}$$

$$= \sqrt{9.5^2 + 12^2 + 6.8^2} \quad = \sqrt{279}$$

$$\bar{e}_N = 17.4 \, nV/\sqrt{Hz}$$

This is total rms noise at the input in one Hertz bandwidth at 1 kHz. If total noise in a given bandwidth is desired, one must integrate the noise over a bandwidth as specified. This is most easily done in a noise measurement set-up, but may be approximated as follows:

1. If the frequency range of interest is in the flat band; i.e., between 1 kHz and 10 kHz in Figure 2, it is simply a matter of multiplying $\bar{e}_N$ by the square root of the bandwidth. Then, in the 1 kHz–10 kHz band,

total noise is

$$\overline{e}_N = 17.4\sqrt{9000}$$
$$= 1.65 \ \mu V$$

2. If the frequency band of interest is not in the flat band of Figure 2, one must break the band into sections, calculating average noise in each section, squaring, multiplying by section bandwidth, summing all sections, and finally taking square root of the sum as follows:

$$\overline{e}_N = \sqrt{\overline{e_R^2}B + \sum_{1}^{i} \overline{(e_n^2 + i_n^2 R_{gen}^2)}_i B_i} \qquad (5)$$

where: i is the total number of sub-blocks.

For most purposes a sub-block may be one or two octaves. Example 2 details such a calculation.

Example 2: Determine the rms noise level in the frequency band 50 Hz to 10 kHz for the amplifier of Figure 2 operating from $R_{gen} = 2k$.

1. Read $\overline{e}_R$ from Figure 4 at 2k, square the value, and multiply by the entire bandwidth. Easiest way is to construct a table as shown on the next page.
2. Read the median value of $\overline{e}_n$ in a relatively small frequency band, say 50 Hz–100 Hz, from Figure 2, square it and enter into the table
3. Read the median value of $\overline{i}_n$ in the 50 Hz–100 Hz band from Figure 2, multiply by $R_{gen} = 2k$, square the result and enter in the table.
4. Sum the squared results from steps 2 and 3, multiply the sum by Δf = 100–50 = 50 Hz, and enter in the table.
5. Repeat steps 2–4 for band sections of 100 Hz–300 Hz, 300 Hz–1000 Hz and 1 kHz–10 kHz. Enter results in the table.
6. Sum all entries in the last column, and finally take the square root of this sum for the total rms noise in the 50 Hz–10,000 Hz band.
7. Total $\overline{e}_n$ is 1.62 µV in the 50 Hz–10,000 Hz band.

1.3 *Calculating S/N and NF*

Signal-to-noise ratio can be easily calculated from known signal levels once total rms noise in the band is determined. Example 3 shows this rather simple calculation from Equation 6 for the data of Example 2.

$$S/N = 20 \log \frac{e_{sig}}{\overline{e}_N} \qquad (6)$$

Example 3: Determine S/N for an rms $e_{sig} = 4$ mV at the input to the amplifier operated in Example 2.

1. RMS signal is $e_{sig} = 4$ mV
2. RMS noise from Example 2 is 1.62 µV
3. Calculate S/N from Equation 6

$$S/N = 20 \log \frac{4 \text{ mV}}{1.62 \ \mu V}$$

$$= 20 \log (2.47 \times 10^3)$$

$$= 20 (\log 10^3 + \log 2.47)$$

$$= 20 (3 + 0.393)$$

$$S/N = 68 \text{ dB}$$

It is also possible to plot NF vs frequency at various R_{gen} for any given plot of $\overline{e}_n$ and $\overline{i}_n$. However there is no specific all-purpose conversion plot relating NF, $\overline{e}_n$, $\overline{i}_n$, R_{gen} and f. If either $\overline{e}_n$ or $\overline{i}_n$ is neglected, a reference chart can be constructed. Figure 5 is such a plot when only $\overline{e}_n$ is considered. It is useful for most op amps when R_{gen} is less than about 200Ω and for FETs at any R_{gen} (because there is no significant $\overline{i}_n$ for FETs), however actual NF for op amps with $R_{gen} > 200Ω$ is higher than indicated on the chart. The graph of Figure 5 can be used to find spot NF if $\overline{e}_n$ and R_{gen} are known, or to find $\overline{e}_n$ if NF and R_{gen} are known. It can also be used to find max R_{gen} allowed for a given max NF when $\overline{e}_n$ is known. In any case, values are only valid if $\overline{i}_n$ is negligible and at the specific frequency of interest for NF and $\overline{e}_n$, and for 1 Hz bandwidth. If bandwidth increases, the plot is valid so long as $\overline{e}_n$ is multiplied by $\sqrt{B}$.
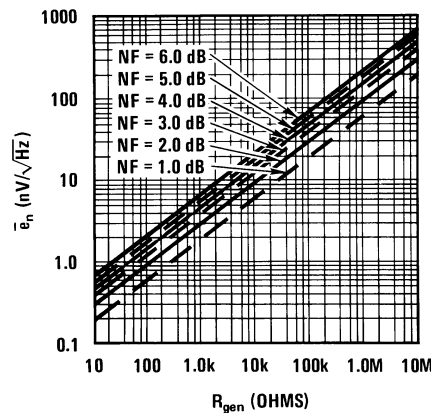

Figure 5. Spot NF vs R_{gen} when Considering *Only* $\overline{e}_n$ and $\overline{e}_R$ (not valid when $\overline{i}_n$ R_{gen} is significant)

1.4 *The Noise Figure Myth*

Noise figure is easy to calculate because the signal level need not be specified (note that e_{sig} drops out of Equation 4). Because NF is so easy to handle in calculations, many designers tend to lose sight of the fact that signal-to-noise ratio $(S/N)_{out}$ is what is important in the final analysis, be it an audio, video, or digital data system. One can, in fact, choose a high R_{gen} to reduce NF to near zero if $\overline{i}_n$ is very small. In this case $\overline{e}_R$ is the major source of noise, overshadowing $\overline{e}_n$ completely. The result is very low NF, but very low S/N as well because of very high noise. Don't be fooled into believing that low NF means low noise *per se*!

Another term is worth considering, that is optimum source resistance R_{OPT}. This is a value of R_{gen} which produces the lowest NF in a given system. It is calculated as

$$R_{OPT} = \frac{\overline{e}_n}{\overline{i}_n}$$

(7)

This has been arrived at by differentiating Equation 4 with respect to R_{gen} and equating it to zero (see Appendix). *Note that this does not mean lowest noise.*

For example, using Figure 2 to calculate R_{OPT} at say 600 Hz,

$$R_{OPT} = \frac{10 \text{ nV}}{0.7 \text{ pA}} = 14 \text{ k}Ω$$

Table 1. Noise Calculations for Example 2[1]

B (Hz)	Δf (Hz)	$\bar{e}_n2$ (nV/Hz)	+ $\bar{i}_n2\ R_{gen}2$			SUM x Δf	=	(nV²)
50–100	50	$(20)^2 = 400$	$(8.7 \times 2.0k)^2$	=	302	702* x 50		35,000
100–300	200	$(13)^2 = 169$	$(8 \times 2.0k)^2$	=	256	425 x 200		85,000
300–1000	700	$(10)^2 = 100$	$(7 \times 2.0k)^2$	=	196	296 x 700		207,000
1.0k–10k	9000	$(9)^2 = 81$	$(6 \times 2.0k)^2$	=	144	225 x 9000		2,020,000
50–10,000	9950	$\bar{e}_R2 = (5.3)^2 = 28$				28 x 9950		279,000
Total $\bar{e}_N = \sqrt{2,626,000}$		= 1620 nV = 1.62 µV						

*The units are as follows: $(20\ nV/\sqrt{Hz})^2 = 400\ (nV)^2/Hz$

$(8.7\ pA/\sqrt{Hz} \times 2.0\ k\Omega)^2 = (17.4\ nA/\sqrt{Hz})^2 = 302\ (nV)^2/Hz$

(1) $Sum = 702\ (nV)^2/Hz \times 50\ Hz = 35,000\ (nV)^2$

Then note in Figure 3, that $\bar{e}_N$ is in the neighborhood of $20\ nV/\sqrt{Hz}$ for R_{gen} of 14k, while $\bar{e}_N = 10\ nV/\sqrt{Hz}$ for $R_{gen} = 0–100\Omega$. STOP! Do not pass GO. Do not be fooled. Using $R_{gen} = R_{OPT}$ does not guarantee lowest noise UNLESS $e_{sig}^2 = kR_{gen}$ as in the case of transformer coupling. When $e_{sig}^2 > kR_{gen}$, as is the case where signal level is proportional to R_{gen} ($e_{sig} = kR_{gen}$), it makes sense to use the highest practical value of R_{gen}. When $e_{sig}^2 < kR_{gen}$, it makes sense to use a value of $R_{gen} < R_{OPT}$. These conclusions are verified in the Appendix.

This all means that it does not make sense to tamper with the R_{gen} of existing signal sources in an attempt to make $R_{gen} = R_{OPT}$. Especially, do not add series resistance to a source for this purpose. It does make sense to adjust R_{gen} in transformer coupled circuits by manipulating turns ratio or to design R_{gen} of a magnetic pick-up to operate with pre-amps where R_{OPT} is known. It does make sense to increase the design resistance of signal sources to match or exceed R_{OPT} so long as the signal voltage increases with R_{gen} in at least the ratio $e_{sig}^2 <5°C\ R_{gen}$. It does not necessarily make sense to select an amplifier with R_{OPT} to match R_{gen} because one amplifier operating at $R_{gen} = R_{OPT}$ may produce lower S/N than another (quieter) amplifier operating with $R_{gen} \neq R_{OPT}$.

With some amplifiers it is possible to adjust R_{OPT} over a limited range by adjusting the first stage operating current (the LM121 and LM381 for example). With these, one might increase operating current, varying R_{OPT}, to find a condition of minimum S/N. Increasing input stage current decreases R_{OPT} as $\bar{e}_n$ is decreased and $\bar{i}_n$ is simultaneously increased.

Let us consider one additional case of a fairly complex nature just as a practical example which will point up some factors often overlooked.

Example 4: Determine the S/N *apparent to the ear* of the amplifier of Figure 2 operating over 50-12,800 Hz when driven by a phonograph cartridge exhibiting $R_{gen} = 1350\Omega$, $L_{gen} = 0.5H$, and average $e_{sig} = 4.0$ mVrms. The cartridge is to be loaded by 47k as in Figure 7. This is equivalent to using a Shure V15, Type 3 for average level recorded music.

1. Choose sectional bandwidths of 1 octave each, listed in the following table.
2. Read $\bar{e}_n$ from Figure 2 as average for each octave and enter in the table.
3. Read $\bar{i}_n$ from Figure 2 as average for each octave and enter in the table.
4. Read $\bar{e}_R$ for the $R_{gen} = 1350\Omega$ from Figure 4 and enter in the table.
5. Determine the values of Z_{gen} at the midpoint of each octave and enter in the table.
6. Determine the amount of $\bar{e}_R$ which reaches the amplifier input; this is

$$\bar{e}_R \frac{R1}{R1 + Z_{gen}}$$

7. Read the noise contribution $\bar{e}_{47k}$ of R1 = 47k from Figure 4.
8. Determine the amount of $\bar{e}_{47K}$ which reaches the amplifier input; this is
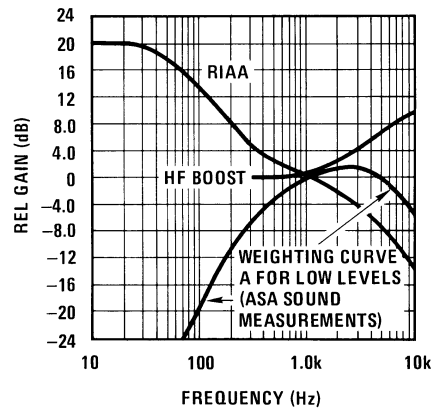
$$\bar{e}_{47k} \frac{Z_{gen}}{R1 + Z_{gen}}$$

Figure 6. Relative Gain for RIAA,
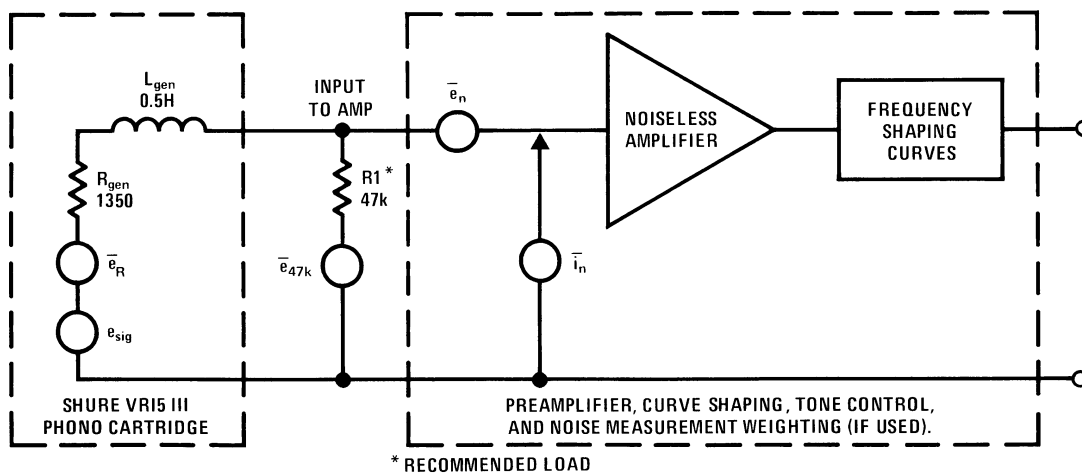**ASA Weighting A, and H-F Boost Curves**

Figure 7. Phono Preamp Noise Sources

1. Determine the effective noise contributed by $\bar{i}_n$ flowing through the parallel combination of R1 and Z_{gen}. This is

2. $$\bar{i}_n \frac{Z_{gen}\,R1}{Z_{gen} + R1}$$

1. Square all noise voltage values resulting from steps 2, 6, 8 and 9; and sum the squares.

2. Determine the relative gain at the midpoint of each octave from the RIAA playback response curve of Figure 6.

3. Determine the relative gain at these same midpoints from the A weighted response curve of Figure 6 for sound level meters (this roughly accounts for variations in human hearing).

4. Assume a tone control high frequency boost of 10 dB at 10 kHz from Figure 6. Again determine relative response of octave midpoints.

5. Multiply all relative gain values of steps 11-13 and square the result.

6. Multiply the sum of the squared values from step 10 by the resultant relative gain of step 14 and by the bandwidth in each octave.

7. Sum all the values resultant from step 15, and find the square root of the sum. This is the total audible rms noise apparent in the band.

8. Divide $e_{sig} = 4$ mV by the total noise to find S/N = 69.4 dB.

1.5   Steps for Example

1	Frequency Band (Hz)	50–100	100–200	200–400	400–800	800–1600	1.6–3.2k	3.2–6.4k	6.4–12.8k
	Bandwidth, B (Hz)	50	100	200	400	800	1600	3200	6400
	Bandcenter, f (Hz)	75	150	300	600	1200	2400	4800	9600
5	Z_{gen} at f (Ω)	1355	1425	1665	2400	4220	8100	16k	32k
	Z_{gen} R1 (Ω)	1300	1360	1600	2270	3900	6900	11.9k	19k
	$Z_{gen}(R1 + Z_{gen})$	0.028	0.030	0.034	0.485	0.082	0.145	0.255	0.400
	$R1/(R1 + Z_{gen})$	0.97	0.97	0.97	0.95	0.92	0.86	0.74	0.60
11	RIAA Gain, A_{RIAA}	5.6	3.1	2.0	1.4	1	0.7	0.45	0.316
12	Corr for Hearing, A_A	0.08	0.18	0.45	0.80	1	1.26	1	0.5
13	H-F Boost, A_{boost}	1	1	1	1	1.12	1.46	2.3	3.1
14	Product of Gains, A	0.45	0.55	0.9	1.12	1.12	1.28	1.03	0.49
	A^2	0.204	0.304	0.81	1.26	1.26	1.65	1.06	0.241
4	$\bar{e}_R$ (nV/√Hz)	4.74	4.74	4.74	4.74	4.74	4.74	4.74	4.74
7	$\bar{e}_{47k}$ (nV/√Hz)	29	29	29	29	29	29	29	29
3	$\bar{i}_n$ (pA/√Hz)	0.85	0.80	0.77	0.72	0.65	0.62	0.60	0.60
2	$\bar{e}_n$ (nV/√Hz)	19	14	11	10	9.5	9	9	9
9	$\bar{e}_1 = \bar{i}_n (Z_{gen} R1)$	1.1	1.09	1.23	1.63	2.55	4.3	7.1	11.4
6	$\bar{e}_2 = \bar{e}_R R1/(R1 + Z_{gen})$	4.35	4.35	4.35	4.25	4.15	3.86	3.33	2.7
8	$\bar{e}_3 = \bar{e}_{47k} Z_{gen}/(R1 + Z_{gen})$	0.81	0.87	0.98	1.4	2.4	4.2	7.4	11.6
10	$\bar{e}_n^2$	360	195	121	100	90	81	81	81
	$\bar{e}_1^2$ (from $\bar{i}_n$)	1.21	1.2	1.5	2.65	6.5	18.5	50	150
	$\bar{e}_2^2$ (from $\bar{e}_R$)	19	19	19	18	17	15	11	7.2
	$\bar{e}_3^2$ (from $\bar{e}_{47k}$)	0.65	0.76	0.96	2	5.8	18	55	135
	$\sum \bar{e}_n^2$ (nV²/Hz)	381	216	142	122	120	133	147	373
15	BA^2 (Hz)	10.2	30.4	162	504	1010	2640	3400	1550
	$BA^2\sum\bar{e}^2$ (nV²)	3880	6550	23000	61500	121000	350000	670000	580000
16	$\sum(\bar{e}_{ni}^2 + \bar{e}_{1i}^2 + \bar{e}_{2i}^2 + \bar{e}_{3i}^2) B_i A_i 2 = 1{,}815{,}930$ nV²								
	$\bar{e}_N = \sqrt{\Sigma} = 1.337\ \mu V$								
17	S/N = 20 log (4.0 mV/1.337 µV) = 69.4 dB								

Note the significant contributions of $\bar{i}_n$ and the 47k resistor, especially at high frequencies. Note also that there will be a difference between calculated noise and that noise measured on broadband meters because of the A curve employed in the example. If it were not for the A curve attenuation at low frequencies, the $\bar{e}_n$ would add a very important contribution below 200 Hz. This would be due to the RIAA boost at low frequency. As it stands, 97% of the 1.35 µV would occur in the 800–12.8 kHz band alone, principally because of the high frequency boost and the A measurement curve. If the measurement were made without either the high frequency boost or the A curve, the $\bar{e}_n$ would be 1.25 µV. In this case, 76% of the total noise would arise in the 50 Hz–400 Hz band alone. If the A curve were used, but the high-frequency boost were deleted, $\bar{e}_n$ would be 0.91 µV; and 94% would arise in the 800–12,800 Hz band alone.

The three different methods of measuring would only produce a difference of +3.5 dB in overall S/N, however the prime sources of the largest part of the noise and the frequency character of the noise can vary greatly with the test or measurement conditions. It is, then, quite important to know the method of measurement in order to know which individual noise sources in Figure 7 must be reduced in order to significantly improve S/N.

1.6 Conclusions

The main points in selecting low noise preamplifiers are:

- Don't pad the signal source; live with the existing R_{gen}.
- Select on the basis of low values of $\overline{e}_n$ and especially $\overline{i}_n$ if R_{gen} is over about a thousand Ω.
- Don't select on the basis of NF or R_{OPT} in most cases. NF specs are all right so long as you know precisely how to use them and so long as they are valid over the frequency band for the R_{gen} or Z_{gen} with which you must work.
- Be sure to (root) sum all the noise sources $\overline{e}_n$, $\overline{i}_n$ and $\overline{e}_R$ in your system over appropriate bandwidth.
- The higher frequencies are often the most important unless there is low frequency boost or high frequency attenuation in the system.
- Don't forget the filtering effect of the human ear in audio systems. Know the eventual frequency emphasis or filtering to be employed.

1.7 Appendix I

Derivation of R_{OPT}:

$$NF = 10 \log \frac{\overline{e_R^2} + \overline{e_n^2} + \overline{i_n^2}\, R_{gen}^2}{\overline{e_R^2}}$$

$$10 \log \left(1 + \frac{\overline{e_n^2} + \overline{i_n^2}\, R_{gen}^2}{\overline{e_R^2}}\right)$$

$$\frac{\delta NF}{\delta R} = \frac{0.435}{(4\,kTRB)^2} \frac{4\,kTRB\,(2R\,\overline{i_n^2}) - (\overline{e_n^2} + \overline{i_n^2}\,R^2)4\,kTB}{1 + (\overline{e_n^2} + \overline{i_n^2}\,R^2)/4\,kTRB} \tag{8}$$

where: $R = R_{gen}$

Set this = 0, and

$$4\,kTRB(2R\,\overline{i_n^2}) = 4\,kTB\,(\overline{e_n^2} + \overline{i_n^2}\,R^2)$$

$$2\,\overline{i_n^2}\,R^2 = \overline{e_n^2} + \overline{i_n^2}\,R^2$$

$$\overline{i_n^2}\,R^2 = \overline{e_n^2}$$

$$R^2 = \overline{e_n^2}/\overline{i_n^2}$$

$$R_{OPT} = \frac{\overline{e}_n}{\overline{i}_n} \tag{9}$$

1.8 Appendix II

Selecting R_{gen} for highest S/N.

$$S/N = \frac{e_{sig}^2}{B(\overline{e_R^2} + \overline{e_n^2} + \overline{i_n^2}\,R^2)} \tag{10}$$

For S/N to increase with R,

$$\frac{\delta S/N}{\delta R} > 0$$

$$\frac{\delta S/N}{\delta R} = \frac{2e_{sig}\,(\delta e_{sig}/\delta R)\,(\overline{e_R^2} + \overline{e_n^2} + \overline{i_n^2}\,R^2) - e_{sig}^2\,(4\,kT + 2\,\overline{i_n^2}R)}{B(\overline{e_R^2} + \overline{e_n^2} + \overline{i_n^2}\,R^2)^2} \tag{11}$$

If we set > 0, then

$$2 \, (\delta e_{sig}/\delta \, R) \, (\overline{e_R{}^2} + \overline{e_n{}^2} + \overline{i_n{}^2} \, R^2) > e_{sig} \, (4 \, kT + 2 \, \overline{i_n{}^2} \, R)$$

$$\text{For } e_{sig} = k_1 \, \sqrt{R}, \, \delta e_{sig}/\delta R = \frac{k_1}{2\sqrt{R}}$$

$$(2 \, k_1/2\sqrt{R}) \, (\overline{e_R{}^2} + \overline{e_n{}^2} + \overline{i_n{}^2} \, R2) > k_1\sqrt{R} \, (4 \, kT + 2 \, \overline{i_n{}^2} \, R)$$

$$\overline{e_R{}^2} + \overline{e_n{}^2} + \overline{i_n{}^2} \, R^2 > 4 \, kTR + 2 \, \overline{i_n{}^2} \, R^2$$

$$\overline{e_n{}^2} > \overline{i_n{}^2} \, R^2$$

$$R < \overline{e}_n/\overline{i}_n \tag{12}$$

Therefore S/N increases with R_{gen} so long as $R_{gen} \leq R_{OPT}$

For $e_{sig} = k_1 \, R$, $\delta e_{sig}/\delta R = k_1$

$$2 \, k_1 \, (\overline{e}_R{}^2 + \overline{e}_n{}^2 + \overline{i}_n{}^2 \, R^2) > k_1 R \, (4 \, kT + 2 \, \overline{i}_n{}^2 \, R)$$

$$2 \, \overline{e}_R{}^2 + 2 \, \overline{e}_n{}^2 + 2 \, \overline{i}_n{}^2 \, R^2 > 4 \, kTR + 2 \, \overline{i}_n{}^2 \, R^2$$

$$\overline{e}_R{}^2 + 2 \, \overline{e}_n{}^2 > 0$$

Then S/N increases with R_{gen} for any amplifier.

For any $e_{sig} < k_1 \sqrt{R}$, an optimum R_{gen} may be determined. Take, for example, $e_{sig} = k_1 \, R^{0.4}$, $\delta e_{sig}/\delta R = 0.4 k_1 R^{-0.6}$

$$(0.8 \, k_1/R^{0.6}) \, (\overline{e}_R{}^2 + \overline{e}_n{}^2 + \overline{i}_n{}^2 \, R^2) > k_1 \, R^{0.4} \, (4 \, kT + 2 \, \overline{i}_n{}^2 \, R)$$

$$0.8 \, \overline{e}_R{}^2 + 0.8 \, \overline{e}_n{}^2 + 0.8 \, \overline{i}_n{}^2 \, R^2 > 4 \, kTR + 2 \, \overline{i}_n{}^2 \, R^2$$

$$0.8 \, \overline{e}_n{}^2 > 0.2 \, \overline{e}_R{}^2 + 1.2 \, \overline{i}_n{}^2 \, R^2$$

Then S/N increases with R_{gen} until

$$0.25 \, \overline{e}_R{}^2 + 1.5 \, \overline{i}_n{}^2 \, R^2 = \overline{e}_n{}^2$$

Revision History

Changes from B Revision (April 2013) to C Revision **Page**

NOTE: Page numbers for previous revisions may differ from page numbers in the current version.

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