APPENDIX B

BODE PLOTS

The Bode plot is a method of displaying complex values of circuit gain (or impedance). The gain magnitude in dB is plotted vs. log frequency. The phase angle is plotted separately against the same log frequency scale.

Bode plots are an excellent vehicle for designing switching power supply closed loop systems. They provide good visibility into the gain/phase characteristics of the various loop elements. Calculation of the overall loop is made simply by adding gain in dB and phase in degrees.

The process is further simplified by using straight line approximations of the actual curves, called asymptotes. Calculations are then made only at the frequencies where the asymptotes change direction.

Bode's theorem for simple systems, which includes most switching power supplies: The phase angle of the gain at any frequency is dependent upon the rate of change of gain magnitude vs. frequency. A single pole (simple RC lowpass filter) has a gain slope of −20 dB/decade above its corner frequency and has a corresponding −90 degree phase shift.

FIRST ORDER FILTERS (RC or LR)

Single pole or single zero first order filters all have a gain slope of 20 dB/decade above the corner frequency. The phase shift asymptotes have a slope of 45°/decade, extending 1 decade each side of the corner frequency for a total 90° phase shift (see Figure B-1).

Maximum error between exact values (curved lines) and the straight line approximations are:

Gain — 3 dB
Phase — 5.7°

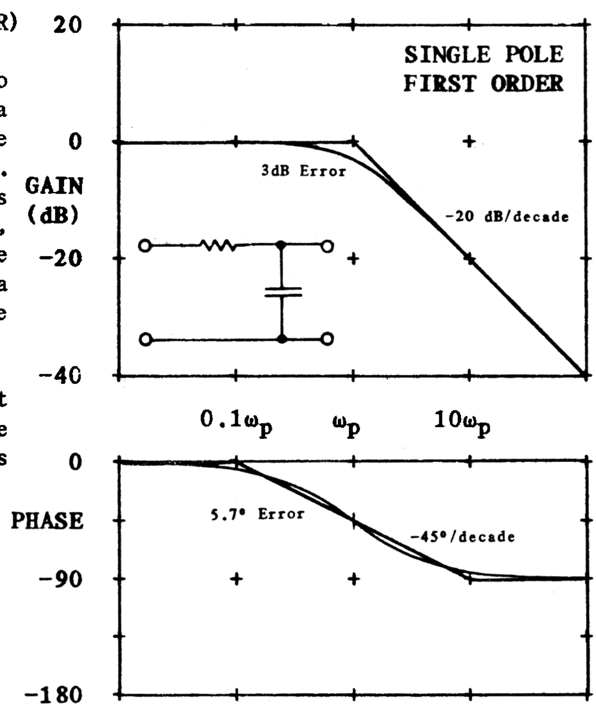

Figure B-1

Low Pass — Single Pole: Figure B-1

$$F(s) = \frac{1}{1+s/\omega_p}, \qquad \omega_p = 1/RC \text{ or } L/R, \qquad s = j\omega$$

Gain: −20 dB/decade slope. Phase: −90° total lag

Single Zero: Has the same gain and phase characteristic as the single pole shown in Figure II-1, except gain increases with frequency. Gain and phase slopes are both positive.

$$F(s) = 1+s/\omega_z, \qquad \omega_z = 1/RC \text{ or } L/R$$

Gain: +20 dB/decade slope. Phase: +90° total lead

Right-Half-Plane Zero. Refers to its location on the complex s-plane. The RHP zero has the same positive gain slope as the conventional (left-half-plane) zero, but the phase slope is negative, like a single pole. The RHP zero holds the loop gain up while adding additional phase lag, making it virtually impossible to provide compensation above its corner frequency. Fortunately, the right-half-plane zero is encountered only in boost and flyback regulators operated in continuous inductor current mode.

$$F(s) = 1-s/\omega_z$$

Gain: +20 dB/decade slope. Phase: −90° total lag

SECOND ORDER FILTERS (Resonant LC)

Low Pass — 2 Pole: Figure B-2

The second order 2 pole resonant LC filter characteristic of Figure B-2 has a −40 dB/decade slope in gain magnitude above the corner (resonant) frequency and a total phase lag of 180 degrees, the same as two single pole first order filters in cascade. Here the similarities end. The gain characteristic has a peak which varies with the Q of the resonant circuit, as shown in Figure B-3. This is of little importance unless the resonant frequency is close to the loop gain crossover frequency, when it could eliminate the gain margin and cause instability.

The phase characteristic slope is −45 degrees/decade with a Q of 0.5. At higher values of Q, Figure B-4 shows the phase slope becomes much more rapid, making compensation by means of first order zeros more difficult.

Gain:	−40 dB/decade slope
Gain peak at ω_o:	20 log Q
Phase:	−180° total lag
Phase asymptote intercepts:	ω/K, $K\omega$, $K = 5^{\frac{1}{2Q}}$

$$F(s) = \frac{1}{1+(s/\omega_o)/Q+(s/\omega_o)^2}$$

$$\text{where} \quad \omega_o = \frac{1}{\sqrt{LC}}, \qquad Q = \omega_o L/R_s, \qquad R_s = R_c+R_L+R_d+R_r+\frac{L}{CR_o}$$

Effective series resistance R_s determines Q. R_s includes: capacitor ESR: R_c, inductor: R_L, rectifier dynamic: R_d, leakage inductance effective resistance: R_i, and load resistance: R_o, transformed into its equivalent series R. ($L/C = Z_o^2$).

Q seldom reaches more than 4 or 5. At full load, low R_o transforms into high R_s. At light loads, diode R_d limits Q.

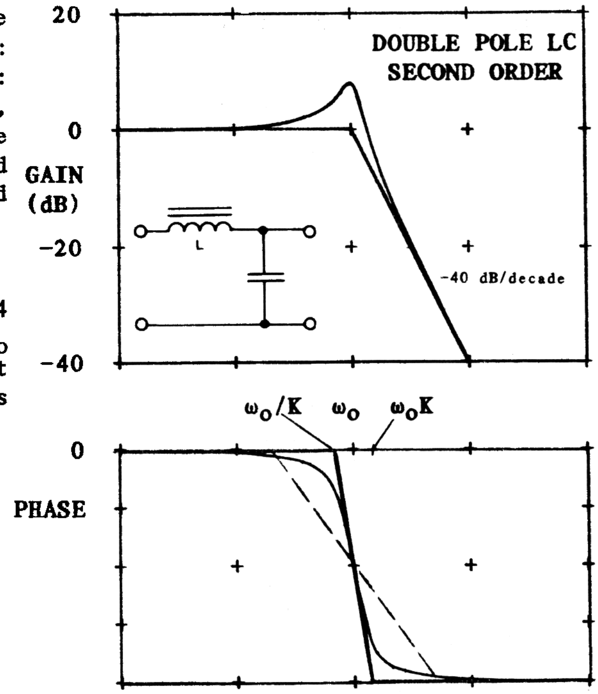

Figure B-2

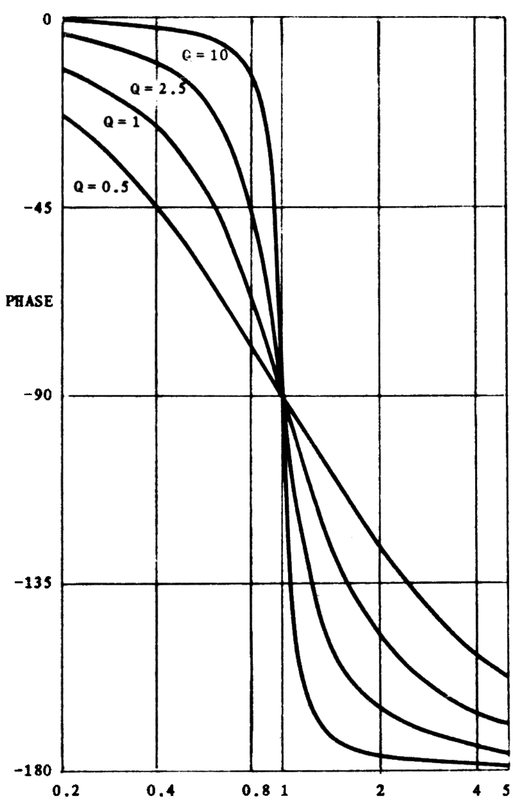

Figure B-4

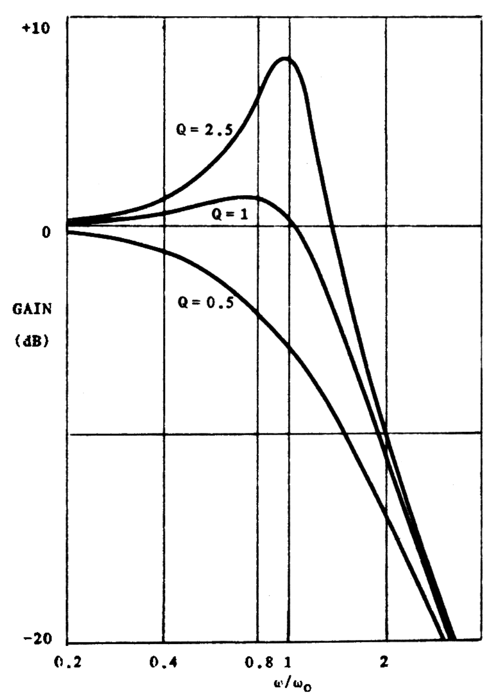

Figure B-3

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