ABSTRACT

This application note describes the methodology used to calculate confidence intervals for radiation-hardened devices that exhibit few or zero radiation-induced events.

Contents

1 Introduction ........................................................................................................................................ 1
2 References ......................................................................................................................................... 3

List of Tables

1 Experimental Example Calculation of Mean-Fluence-to-Failure (MFTF) and, \( \sigma \) Using a 95% Confidence Interval .................................................................................................................... 2

1 Introduction

Determining the SEE cross-section of robustly radiation-hardened devices becomes more difficult since often few, or possibly even no events are observed during an entire exposure. Determining the cross-section using an average event rate with standard deviation is no longer a viable option, and the common practice of assuming a single error occurred at the conclusion of a null-result can end up in a greatly underestimated cross-section.

In cases where observed events are rare or non-existent, the use of confidence intervals and the Chi-Squared distribution is indicated. The Chi-Squared distribution is particularly well-suited for determining a reliability level when the events occur at a constant rate. In the case of SEE testing where the ion events are random in time and position within the irradiation area, it is expected that the event rate is independent of time (presuming that parametric shifts induced by the total ionizing dose do not affect the failure rate), so the use of Chi-Squared statistical techniques is valid (since events are rare an exponential or Poisson distribution is usually used).

In a typical SEE experiment, the device-under-test (DUT) is exposed to a known, fixed fluence (ions/cm\(^2\)) while the DUT is monitored for events. This is analogous to fixed-time reliability testing and, more specifically, time-terminated testing, where the reliability test is terminated after a fixed amount of time whether or not a failure has occurred (in the case of SEE tests, fluence is substituted for time and hence it is a fixed fluence test) [1]. Calculating a confidence interval specifically provides a range of values which is likely to contain the parameter of interest (the actual number of events/fluence). Confidence intervals are constructed at a specific confidence level. For example, a 95% confidence level implies that if a given number of units was sampled numerous times and a confidence interval was estimated for each test, the resulting set of confidence intervals would bracket the true population parameter in about 95% of the cases.

To estimate the cross-section from a null-result (no events observed for a given fluence) with a confidence interval, start with the standard reliability determination of lower-bound (minimum) mean-time-to-failure for fixed-time testing (an exponential distribution is assumed):
Single-Event Effects Confidence Interval Calculations

\[ MTTF = \frac{2nT}{\chi^2_{2(d+1);100\left(1 - \frac{\alpha}{2}\right)}} \]

where
- \( MTTF \) is the minimum (lower-bound) mean-time-to-failure
- \( n \) is the number of units tested (presuming each unit is tested under identical conditions)
- \( T \) is the test time
- \( \chi^2 \) is the chi-square distribution evaluated at \( 100(1 - \frac{\alpha}{2}) \) confidence level
- \( d \) is the degrees-of-freedom (the number of events observed) \((1)\)

With slight modification for this purpose, invert the inequality and substitute \( F \) (fluence) in the place of \( T \):

\[ MFTF = \frac{2nF}{\chi^2_{2(d+1);100\left(1 - \frac{\alpha}{2}\right)}} \]

where
- \( MFTF \) is mean-fluence-to-failure
- \( F \) is the test fluence
- \( \chi^2 \) is the chi-square distribution evaluated at \( 100(1 - \frac{\alpha}{2}) \) confidence
- \( d \) is the degrees-of-freedom (the number of failures observed) \((2)\)

The inverse relation between \( MTTF \) and event rate is mirrored with the \( MFTF \). Thus the upper-bound cross section is obtained by inverting the \( MFTF \):

\[ \sigma = \frac{\chi^2_{2(d+1);100\left(1 - \frac{\alpha}{2}\right)}}{2nF} \]

\((3)\)

Assume that all tests are terminated at a total fluence of \( 10^6 \) ions/cm\(^2\). Also assume that you have a number of devices with very different performances that are tested under identical conditions. Assume a 95% confidence level (\( \alpha = 0.05 \)). Note that as \( d \) increases from zero events to 100 events, the actual confidence interval becomes smaller, indicating that the range of values of the true value of the population parameter (in this case, the cross section) is approaching the mean value + 1 standard deviation. This makes sense when you consider that as more events are observed, the statistics are improved such that uncertainty in the actual device performance is reduced.

### Table 1. Experimental Example Calculation of Mean-Fluence-to-Failure (MFTF) and, \( \sigma \) Using a 95% Confidence Interval\(^{(1)}\)

<table>
<thead>
<tr>
<th>DEGREES-OF-FREEDOM (d)</th>
<th>2(d + 1)</th>
<th>( \chi^2 ) @ 95%</th>
<th>CALCULATED CROSS SECTION (cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>UPPER-BOUND @ 95% CONFIDENCE</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>7.38</td>
<td>3.69E–06</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>11.14</td>
<td>5.57E–06</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>14.45</td>
<td>7.22E–06</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>17.53</td>
<td>8.77E–06</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>20.48</td>
<td>1.02E–05</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>23.34</td>
<td>1.17E–05</td>
</tr>
<tr>
<td>10</td>
<td>22</td>
<td>36.78</td>
<td>1.84E–05</td>
</tr>
<tr>
<td>50</td>
<td>102</td>
<td>131.84</td>
<td>6.59E–05</td>
</tr>
<tr>
<td>100</td>
<td>202</td>
<td>243.25</td>
<td>1.22E–04</td>
</tr>
</tbody>
</table>

\(^{(1)}\) Using a 95% confidence for several different observed results (\( d = 0, 1, 2...100 \) observed events during fixed-fluence tests) assuming \( 10^6 \) ion/cm\(^2\) for each test.
2 References

IMPORTANT NOTICE AND DISCLAIMER

TI PROVIDES TECHNICAL AND RELIABILITY DATA (INCLUDING DATASHEETS), DESIGN RESOURCES (INCLUDING REFERENCE DESIGNS), APPLICATION OR OTHER DESIGN ADVICE, WEB TOOLS, SAFETY INFORMATION, AND OTHER RESOURCES “AS IS” AND WITH ALL FAULTS, AND DISCLAIMS ALL WARRANTIES, EXPRESS AND IMPLIED, INCLUDING WITHOUT LIMITATION ANY IMPLIED WARRANTIES OF MERCHANTABILITY, FITNESS FOR A PARTICULAR PURPOSE OR NON-INFRINGEMENT OF THIRD PARTY INTELLECTUAL PROPERTY RIGHTS.

These resources are intended for skilled developers designing with TI products. You are solely responsible for (1) selecting the appropriate TI products for your application, (2) designing, validating and testing your application, and (3) ensuring your application meets applicable standards, and any other safety, security, or other requirements. These resources are subject to change without notice. TI grants you permission to use these resources only for development of an application that uses the TI products described in the resource. Other reproduction and display of these resources is prohibited. No license is granted to any other TI intellectual property right or to any third party intellectual property right. TI disclaims responsibility for, and you will fully indemnify TI and its representatives against, any claims, damages, costs, losses, and liabilities arising out of your use of these resources.

TI’s products are provided subject to TI’s Terms of Sale (www.ti.com/legal/termsofsale.html) or other applicable terms available either on ti.com or provided in conjunction with such TI products. TI’s provision of these resources does not expand or otherwise alter TI’s applicable warranties or warranty disclaimers for TI products.

Mailing Address: Texas Instruments, Post Office Box 655303, Dallas, Texas 75265
Copyright © 2020, Texas Instruments Incorporated