Calculating noise figure and third-order intercept in ADCs

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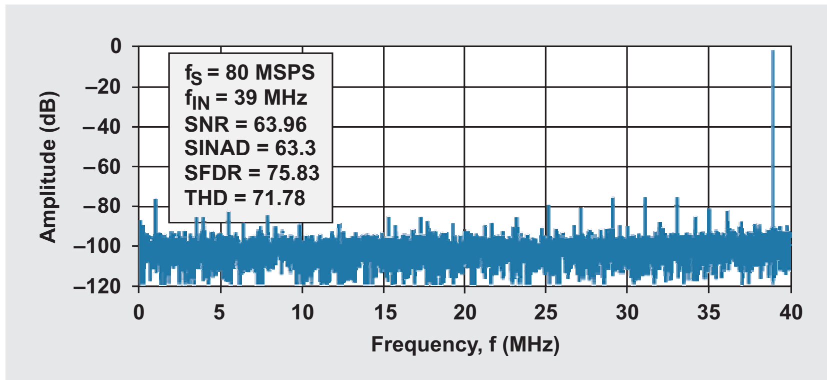
Introduction

Noise figure (NF) and the third-order intercept point (IP_3) are used in radio receiver link budget analysis as a means to quantify the effects of device noise and nonlinearity on the sensitivity of the radio. Analog-to-digital converters (ADCs) are used in radio receivers to convert the signal from the analog domain to the digital domain. NF and IP_3 typically are not specified for the device, but equivalent parameters are given whereby they can be calculated.

ADCs specify signal-to-noise ratio (SNR) and two-tone, third-order intermodulation distortion (IMD_3) under certain input signal and clocking conditions. With this information, NF and IP_3 can be calculated.

In general, a low NF and high IP_3 are desired. The actual values required to meet the design goals depend on the architecture of the system.

Figure 1. ADS5410 FFT or spectral plot from Reference 1



Review of noise figure

Noise figure (NF) is the decibel equivalent of noise factor (F): NF (dB) = 10log(F).

Noise factor of a device is the power ratio of the SNR at the input (SNR_I) divided by the SNR at the output (SNR_O):

$$F = \frac{SNR_I}{SNR_O}. \tag{1}$$

The output signal (S_O) is equal to the input signal (S_I) times the gain: $S_O = S_I \times G$. The output noise is equal to the noise delivered to the input (N_I) from the source plus the input noise of the device (N_A) times the gain: $N_O = (N_I + N_A) \times G$. Substituting into Equation 1 and simplifying, we get

$$F = \frac{SNR_I}{SNR_O} = \left[\frac{\dfrac{S_I}{N_I}}{\dfrac{G \times S_I}{G \times (N_I + N_A)}}\right] = 1 + \frac{N_A}{N_I}. \tag{2}$$

Assuming that the input is terminated in the same impedance as the source, N_I = kT = –174 dBm/Hz, where k is Boltzman's constant and T = 300 Kelvin). Once we find the input noise spectral density of the device, it is a simple matter to plug it into Equation 2 and calculate F.

NF in ADCs

There are a couple of ways to go about calculating the input noise spectral density of an ADC, but using the SNR specification is easy.

To measure SNR, a low-noise signal is input to the ADC, and the output is examined by taking a fast Fourier transform (FFT) or spectral plot. Figure 1 shows such a plot from Reference 1. The ratio of the signal to the noise integrated over half the sampling frequency ($f_S/2$) is the SNR. Since the noise of the ADC is—to first-order approximation—independent of signal level, the higher the input level the better the SNR, up to a point. As the signal approaches full scale (FS), spurious behavior begins to degrade the SNR. An input signal level 1 dB below full-scale input (–1 dBFS) seems to give good results and is commonly used.

To find the input noise spectral density, we divide the signal level by the SNR divided by half the sampling frequency (since SNR is calculated by dividing the signal by the noise integrated over $f_S/2$):

N_A (dBm/Hz) = –1 dBFS (dBm) + SNR (dBc) – $f_S/2$ (dBHz).

An ADC is a voltage-driven device, so we must choose an input resistance to find the signal power with the formula $P = V^2/R$. Assuming that FS = $2V_{p-p}$ and R = 50 Ω, the full-scale input is +10 dBm.

As an example of how to calculate, consider the following for the ADS5410, a 12-bit ADC. Given that f_S = 80 MSPS, R_{IN} = 50 Ω, FS = $2V_{p-p}$, and SNR = 63.96, then

N_A (dBm/Hz) = +9 dBm – 63.96 dBc – 76.02 dBHz
= –130.98 dBm/Hz.

To use Equation 2, we need to use the linear equivalents of N_I and N_A:

$$F = 1 + 10E\left(\frac{-130.98 + 174}{10}\right) = 20045,$$

or NF (dB) = 43.02 dB.

Looking at the result, we see that adding the 1 in Equation 2 makes very little difference since the noise figure is so high. Therefore, using

$$NF\ (dB) = N_A\ (dBm/Hz) - N_I\ (dBm/Hz)$$

introduces little error.

It is common practice to use a transformer or a fully differential op amp to drive high-performance ADCs differentially. This gives us the opportunity to use higher ADC input resistance. If NF is calculated based on 50 Ω, it is reduced by $\log_{10}$(impedance ratio).

For example, if we use a 1:4 impedance ratio (1:2 turns ratio) transformer, the input resistance is 200 Ω to match to a 50-Ω drive amplifier. The NF is reduced by $10 \times \log_{10}(200/50)$ = 6 dB. Or, if we use a 1:16 impedance ratio transformer with 800-Ω input resistance, NF is 12 dB lower.

Review of third-order intercept point (IP_3)

Due to nonlinearity in the transfer function of all electronics, distortion is generated. With reference to the formula of a straight line, y = b + mx, nonlinearity is any deviation that the output (y) may have from a constant multiple (m) of the input (x) plus any constant offset (b).

Expanding the nonlinear transfer functions of basic transistor circuits into a power series is a typical way to quantify distortion products (see Reference 2). For example, transistors typically have an exponential transfer function (i.e., collector current vs. base emitter voltage), $y = e^x$, where x is the input and y is the output. Expanding e^x into a power series around x = 0 results in

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots + \frac{x^n}{n!}.$$

Figure 2 shows the function $y = e^x$ along with estimates that use progressively more terms of the power series.

The farther x is from 0, the more terms are required to estimate the value of e^x properly. If x < 0.25, the linear term 1 + x provides a close estimate of the actual function, and the circuit is linear. As x becomes larger, progressively more terms (quadratic, cubic, and higher-order distortion terms) are required to estimate e^x properly.

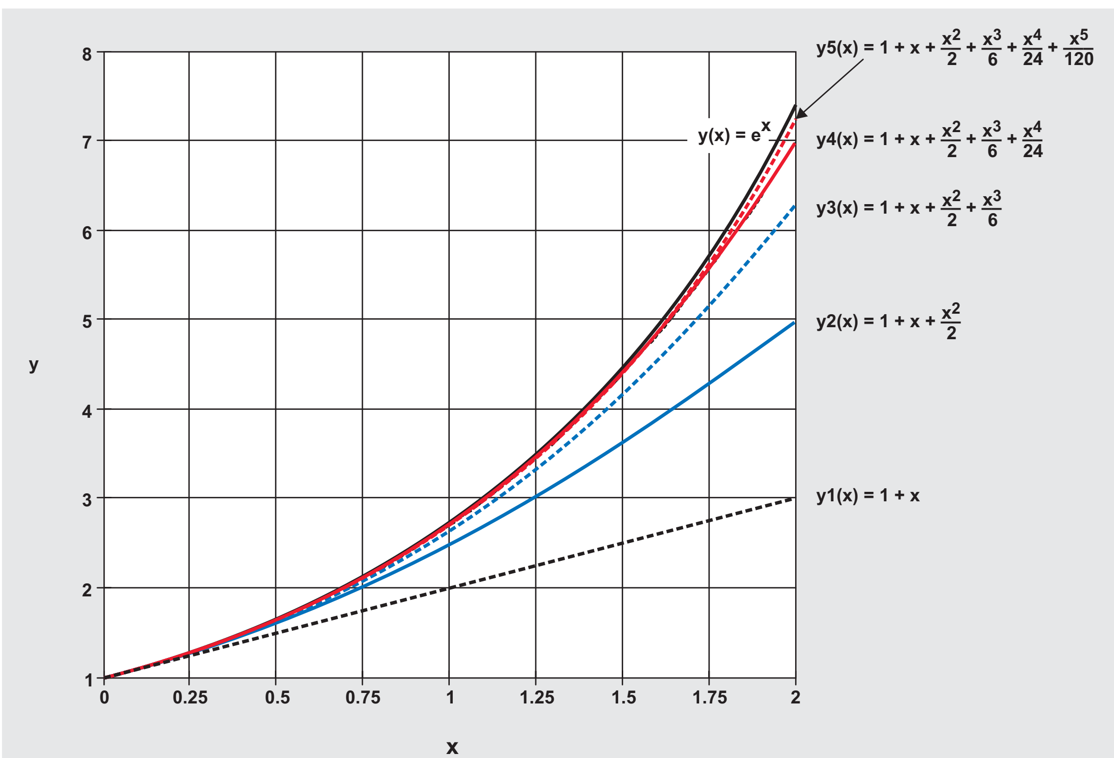
If the input to this circuit is a sinusoid—i.e., $x = A\sin(\omega t)$—then the output

$$y = K_0 + K_1 A\sin(\omega t) + K_2 A^2 \sin^2(\omega t) + K_3 A^3 \sin^3(\omega t) + \cdots,$$

where K_0, K_1, etc. are constant scaling factors. Using the trigonometric identities

$$\sin^2(\omega t) = \frac{1-\cos(2\omega t)}{2} \quad \text{and}$$

Figure 2. Function $y = e^x$ and its power series estimates



$$\sin^3(\omega t) = \frac{3\sin(\omega t) - \sin(3\omega t)}{4}$$

shows that the quadratic and cubic terms give rise to second- and third-order harmonic distortion (HD_2 and HD_3, respectively). Similarly, higher-order terms give rise to higher-order harmonic distortion.

If the input is comprised of two tones—i.e., $x = A_1\sin(\omega_1 t) + A_2\sin(\omega_2 t)$—then the output

$$\begin{aligned} y = K_0 &\\ &+ K_1[A_1\sin(\omega_1 t) + A_2\sin(\omega_2 t)] \\ &+ K_2[A_1\sin(\omega_1 t) + A_2\sin(\omega_2 t)]^2 \\ &+ K_3[A_1\sin(\omega_1 t) + A_2\sin(\omega_2 t)]^3 \\ &+ \cdots, \end{aligned}$$

where K_0, K_1, etc. are constant scaling factors.

Expanding the third term, we get

$$\begin{aligned} &K_2[A_1\sin(\omega_1 t) + A_2\sin(\omega_2 t)]^2 \\ &= K_2[A_1^2\sin^2(\omega_1 t) + 2A_1A_2\sin(\omega_1 t)\sin(\omega_2 t) + A_2^2\sin^2(\omega_2 t)]. \end{aligned}$$

Using the trigonometric identities

$$\sin^2(\omega t) = \frac{1 - \cos(2\omega t)}{2} \text{ and}$$

$$\sin(\omega_1 t)\sin(\omega_2 t) = \frac{\cos(\omega_1 t - \omega_2 t) - \cos(\omega_1 t + \omega_2 t)}{2}$$

shows that the quadratic terms give rise to HD_2 and second-order intermodulation distortion (IMD_2).

Expanding the fourth term, we get

$$\begin{aligned} &K_3[A_1\sin(\omega_1 t) + A_2\sin(\omega_2 t)]^3 \\ &= K_3[A_1^3\sin^3(\omega_1 t) + 3A_1A_2\sin^2(\omega_1 t)\sin(\omega_2 t) \\ &\quad + 3A_1A_2\sin(\omega_1 t)\sin^2(\omega_2 t) + A_2^3\sin^3(\omega_2 t)]. \end{aligned}$$

Using the trigonometric identities

$$\sin^3(\omega t) = \frac{3\sin(\omega t) - \sin(3\omega t)}{4} \text{ and}$$

$$\sin^2(\omega_1 t)\sin(\omega_2 t) = \frac{2\sin(\omega_2 t) - \sin(2\omega_1 t + \omega_2 t) - \sin(2\omega_2 t + \omega_1 t)}{4}$$

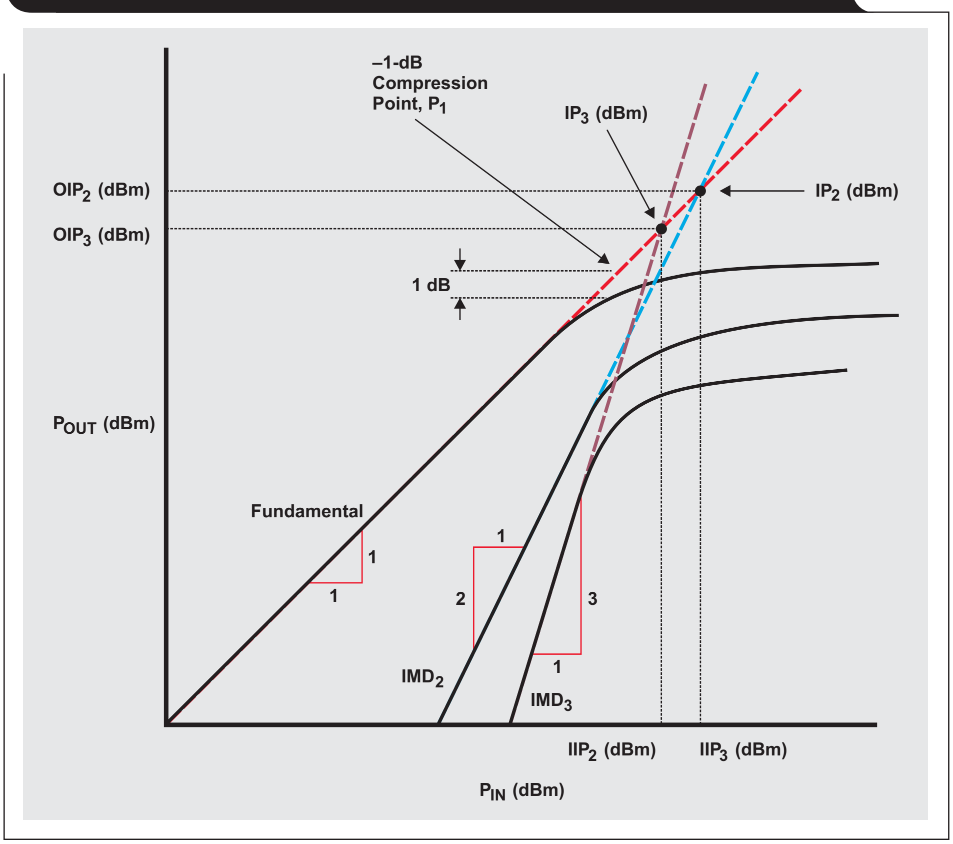
shows that the cubic terms give rise to HD_3 and third-order intermodulation distortion (IMD_3). Similarly, higher-order terms give rise to higher-order harmonic and intermodulation distortion.

Table 1 shows the frequencies of the distortion products that will be generated due to second- and third-order nonlinearity, given a two-tone input at frequencies f_1 and f_2.

Table 1. Distortion product frequencies due to second- and third-order nonlinearity

SECOND-ORDER FREQUENCIES		THIRD-ORDER FREQUENCIES	
$2f_1$	$2f_2$	$3f_1$	$3f_2$
$f_1 - f_2$	$f_1 + f_2$	$2f_1 - f_2$	$2f_2 - f_1$
		$2f_1 + f_2$	$2f_2 + f_1$

Figure 3. Input and output two-tone and intermodulation distortion



So now the question arises: Why is all this important? The answer is that radio specifications for GSM, CDMA2000, WCDMA, and the like all call for sensitivity requirements to be met with two interfering signals spaced in the frequency domain such that their third-order intermodulation product will fall on top of the signal of interest. The third-order intermodulation point is used to quantify how much distortion is generated. Referring this to the antenna input provides an easy method to determine whether or not the spec can be met.

If the input and output power of two tones applied to a device and their intermodulation products are graphed on a log-log scale as shown in Figure 3, the fundamental tones have a slope of 1, the second-order product has a slope of 2, and the third-order products have a slope of 3. The device will go into compression before the lines intersect. The point where the output power is reduced by 1 dB from what is expected is called the 1-dB compression point (P_1). By extending the lines, the second- and third-order intercept points (IP_2 and IP_3, respectively) can be found. If they are referred to the input, they are called input intercept points (IIP_2, IIP_3); and if they are referred to the output, they are called output intercept points (OIP_2, OIP_3).

Since we are interested in intermodulation distortion relative to the carriers, why should we concern ourselves with some fictitious point that the amplifier will never reach? The answer is that there is a mathematical relationship between the two. Given the intercept point, we can calculate the intermodulation product for any input/output power.

Given that the slopes are known, equations for slopes L_1 and L_3 are written as shown in Figure 4.

Subtracting two arbitrary points on each line and rearranging gives us

$$y_2 - y_3 = x_2 - x_3 \quad \Rightarrow y_2 = y_3 + x_2 - x_3 \text{ for L1, and}$$
$$y_1 - y_3 = 3x_2 - 3x_3 \quad \Rightarrow y_1 = y_3 + 3x_2 - 3x_3 \text{ for L3.}$$

Subtracting again results in

$$\begin{array}{l} y_2 = y_3 + x_2 - x_3 \\ \underline{-(y_1 = y_3 + 3x_2 - 3x_3)} \\ \underbrace{y_2 - y_1}_{\text{IMD}_3\text{ (dBc)}} = 2(\underbrace{x_3}_{\text{IIP}_3\text{ (dBm)}} - \underbrace{x_2}_{\text{P}_{\text{IN}}}). \end{array}$$

From this it is seen that

$$\text{IIP}_3\text{ (dBm)} = \text{P}_{\text{IN}}\text{ (dBm)} - \frac{\text{IMD}_3\text{ (dBc)}}{2}. \tag{3}$$

Once we find IMD_3 and know the input power, it is a simple matter to plug them into Equation 3 and calculate IIP_3.

Figure 4. Straight-line relationship between IMD_3 and the fundamental

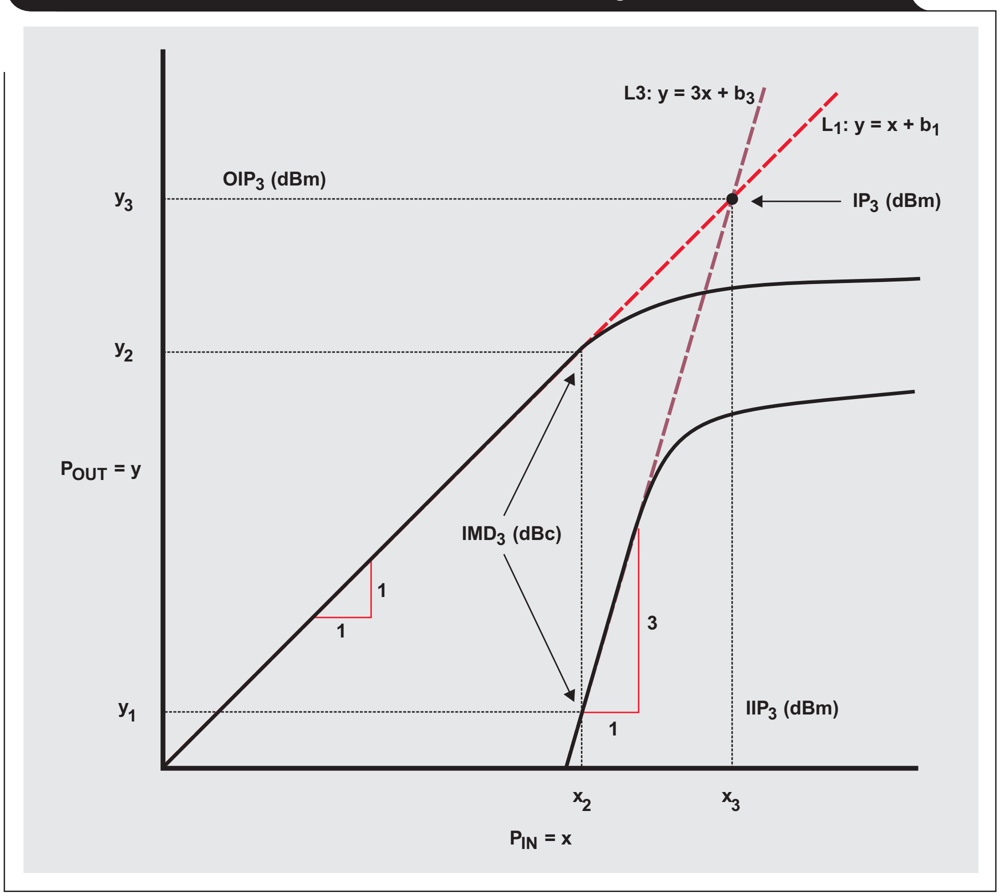
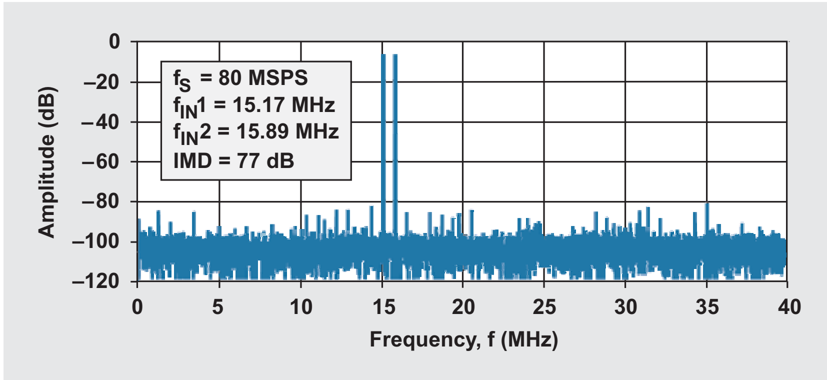


Figure 5. Two-tone FFT or spectral plot from Reference 1



IP_3 in ADCs

In ADC testing, two tones are applied to the ADC, and the output is examined by taking an FFT or spectral plot to find the two-tone IMD_3. Figure 5 shows such a plot from Reference 1. The ratio of each of the tones to the IMD_3 product(s) is the two-tone IMD_3 in dBc. IMD_3 depends on signal level. Too high a signal level results in excessive distortion, and too low a signal level makes distortion hard to detect in the presence of noise and other spurious components. An input signal level 7 dB below full-scale input (–7 dBFS) seems to give good results and is commonly used.

Since an ADC is a voltage-driven device, we must choose an input resistance to find the signal power with the formula $P = V^2/R$. Assuming that FS = $2V_{p-p}$ and R = 50 Ω, the full-scale input is +10 dBm. With the input power and the IMD_3, Equation 3 is used to find IIP_3.

As an example of how to calculate, consider the following for the ADS5410, a 12-bit ADC. Given that FS = $2V_{p-p}$, R_{IN} = 50 Ω, and IMD_3 = 77 dBc, then

$$IIP_3 = 3\text{ dBm} - \frac{-77\text{ dBc}}{2} = 41.5\text{ dBm}.$$

As mentioned earlier, it is common practice to use a transformer or a fully differential op amp to drive high-performance ADCs differentially. If 50 Ω is originally used as shown in the example, then the IIP_3 is reduced by $10 \times \log_{10}$(impedance ratio).

For example, if we use a 1:4 impedance ratio (1:2 turns ratio) transformer, the input resistance is 200 Ω to match to a 50-Ω drive amplifier. The IIP_3 is reduced by $10 \times \log_{10}(200/50) = 6$ dB. Or, if we use a 1:16 impedance ratio transformer with 800-Ω input resistance, IIP_3 is 12 dB lower.

Conclusion

We have examined typical ADC noise and distortion specifications to see how they relate to NF and IP_3. It is seen that the required information to calculate NF and IP_3 is contained in a typical ADC data sheet.

A key point to remember is that an ADC is a voltage-driven device, whereas NF and IP_3 are associated with power. Thus, in order for the calculations to proceed, an impedance is imposed on the ADC input to find the corresponding power levels.

References

For more information related to this article, you can download an Acrobat Reader file at www-s.ti.com/sc/techlit/*litnumber* and replace *"litnumber"* with the **TI Lit. #** for the materials listed below.

Document Title	**TI Lit. #**
1. "12-Bit, 80 MSPS CommsADC™ Analog-to-Digital Converter," Data Sheet, p. 7	slas346
2. Piet Wambacq and Willy Sansen, *Distortion Analysis of Analog Integrated Circuits* (Kluwer Academic Publishers, 1998).	

Related Web sites

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