Analysis of fully differential amplifiers

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Introduction

The August issue of *Analog Applications Journal* introduced the fully differential amplifiers from Texas Instruments and illustrated their basic operation (see Reference 1). This article explores the topic more deeply by analyzing gain and noise. The fully differential amplifier has multiple feedback paths, and circuit analysis requires close attention to detail. Care must be taken to include the V_{OCM} pin for a complete analysis.

Circuit analysis

Circuit analysis of fully differential amplifiers follows the same rules as normal single-ended amplifiers, but subtleties are present that may not be fully appreciated until a full analysis is done. The analysis circuit shown in Figure 1 is used to calculate a generalized circuit formula and block diagram from which specific circuit configurations can be easily solved. The voltage definitions are required to arrive at practical solutions.

A_F is used to represent the open-loop differential gain of the amplifier such that $(V_{OUT}+) – (V_{OUT}–) = A_F(V_P – V_N)$. This assumes that the gains of the two sides of the differential amplifier are well matched and that variations are insignificant. With negative feedback, this is typically the case when $A_F >> 1$.

Input voltage definitions:

$$V_{ID} = (V_{IN}+) - (V_{IN}-) \tag{1}$$

$$V_{IC} = \frac{(V_{IN}+) + (V_{IN}-)}{2} \tag{2}$$

Output voltage definitions:

$$V_{OD} = (V_{OUT}+) - (V_{OUT}-) \tag{3}$$

$$V_{OC} = \frac{(V_{OUT}+) + (V_{OUT}-)}{2} \tag{4}$$

$$(V_{OUT}+) - (V_{OUT}-) = A_F (V_P - V_N) \tag{5}$$

$$V_{OC} = V_{OCM} \tag{6}$$

Figure 1. Analysis circuit

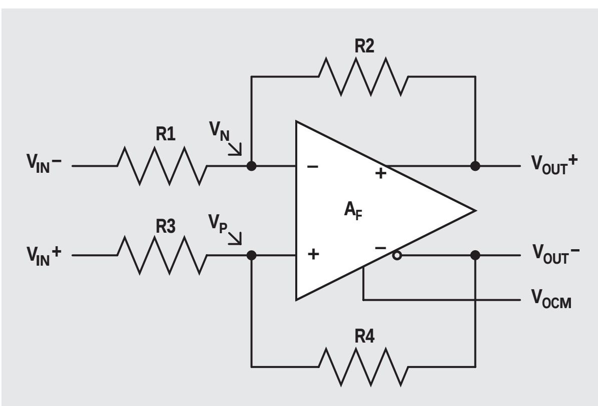
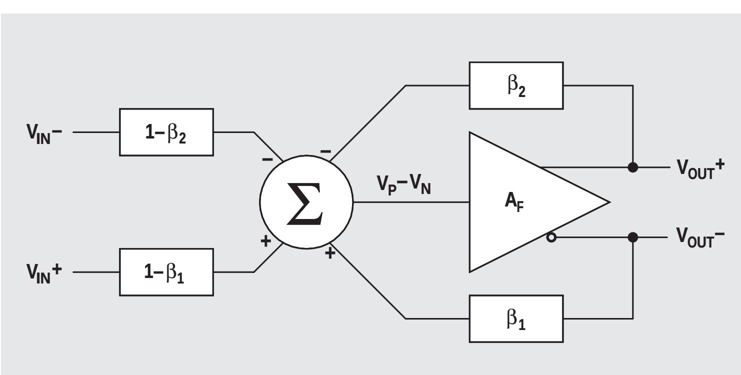


Figure 2. Block diagram



There are two amplifiers: the main differential amplifier (from V_{IN} to V_{OUT}) and the V_{OCM} error amplifier. The operation of the V_{OCM} error amplifier is the simpler of the two and will be considered first. It may help to review the simplified schematic shown in Reference 1.

$V_{OUT}+$ and $V_{OUT}–$ are filtered and summed by an internal RC network. The V_{OCM} amplifier samples this voltage and compares it to the voltage applied to the V_{OCM} pin. An internal feedback loop is used to drive "error" voltage of the V_{OCM} error amplifier (the voltage between the input pins) to zero, so that $V_{OC} = V_{OCM}$. This is the basis of the voltage definition given in Equation 6.

There is no simple way to analyze the main differential amplifier except to sit down and write some node equations, then do the algebra to massage them into practical form. We will first derive a solution based solely on nodal analysis. Then we will make use of the voltage definitions given in Equations 1–6 to derive solutions for the output voltages, looking at them single-ended; i.e., $V_{OUT}+$ and $V_{OUT}–$. These are then used to calculate V_{OD}.

Solving the node equations at V_N and V_P yields

$$V_N = (V_{IN}-)\left(\frac{R2}{R1+R2}\right) + (V_{OUT}+)\left(\frac{R1}{R1+R2}\right) \text{ and } V_P = (V_{IN}+)\left(\frac{R4}{R3+R4}\right) + (V_{OUT}-)\left(\frac{R3}{R3+R4}\right).$$

By setting $\beta_1 = \left(\frac{R3}{R3+R4}\right)$ and $\beta_2 = \left(\frac{R1}{R1+R2}\right)$, V_N and V_P can be rewritten as

$$V_N = (V_{IN}-)(1-\beta_2) + (V_{OUT}+)(\beta_2), \text{ and} \quad (7)$$

$$V_P = (V_{IN}+)(1-\beta_1) + (V_{OUT}-)(\beta_1). \quad (8)$$

With Equations 7 and 8, a block diagram of the main differential amplifier can be constructed, like that shown in Figure 2. Block diagrams are useful tools for understanding circuit operation and investigating other variations.

By using the block diagram, or combining Equations 7 and 8 with Equation 5, we can find the input-to-output relationship:

$$(V_{OUT}+)(1+A_F\beta_2) - (V_{OUT}-)(1+A_F\beta_1) = A_F[(V_{IN}+)(1-\beta_1) - (V_{IN}-)(1-\beta_2)]. \quad (9)$$

Although accurate, Equation 9 is somewhat cumbersome when the feedback paths are not symmetrical. By using the voltage definitions given in Equations 1–4 and Equation 6, we can derive more practical formulas.

Substituting $(V_{OUT}-) = 2V_{OC} - (V_{OUT}+)$, and $V_{OC} = V_{OCM}$, we can write

$$(V_{OUT}+)(2+A_F\beta_1+A_F\beta_2) - 2V_{OCM}(1+A_F\beta_1) = A_F[(V_{IN}+)(1-\beta_1) - (V_{IN}-)(1-\beta_2)], \text{ or}$$

$$(V_{OUT}+) = \frac{1}{(\beta_1+\beta_2)} \frac{(V_{IN}+)(1-\beta_1) - (V_{IN}-)(1-\beta_2) + 2V_{OCM}\left(\frac{1}{A_F}+\beta_1\right)}{\left(1+\frac{2}{A_F\beta_1+A_F\beta_2}\right)}. \quad (10)$$

With the "ideal" assumption $A_F\beta_1 >> 1$ and $A_F\beta_2 >> 1$, this reduces to

$$(V_{OUT}+) = \frac{(V_{IN}+)(1-\beta_1) - (V_{IN}-)(1-\beta_2) + 2V_{OCM}\beta_1}{(\beta_1+\beta_2)}. \quad (11)$$

$V_{OUT}-$ is derived in a similar manner:

$$(V_{OUT}-) = \frac{1}{(\beta_1+\beta_2)} \frac{-(V_{IN}+)(1-\beta_1) + (V_{IN}-)(1-\beta_2) + 2V_{OCM}\left(\frac{1}{A_F}+\beta_2\right)}{\left(1+\frac{2}{A_F\beta_1+A_F\beta_2}\right)}. \quad (12)$$

Again, assuming $A_F\beta_1 >> 1$ and $A_F\beta_2 >> 1$, this reduces to

$$(V_{OUT}-) = \frac{-(V_{IN}+)(1-\beta_1) + (V_{IN}-)(1-\beta_2) + 2V_{OCM}(+\beta_2)}{(\beta_1+\beta_2)}. \quad (13)$$

To calculate $V_{OD} = (V_{OUT}+) - (V_{OUT}-)$, subtract Equation 12 from Equation 10:

$$V_{OD} = \frac{1}{(\beta_1+\beta_2)} \frac{2[(V_{IN}+)(1-\beta_1) - (V_{IN}-)(1-\beta_2)] + 2V_{OCM}(\beta_1-\beta_2)}{\left(1+\frac{2}{A_F\beta_1+A_F\beta_2}\right)} \quad (14)$$

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Again, assuming $A_F\beta_1 >> 1$ and $A_F\beta_2 >> 1$, this reduces to

$$V_{OD} = \frac{2[(V_{IN}+)(1-\beta_1)-(V_{IN}-)(1-\beta_2)]+2V_{OCM}(\beta_1-\beta_2)}{(\beta_1+\beta_2)}. \quad \textbf{(15)}$$

It can be seen from Equations 11, 13, and 15 that even though the obvious use of a fully differential amplifier is with symmetrical feedback, the gain can be controlled with only one feedback path.

Using matched resistors R1 = R3 and R2 = R4 in the analysis circuit of Figure 1 balances the feedback paths so that $\beta_1 = \beta_2 = \beta$, and the transfer function is

$$\frac{(V_{OUT}+)-(V_{OUT}-)}{(V_{IN}+)-(V_{IN}-)} = \frac{A_F}{(1+A_F\beta)} = \frac{1-\beta}{\beta} \times \frac{1}{\left(1+\frac{1}{A_F\beta}\right)}.$$

The common-mode voltages at the input and output do not enter into the equation, V_{IC} is rejected, and V_{OC} is set by the voltage at V_{OCM}. The ideal gain (assuming $A_F\beta >> 1$) is set by the ratio

$$\frac{1-\beta}{\beta} = \frac{R2}{R1}.$$

Note that the normal inversion we might expect, given two balanced inverting amplifiers, is accounted for by the output voltage definitions, resulting in a positive gain.

Many applications require that a single-ended signal be converted to a differential signal. The circuits in Figures 3–7 show various approaches. Using Equations 11, 13, and 15, we can easily derive circuit solutions.

With a slight variation of Figure 1 as shown in Figure 3, single-ended signals can be amplified and converted to differential signals. $V_{IN}-$ is now grounded and the signal is applied to $V_{IN}+$. Substituting $V_{IN}- = 0$ in Equations 11, 13, and 15 results in

$$(V_{OUT}+) = \frac{(V_{IN}+)(1-\beta_1)+2V_{OCM}\beta_1}{(\beta_1+\beta_2)},$$

$$(V_{OUT}-) = \frac{2V_{OCM}\beta_2-(V_{IN}+)(1-\beta_1)}{(\beta_1+\beta_2)}, \quad \text{and}$$

$$V_{OD} = \frac{2(V_{IN}+)(1-\beta_1)+2V_{OCM}(\beta_1-\beta_2)}{(\beta_1+\beta_2)}.$$

If the signal is not referenced to ground, the reference voltage will be amplified along with the desired signal, reducing the dynamic range of the amplifier. To strip unwanted dc offsets, use a capacitor to couple the signal to $V_{IN}+$. Keeping $\beta_1 = \beta_2$ will prevent V_{OCM} from causing an offset in V_{OD}.

The circuits in Figures 4–7 have nonsymmetrical feedback. This causes V_{OCM} to influence $V_{OUT}+$ and $V_{OUT}-$ differently, making V_{OCM} show up in V_{OD}. This will change the operating points between the internal nodes in the

Figure 3. Single-ended to differential amplifier

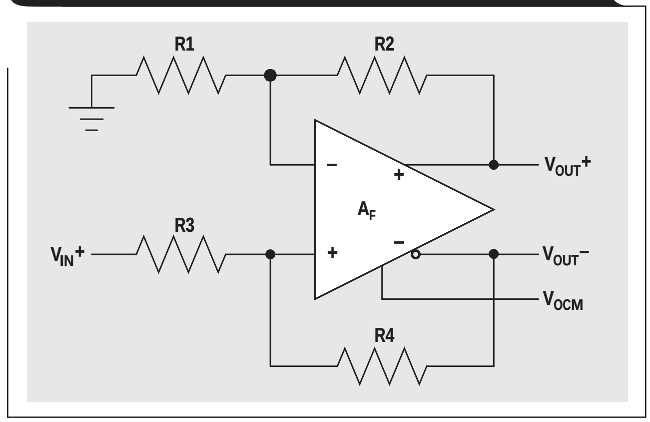


Figure 4. $\beta_1 = 0$

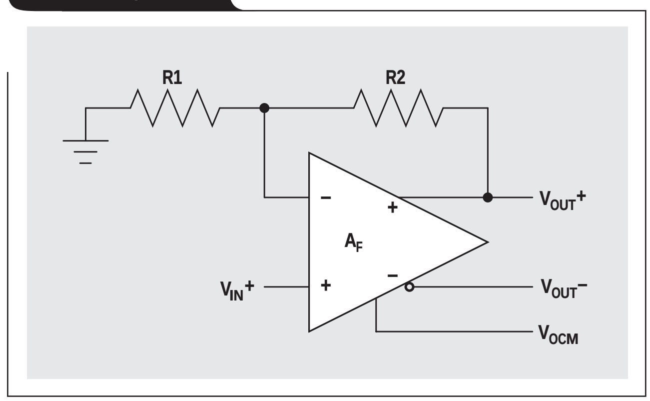


Figure 5. $\beta_2 = 0$

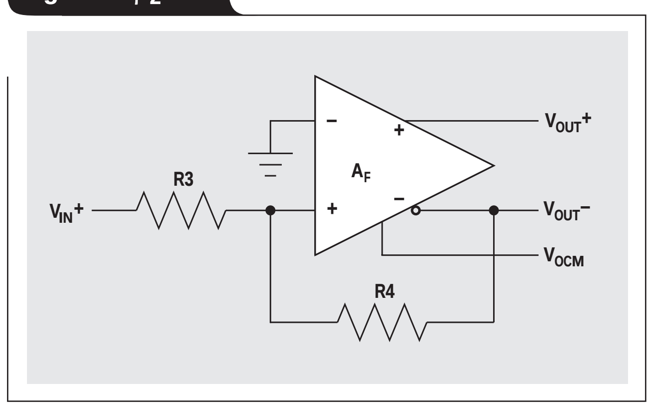


Figure 6. $\beta_2 = 1$

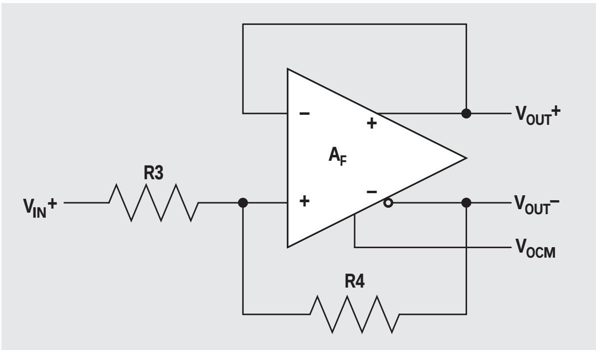
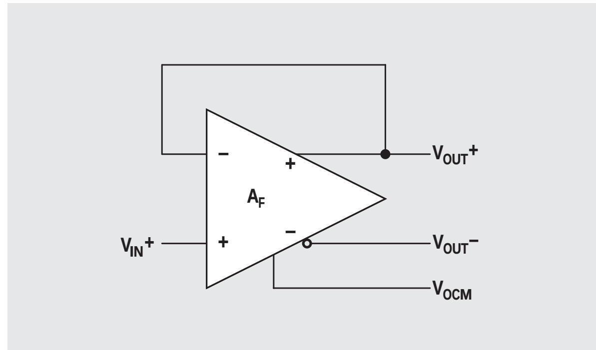


Figure 7. $\beta_1 = 0$, and $\beta_2 = 1$



differential amplifier, and matching of the open-loop gains will degrade. CMRR is not a real issue with single-ended inputs, but the analysis points out that CMRR is severely compromised when nonsymmetrical feedback is used. In the discussion of noise analysis that follows, it is shown that nonsymmetrical feedback also increases noise introduced at the V_{OCM} pin. For these reasons, even though the circuits shown in Figures 4–7 have been tested to prove they work in accordance with the equations given, they are presented mainly for instructional purposes. They are not recommended without extensive lab testing to prove their worthiness in your application.

In the circuit shown in Figure 4, $V_{IN}- = 0$ and $\beta_1 = 0$. The output voltages are

$$(V_{OUT}+) = \frac{(V_{IN}+)}{\beta_2},$$

$$(V_{OUT}-) = 2V_{OCM} - \frac{(V_{IN}+)}{\beta_2}, \quad \text{and}$$

$$V_{OD} = \frac{2(V_{IN}+)}{\beta_2} - 2V_{OCM}.$$

With $\beta_1 = 0$, this circuit is similar to a noninverting amplifier.

In the circuit shown in Figure 5, $V_{IN}- = 0$ and $\beta_2 = 0$. The output voltages are

$$(V_{OUT}+) = \frac{(V_{IN}+)(1-\beta_1)}{\beta_1} + 2V_{OCM},$$

$$(V_{OUT}-) = \frac{-(V_{IN}+)(1-\beta_1)}{\beta_1}, \quad \text{and}$$

$$V_{OD} = \frac{2(V_{IN}+)(1-\beta_1)}{\beta_1} + 2V_{OCM}.$$

With $\beta_2 = 0$, the gain is twice that of an inverting amplifier (without the minus sign).

In the circuit shown in Figure 6, $V_{IN}- = 0$ and $\beta_2 = 1$. The output voltages are

$$(V_{OUT}+) = \frac{(V_{IN}+)(1-\beta_1) + 2V_{OCM}\beta_1}{\beta_1 + 1},$$

$$(V_{OUT}-) = \frac{2V_{OCM} - (V_{IN}+)(1-\beta_1)}{\beta_1 + 1}, \text{ and}$$

$$V_{OD} = \frac{2(V_{IN}+)(1-\beta_1) + 2V_{OCM}(\beta_1 - 1)}{(\beta_1 + 1)}.$$

The gain is 1 with $\beta_1 = 0.333$; or, with $\beta_1 = 0.6$, the gain is 1/2.

In the circuit shown in Figure 6, $V_{IN}- = 0$, $\beta_1 = 0$, and $\beta_2 = 1$. The output voltages are

$$(V_{OUT}+) = (V_{IN}+), \quad (V_{OUT}-) = 2V_{OCM} - (V_{IN}+),$$

and $V_{OD} = 2[(V_{IN}+) - V_{OCM}]$.

This circuit realizes a gain of 2 with no resistor.

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Noise analysis

The noise sources are identified in Figure 8, which will be used for analysis with the following definitions.

E_{IN} is the input-referred RMS noise voltage of the amplifier: $E_{IN} \approx e_{IN} \times \sqrt{ENB}$ (assuming the 1/f noise is negligible), where e_{IN} is the input white noise spectral density in volts per square root of the frequency in Hertz, and ENB is the effective noise bandwidth. E_{IN} is modeled as a differential voltage at the input.

$I_{IN}+$ and $I_{IN}-$ are the input-referred RMS noise currents that flow into each input. They are taken as equal and called I_{IN}. $I_{IN} \approx i_{IN} \times \sqrt{ENB}$ (assuming the 1/f noise is negligible), where i_{IN} is the input white noise spectral density in amps per square root of the frequency in Hertz, and ENB is the effective noise bandwidth. I_{IN} develops a voltage in proportion to the equivalent input impedance seen from the input nodes. Assume the equivalent input impedance is dominated by the parallel combination of the gain setting resistors:

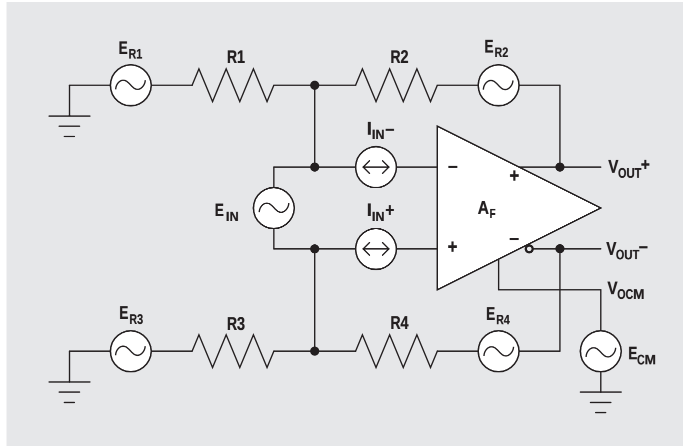
$$R_{EQ1} = \frac{R1R2}{R1+R2} \quad \text{and} \quad R_{EQ2} = \frac{R3R4}{R3+R4}.$$

E_{CM} is the RMS noise at the V_{OCM} pin, taking into account the spectral density and bandwidth as with the input-referred noise sources.

Noise current into the V_{OCM} pin will develop a noise voltage across the impedance seen from the node. It is assumed that proper bypassing of the V_{OCM} pin is done to reduce the effective bandwidth, so this voltage is negligible. If this is not the case, the added noise should be added to E_{CM} in a similar manner, as shown below.

E_{R1} through E_{R4} are the RMS noise voltages from the resistors, calculated by $E_{Rn} = \sqrt{4kTR \times ENB}$, where n is the resistor number, k is Boltzmann's constant (1.38×10^{-23}j/K), T is the absolute temperature in Kelvin (K), R is the resistance in ohms (Ω), and ENB is the effective noise bandwidth.

Figure 8. Noise analysis circuit



E_{OD} is the differential RMS output noise voltage. $E_{OD} = A(E_{ID})$, where E_{ID} is the input noise source, and A is the gain from the source to the output. Half of E_{OD} is attributed to the positive output ($+E_{OD}/2$), and half is attributed to the negative output ($-E_{OD}/2$). Therefore, ($+E_{OD}/2$) and ($-E_{OD}/2$) are correlated to one another and to the input source, and can be directly added together; i.e.,

$$\left(\frac{+E_{OD}}{2}\right) - \left(\frac{-E_{OD}}{2}\right) = E_{OD} = A(E_{ID}).$$

Independent noise sources typically are not correlated. To combine noncorrelated noise voltages, a sum-of-squares technique is used. The total RMS voltage squared is equal to the square of the individual RMS voltages added together. The output noise voltages from the individual noise sources are calculated one at a time and then combined in this fashion.

The block diagram shown in Figure 9 helps in analyzing the amplifier's noise sources.

Considering only E_{IN}, from the block diagram we can write:

$$E_{OD} = A_F\left[E_{IN} + \frac{(-E_{OD})\beta_1}{2} - \frac{(+E_{OD})\beta_2}{2}\right].$$

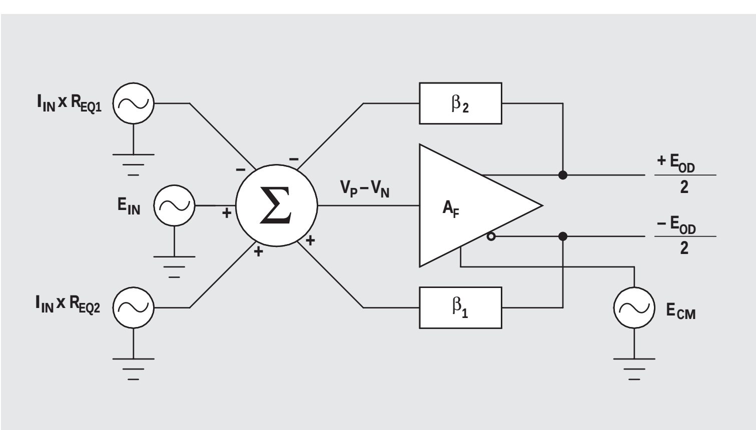
Solving yields

$$E_{OD} = \left(\frac{2E_{IN}}{\beta_1+\beta_2}\right)\left(\frac{1}{1+\frac{2}{A_F(\beta_1+\beta_2)}}\right).$$

Assuming $A_F\beta_1 >> 1$ and $A_F\beta_2 >> 1$,

$$E_{OD} = \frac{2E_{IN}}{(\beta_1+\beta_2)}.$$

Figure 9. Block diagram of the amplifier's input-referred noise



Given $\beta_1 = \beta_2 = \beta$ (symmetrical feedback),

$$E_{OUT} = \frac{E_{IN}}{\beta},$$

the same as a standard single-ended voltage feedback op amp.

Similarly, the noise contributions from I_{IN} x R_{EQ1} and I_{IN} x R_{EQ2} will be

$$\frac{2I_{IN} \times R_{EQ1}}{(\beta_1 + \beta_2)} \quad \text{and} \quad \frac{2I_{IN} \times R_{EQ2}}{(\beta_1 + \beta_2)}, \text{ respectively.}$$

The V_{OCM} error amplifier will produce a common-mode noise voltage at the output equal to E_{CM}. Due to the feedback paths, β_1 and β_2, a noise voltage is seen at the input that is equal to $E_{CM}(\beta_1 - \beta_2)$. This is amplified, just as an input, and seen at the output as a differential noise voltage equal to

$$\frac{2E_{CM}(\beta_1 - \beta_2)}{(\beta_1 + \beta_2)}.$$

Noise gain from the V_{OCM} pin ranges from 0 (given $\beta_1 = \beta_2$) to a maximum absolute value of 2 (given $\beta_1 = 1$ and $\beta_2 = 0$, or $\beta_1 = 0$ and $\beta_2 = 1$).

Noise from resistors R1 and R3 appears like signals at $V_{IN}+$ and $V_{IN}-$ in Figure 1. From the circuit analysis presented earlier, the differential output noise contribution is

$$\frac{2(E_{R1})(1-\beta_2)}{(\beta_1 + \beta_2)} \quad \text{and} \quad \frac{2(E_{R3})(1-\beta_1)}{(\beta_1 + \beta_2)}$$

for each resistor respectively.

Noise from resistors R2 and R4 (E_{R2} and E_{R4}, respectively) is imposed directly on the output with no amplification.

Adding the individual noise sources yields the total output differential noise:

$$E_{OD} = \sqrt{\frac{(2E_{IN})^2 + (2I_{IN} \times R_{EQ1})^2 + (2I_{IN} \times R_{EQ2})^2 + [2E_{CM}(\beta_1 - \beta_2)]^2 + [2(E_{R1})(1-\beta_2)]^2 + [2(E_{R3})(1-\beta_1)]^2}{(\beta_1 + \beta_2)^2} + E_{R2}{}^2 + E_{R4}{}^2}.$$

The individual noise sources are added in sum-of-squares fashion. Input-referred terms are amplified by the noise gain of the circuit:

$$G_n = \frac{2}{\beta_1 + \beta_2}.$$

If symmetrical feedback is used where $\beta_1 = \beta_2 = \beta$, the noise gain is

$$G_n = \frac{1}{\beta} = 1 + \frac{R_F}{R_G},$$

where R_F is the feedback resistor and R_G is the input resistor, the same as a standard single-ended voltage feedback amplifier.

Reference

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Document Title	**TI Lit. #**
1. Jim Karki, "Fully differential amplifiers," *Analog Applications Journal* (August 2000), pp. 38-41	slyt018

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