AN EXTENDED GATHERER-POLLEY METHOD FOR OFDM AND DMT PEAK POWER REDUCTION

Douglas L. Jones

Department of Electrical and Computer Engineering & Coordinated Science Laboratory
University of Illinois at Urbana-Champaign
1308 W. Main Street
Urbana, IL 61801
dl-jones@uiuc.edu

ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) and Discrete MultiTone (DMT) modulation offer many advantages for digital data transmission and have been adopted for several important standards. Their primary drawback is a high peak-to-average power ratio (PAR). Methods which introduce compensation signals in unused channels, first developed by Gatherer and Polley, are among the more promising methods for reducing the PAR. However, these methods apply only to systems with unused channels or require the sacrifice of data rate for PAR reduction. We present a new method for PAR reduction using active (data- carrying) channels which dynamically moves outer constellation points, within margin-preserving constraints, to minimize the peak magnitude. This scheme simultaneously decreases the bit error rate slightly while substantially reducing the peak magnitude; simulations show peak reductions of slightly more than 6 dB.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM), including variants such as Discrete MultiTone (DMT) modulation, are becoming widely used. They offer several advantages in comparison with other modulation methods, including high capacity, reduced need for equalization, computational efficiency, and modest immunity to noise bursts. Several important standards, such as ADSL and the European digital audio broadcast standard, are OFDM-based, and it is being actively considered for many emerging standards. However, OFDM and DMT have certain drawbacks, of which the most significant is probably high peak-to-average power ratios (PARs). High PARs require a transmitter with greater linearity, larger dynamic range, and higher peak power delivery, thus greatly increasing the cost and the power consumption of the transmitter.

A number of methods have been developed to reduce the PAR of OFDM and DMT. Some methods are based on coding, in which some bits or bit combinations are sacrificed to exclude high-PAR patterns [1, 2, 3]. While peak power is reduced, so is the data rate. Furthermore, these methods are not compatible with existing standards. Other researchers, beginning with Gatherer and Polley, have introduced schemes which reduce peak power by inserting signals in unused channels which partially cancel the time-domain peaks [4, 5, 6, 7]. Since the various FDM channels are orthogonal, these additional signals cause no distortion of the data-bearing channels. Simple projection-onto-convex-sets (POCS) or linear programming algorithms make these approaches practical, and they are compatible with any existing standard, such as the DMT-based standard for ADSL, in which some channels may often go unused.

Methods exploiting unused channels obtain several dB of peak power reduction when several unused channels are available [4, 7]. However, in many OFDM systems all channels carry data; either several channels must be set aside for peak power reduction [6], thus sacrificing data rate, or peak power reduction must be foregone. We present here a method in which, with constraints, the signals in data-bearing channels can be modified to reduce peak power without increasing the bit error rate (BER). This method thus supports peak power reduction in systems with full channel utilization, and also enhances the performance of existing methods exploiting unused channels. Substantial peak power reductions of more than 6 dB are observed in

This work was supported in part by Rice University and Texas Instruments through a Texas Instruments Visiting Professorship at Rice

simulations.

2. EXTENDED METHOD FOR ACTIVE CHANNELS

The new method for reducing peak power by altering the signals in actively transmitting channels, without compromising performance, is most easily explained by considering the specific case of OFDM with QPSK modulation in each orthogonal frequency channel. For a single channel, the four possible constellation points lie in each quadrant of the complex plane equidistant from the real and imaginary axes, as illustrated in Figure 1. The optimal decision regions are the four quadrants, bounded by the real and imaginary axes; that is, an observed received data sample will be assigned to the closest constellation point, which is the one in the same quadrant as the received observation.

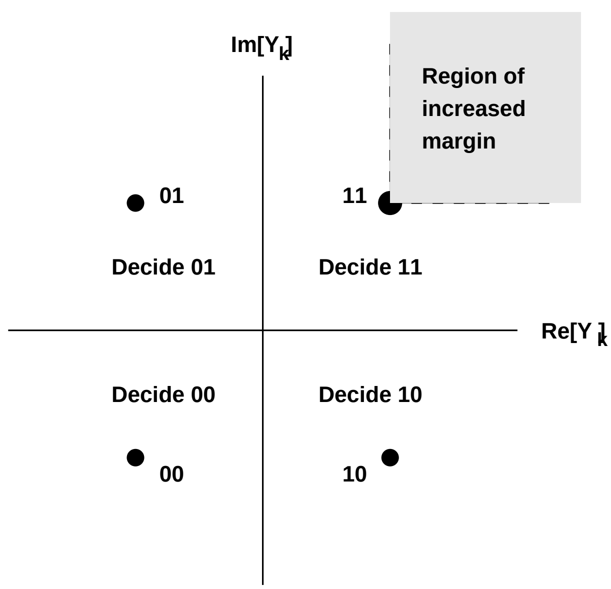

Figure 1: A QPSK constellation with assigned 2-bit codes. The four quadrants bounded by the real and imaginary axes form the decision regions. When 1,1 is transmitted, the constellation point can be moved anywhere in the shaded region, as shown, without decreasing the margin or increasing the probability of error

In traditional OFDM systems, only one of the four constellation points can be transmitted. Note, however, that with no transmission error or noise, any point in the proper quadrant would result in correct data recovery. More significantly, any point which is farther from the decision boundaries in the proper quadrant than the nominal constellation point will offer increased margin and thus equal or lower error rates. We can thus allow modification of the constellation point within the quarter-plane outside of the nominal constellation point with no degradation in performance. This region is shaded in Figure 1 for a constellation point in the first quadrant, and a modified symbol with increased margin is also illustrated.

For an OFDM system, the effect of this change is to add, subject to the quarter-plane constraints, an additional cosinusoid and/or a sinusoid at that particular channel's frequency to the transmitted signal. If adjusted properly, a combination of such components can be used to partially cancel the time-domain peaks in the composite OFDM signal.

This extension is not limited to QPSK constituent channel modulations. For BPSK modulation, alteration of the constellation point within the half-plane of distance equal or greater from the decision boundary is acceptable. For larger constellations, the outer constellation points are free to be adjusted in a similar manner. That is, whenever the data to be transmitted in that channel selects one of the constellation points on the periphery of the constellation, any point which maintains or increases the distance from the decision region can be used. While in all cases these modifications increase the transmitted power for that data block, in practice peak exceedances occur rarely enough that these modifications have an almost negligible impact on the total transmitted power, and they appear to be compatible with most implementations of current standards.

3. ALGORITHM FOR OPTIMAL CONSTELLATION MODIFICATION

A fairly efficient projection-onto-convex-sets (POCS) algorithm is easily derived for optimizing the constellation points within the appropriate quarter-plane or half-plane constraints, as well as simultaneously optimizing the non-data-bearing channels. This algorithm parallels the algorithm introduced by Gatherer and Polley, with minor extensions to support the new method.

1. Assign frequency-domain constellation points, X_k, according to the input data

2. Reconstruct the time-domain signal with an inverse FFT

3. Compare the magnitude of all time samples with the peak level constraint, $L_{\max}$

$$|x_n| \leq L_{\max} \quad , \qquad 0 \leq n < N \qquad (1)$$

4. For all time samples for which the peak level constraint is exceeded, scale (project) the magnitude

to the peak level;

$$\hat{x}_n = L_{\max} \exp j\theta_n \quad (2)$$

where

$$x_n = |x_n| \exp j\theta_n \quad (3)$$

5. Forward transform the modified signal via the FFT

6. Enforce all frequency constraints; i.e., restore all interior constellation points to original values, and project exterior points onto the region of increased margin. For the $1, 1$ QPSK example in Figure 1, this corresponds to enforcing the constraints

$$\mathrm{Re}\left[\hat{X}_k\right] \geq \mathrm{Re}\left[X_k\right] \quad (4)$$

$$\mathrm{Im}\left[\hat{X}_k\right] \geq \mathrm{Im}\left[X_k\right] \quad (5)$$

   Similar constraints are applied for the other constellation points or with other constellations

7. Return to Step 2 and iterate until the time-domain constraint is achieved for all time samples or the maximum allowed number of iterations is exceeded

This algorithm closely resembles the Gatherer-Polley method, except that Step 6 includes margin-maintaining constraints instead of always restoring all constellation points in active channels to their original values. The extra freedom allows greater peak reduction without compromising performance.

For situations in which the goal is to minimize peak value, rather than achieve a fixed, maximum peak level, a related gradient-project algorithm can be developed. The gradient step which maximizes the time-domain peak reduction is precisely proportional to the change which shrinks the peak by a miniscule amount; thus rather than clipping in the above algorithm, we scale the largest time-domain peak(s) by a small amount and then project exactly as above to enforce the margin-preserving frequency-domain constraints. Such an algorithm is guaranteed to converge to the minimal peak level, due to the convexity of the constraints, for a vanishingly small scaling (gradient) step size; however, a small step size leads to generally slower convergence. Experimental optimization of this parameter is called for in a practical implementation.

4. SIMULATIONS AND RESULTS

Initial simulations for over 100,000 time samples for 256-channel QPSK OFDM with no unused channels yielded a 51% reduction in the peak value, or slightly more than a 6 dB peak power reduction. Note that no improvement is possible with conventional methods. Further results will appear in the final manuscript. These results indicate that this simple, effective extension of current methods to utilize data-bearing channels yields a very significant decrease in peak power while maintaining compatibility with most implementations of existing standards.

5. DISCUSSION

The proposed method for peak power minimization of OFDM and DMT modulations extends the Gatherer-Polley method by allowing margin-increasing modification of exterior constellation points in the constituent frequency channels. This extension offers considerable (several dB) reduction in peak power even when all channels are active, whereas the original Gatherer-Polley method cannot be used. The extensions should also substantially improve performance with a mixture of active and unused channels, although we have not yet performed extensive simulations of this case. The new method shows considerable promise, but many more simulations and a much more thorough investigation are needed to quantify its performance.

The extended algorithm is very similar to the POCS-based Gatherer-Polley optimization, so similar relatively rapid peak power reduction is expected. However, convergence to the absolute minimal peak power is likely to be slower, since many more channels and constraints are used. Each iteration of the algorithm is relatively inexpensive, with the forward and inverse FFTs dominating the cost. With more than a few iterations, though, this operation would dominate the computation in an OFDM transmitter, thus rendering it either impractical or the complexity-limiting component. It is quite possible that faster or less expensive optimization methods can be developed, and this appears to be a promising direction for future research.

ACKNOWLEDGMENT

The author would like to express his thanks to Donald Shaver, Michael Polley, and Alan Gatherer for introducing him to this problem and for several helpful discussions.

6. REFERENCES

[1] E. Jones, T. Wilkinson, and S. Barton, "Block coding scheme for reduction of peak to mean envelope power ratio of multicarrier transmission schemes," Electronics Letters, vol. 30, pp. 2098–2099, December 8, 1994.

[2] M. Friese, “Multicarrier modulation with low peak-to-average power ratio,” Electronics Letters, vol. 32, pp. 713–714, April 11, 1996.

[3] S. Mueller and J. Huber, “OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences,” Electronics Letters, vol. 33, pp. 368–369, February 1997.

[4] A. Gatherer and M. Polley, “Controlling clipping probability in DMT transmission,” in Proceedings of the 31st Asilomar Conference on Signals, Systems, and Computers, pp. 578–584, November 1997.

[5] J. Tellado and J. Cioffi, “PAR reduction in multicarrier transmission systems,” tech. rep., ANSI Document, T1E1.4 Technical Subcommittee, December 1997.

[6] J. Tellado and J. Cioffi, “Peak power reduction for multicarrier transmission,” in Proceedings of the International Symposium on Information Theory, (Cambridge, Massachusetts), August 1998.

[7] J. Tellado and J. Cioffi, “Peak power reduction for multicarrier transmission,” in Proceedings CTMC, Globecom 1998, (Sydney, Australia), November 1998.