Research on Bandwidth Efficient Wireless Communications: CPM

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Overview

- Research on Continuous Phase Modulation.
  - CPM background.
  - Finite state machine model.
  - MLSE.
  - MAP symbol-by-symbol.
  - Phase and frequency estimation.
  - Coherent demodulation for frequency flat Rayleigh fading channels.
  - Symbol timing recovery.
- Investigations into the Bluetooth Standard.
- Summary of Project Status and Future Work.
- Purdue Course on Fixed Point DSP Programming.

Continuous Phase Modulation (CPM)

- CPM is a nonlinear modulation with memory.
- CPM has a constant envelope.
  - good for use with nonlinear power amplifiers.
  - example: satellite communication.
- CPM has better spectral characteristics than MPSK.
  - memory reduces spectral occupancy.
- Optimum detection of CPM is more complex than MPSK.
  - memory increases complexity of receiver.
  - usually sequence detection via Viterbi algorithm.

Signal Model

- The (complex baseband) transmitted signal is:

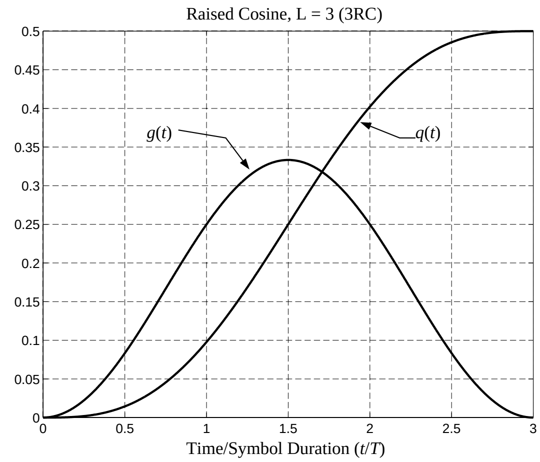
$$s(t;\underline{\mathrm{d}}) = \sqrt{E/T}\exp\left\{j\phi(t;\underline{\mathrm{d}})\right\}.$$

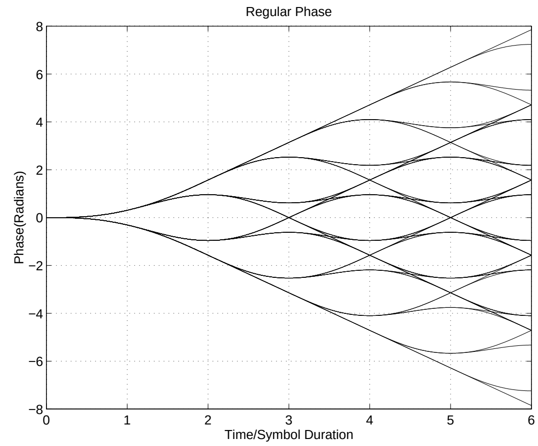
- Information is carried in the phase (nonlinear with memory):

$$\phi(t;\underline{\mathrm{d}}) = 2\pi h \sum_i d_i q(t - iT)$$

  - $h = 2m/p$ is the modulation index.
  - $d_i \in \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$.
  - $q(t)$ is called the phase response; normalized such that $q(t) = 0$ for $t \leq 0$, $q(t) = 1/2$ for $t \geq LT$.
  - L called the memory of the CPM scheme.

Phase Response and Phase Trellis





- Parameters: 3RC, h = 1/2, M = 2.
- Memory is used to control spectral occupancy. Memory introduced through:
  - Phase continuity (smoothness of $q(t)$).
  - Pulse length parameter L.
- Optimum demodulation of CPM is complex because of memory and nonlinearity.

Finite State Description

- For $n \geq L$ the phase in the interval $nT \leq t < (n+1)T$ is

$$\phi(t;\underline{\mathrm{d}}) = \underbrace{\pi h \sum_{i=0}^{n-L} d_i}_{\theta_n} \quad + \quad 2\pi h \sum_{i=n-L+1}^{n} d_i q(t - iT).$$

  - For $h = 2m/p$, θ_n takes at most p values

$$\theta_n \in \{0, 2\pi/p, \ldots, 2(p-1)\pi/p\}.$$

- Phase in $nT \leq t < (n+1)T$ is completely specified by
  - Current symbol d_n and
  - The state $\mathbf{s}_n = (d_{n-1}, \ldots, d_{n-L+1}, \theta_n) \in \mathcal{S}$.
  - $\|\mathcal{S}\| = pM^{L-1}$.

On the Complexity of CPM

- Current CPM applications have been limited to very simple schemes (MSK and generalizations) due to:
  1. Implementation complexity which arises primarily in two ways.
     - Large number of states needed to describe a CPM signal $\Longrightarrow$ MLSE must operate on large trellis.
     - Dimensionality of the signal space can be large esp. with partial response or multilevel $\Longrightarrow$ Filter bank for computation of demodulator metrics is large.
  2. Synchronization difficulties.
     - The standard low complexity approach is to pass the IF signal through a non-linearity $\Longrightarrow$ Generates tones at carrier and clock frequencies.
     - Tones usually extracted by PLLs.
     - However, method fails with smoothed frequency pulses because amplitude of tones too small compared with noise level.

Demodulation of CPM

1. Maximum Likelihood Sequence Detection (MLSD).
   - Computes MAP estimate of the symbol sequence via Viterbi Algorithm.
   - Complexity is $O(pM^L)$.
2. Symbol-by-Symbol Detection (OSA).
   - Produces soft decision metrics.
   - Complexity is $O(KpM^{L+1})$.
3. Complexity reduction techniques.
   - M-Algorithm.
   - T-Algorithm.
   - Reduced state demodulation.

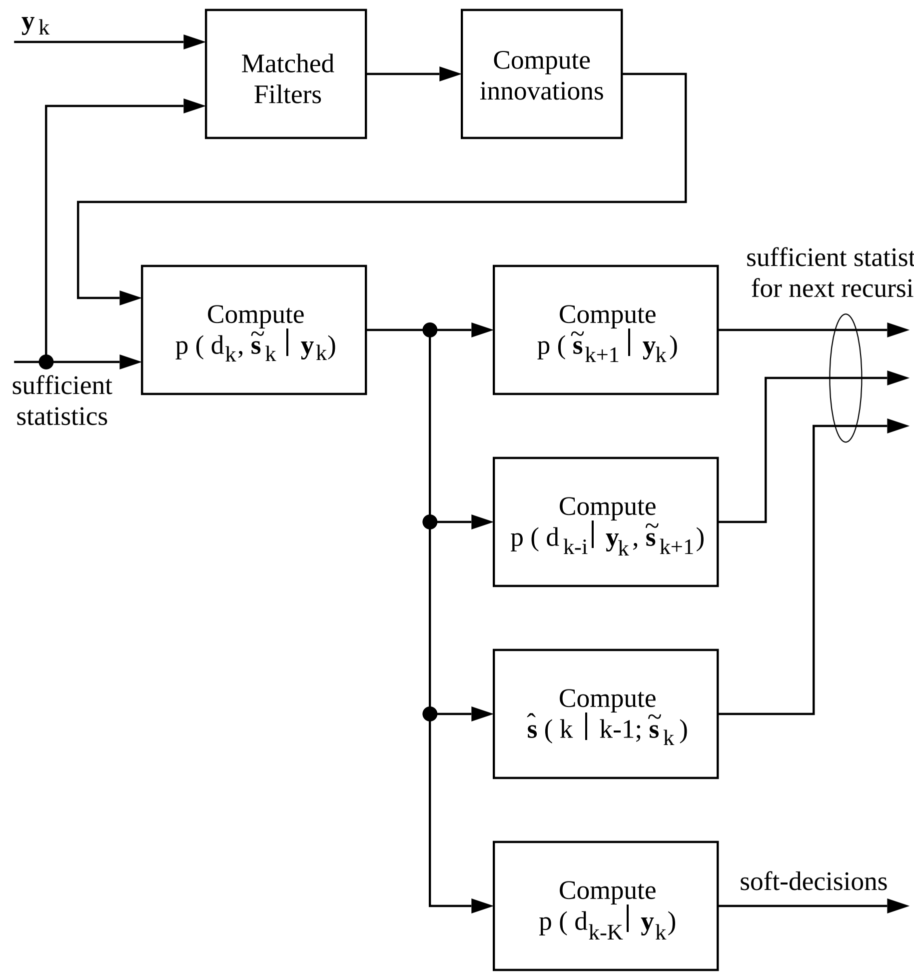
Complexity Reduction Techniques for OSA

- M-Algorithm.
  - retains the largest M_t sufficient statistics.
  - must do a sorting operation.
  - moderately variable complexity.
- T-Algorithm.
  - retains sufficient statistics exceeding a threshold p_t.
  - large variations in complexity.
- Reduced state demodulation.
  - demodulation occurs on a reduced modulation state.
  - no variations in complexity.

Basic Idea of Reduced State Detection

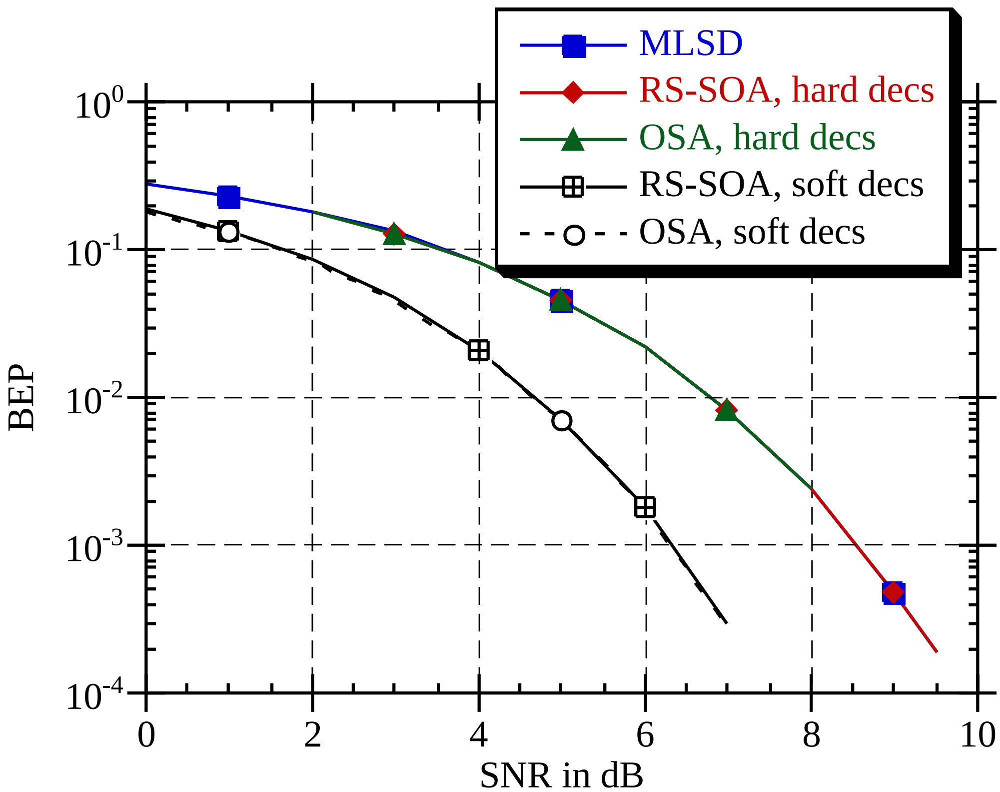
- CPM signal represented by a trellis $\mathcal{S}$.
  - number of states in trellis is $\|\mathcal{S}\| = pM^{L-1}$.
- A reduced state trellis $\tilde{\mathcal{S}}$ is considered instead.
- Partition original trellis to get a reduced state representation.
- Each state in reduced trellis consists of one or more of the original states.
  - e.g., if $\|\mathcal{S}\| = 64$ can have $\|\tilde{\mathcal{S}}\| = 16$ such that each state in reduced trellis contains 4 of the original states.
- Partition such that Euclidean distance between members in the same state in the reduced trellis is maximized.
  - same principle used for reduced state sequence detection by Eyuboglu and Qureshi (1988).

Block Diagram of the Recursions for RS-SOA

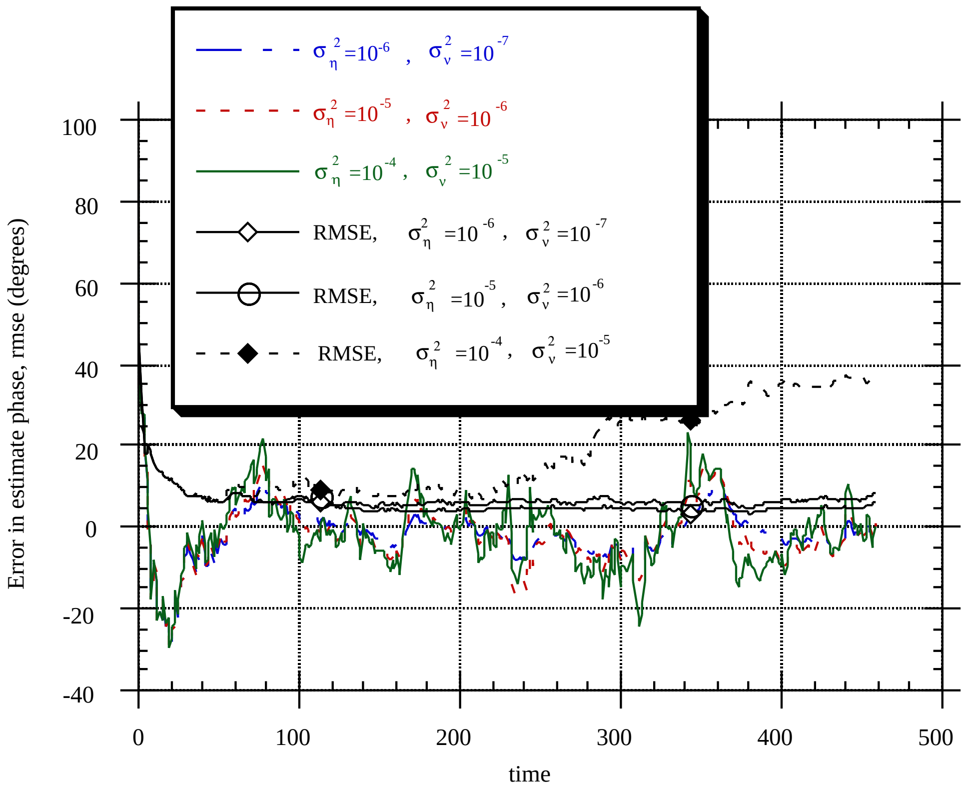


BEP Performance

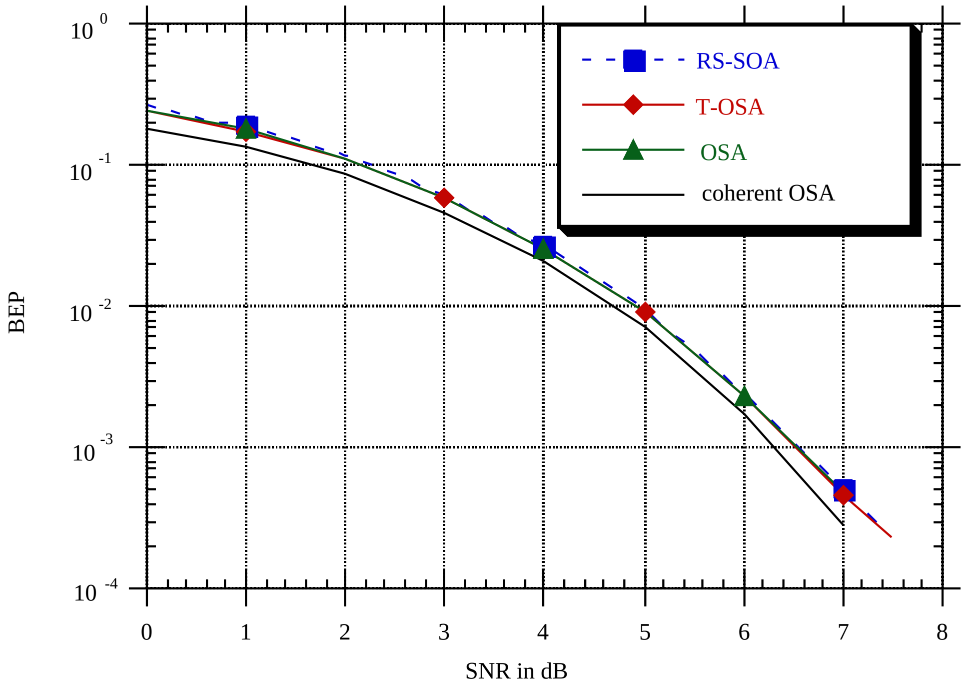
- RC pulse, $L = 3$, $M = 4$, $h = 0.5$.
- Rate 1/2 code, constraint length = 2, 20×20 interleaver.
- Complexity of RS-SOA is 25% of OSA.



Phase Error vs. Time for OSA



BEP vs. SNR for Various Algorithms



Received Signal Model

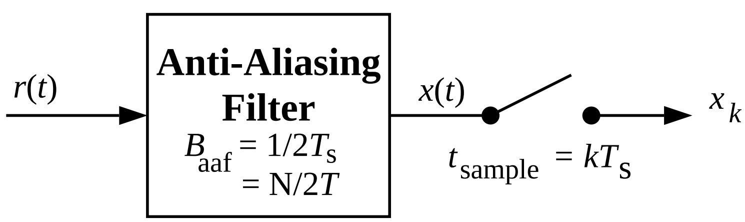
- Complex baseband of the received waveform is

$$\begin{aligned} r(t) &= s(t-\tau;\underline{\mathrm{d}})e^{j\theta} + w(t) \\ &= \sqrt{\frac{E}{T}} e^{j[\theta+\phi(t-\tau;\underline{\mathrm{d}})]} + w(t) \end{aligned}$$

where

  - θ and τ are carrier phase and timing epoch, respectively.
  - $w(\cdot)$ is complex AWGN of spectral height N_0.

- Filter and sample with free running clock



  - Sampling rate is $1/T_s = N/T$.
  - Assume signal is not distorted by AAF:

$$x_k = \sqrt{E/T} e^{j[\theta+\phi(kT_s-\tau;\underline{\mathrm{d}})]} + w_k.$$

Maximum Likelihood Estimation

- Signal component $\{x_k\}$ depends upon:
  - Transmitted symbols $\underline{\mathrm{d}}$.
  - Carrier phase θ.
  - Symbol timing epoch τ.
- We will assume:
  - Symbols $\underline{\mathrm{d}}$ are known (synchronization preamble).
  - Carrier phase θ is nuisance parameter.
- Thus seek **data-aided** and **phase-independent** ML estimate of τ.
  - If known preamble length is K symbols, define

$$\underline{\mathrm{x}}^T = \left[\, x_0 \;\; x_1 \;\; \cdots \;\; x_{NK-1} \,\right].$$

  - Likelihood function:

$$\Lambda(\underline{\mathrm{x}}|\theta,\tau) = \exp\left\{ -\frac{1}{2\sigma^2} \sum_{k=0}^{NK-1} \left| x_k - \sqrt{\frac{E}{T}} e^{j[\theta+\phi(kT_s-\tau;\underline{\mathrm{d}})]} \right|^2 \right\}.$$

Maximum Likelihood Estimation(cont'd.)

- Simplify by dropping terms independent of θ, τ get equivalent likelihood function

$$\Lambda_1(\underline{\mathrm{x}}|\theta,\tau) = \exp\left\{\frac{1}{\sigma^2}\sqrt{\frac{E}{T}}\mathrm{Re}\left[e^{-j\theta}\sum_{k=0}^{NK-1} x_k e^{-j\phi(kT_s-\tau;\underline{\mathrm{d}})}\right]\right\}.$$

- Since θ is nuisance parameter we may average it out of $\Lambda_1(\underline{\mathrm{x}}|\theta,\tau)$ by integrating against pdf of θ (assumed uniform over $(-\pi,\pi]$).

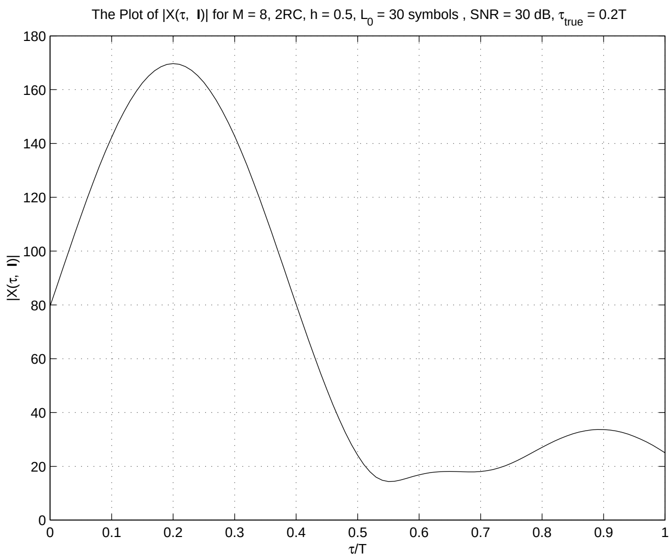
- With $X(\tau) \overset{\text{def}}{=} \sum_{k=0}^{NK-1} x_k e^{-j\phi(kT_s-\tau;\underline{\mathrm{d}})}$, an equivalent likelihood function is

$$\Lambda_2(\underline{\mathrm{x}}|\tau) = I_0\left(\frac{1}{\sigma^2}\sqrt{\frac{E}{T}}|X(\tau)|\right).$$

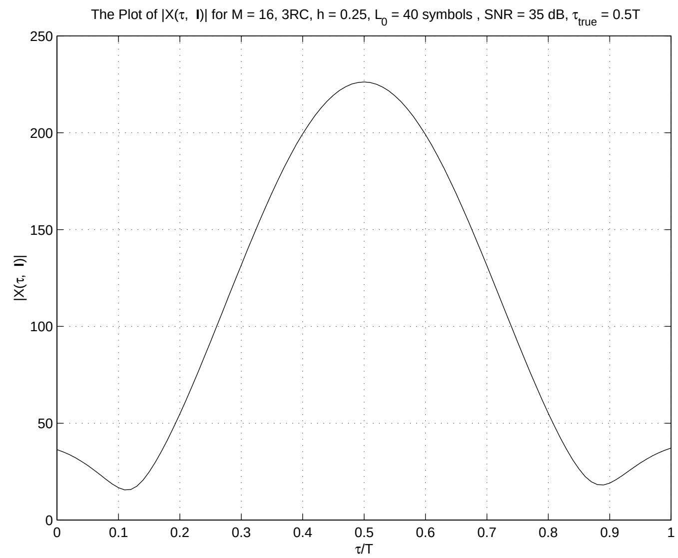
- Since $I_0(\cdot)$ is increasing in the magnitude of its argument, equivalent ML estimation problem for τ is

$$\hat{\tau} = \mathrm{argmax}_\tau |X(\tau)|.$$

Examples of $|X(\tau)|$

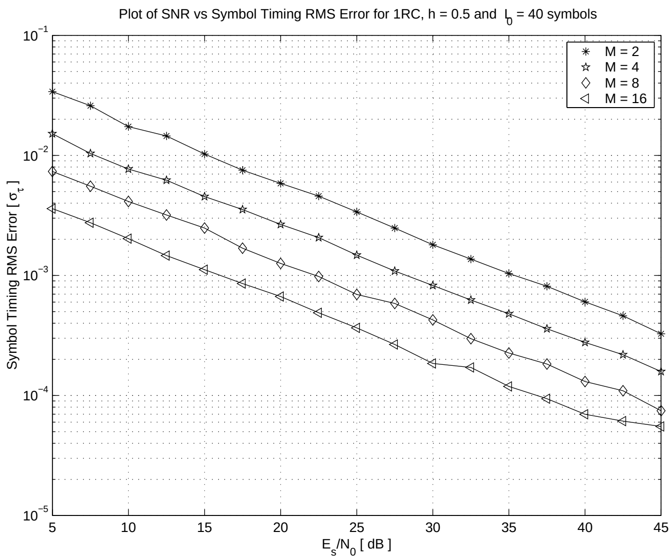


$M = 8$

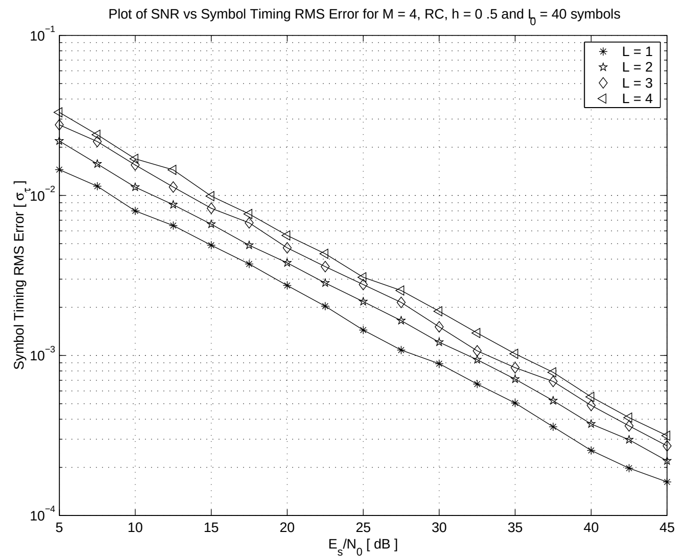


$M = 16$

Plot of RMS Error as a Function of SNR

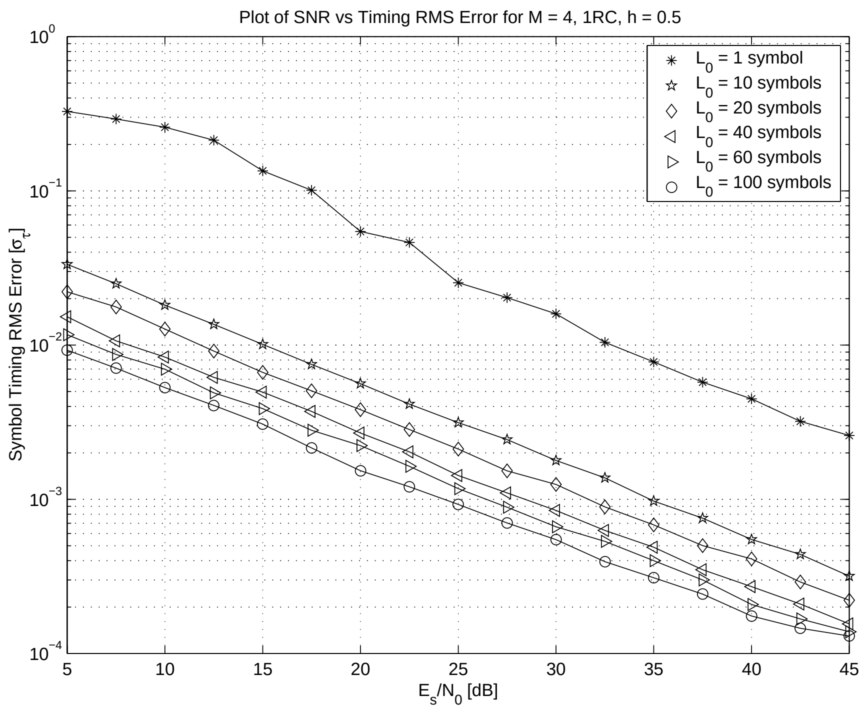


Param. by Size of Alphabet (M)



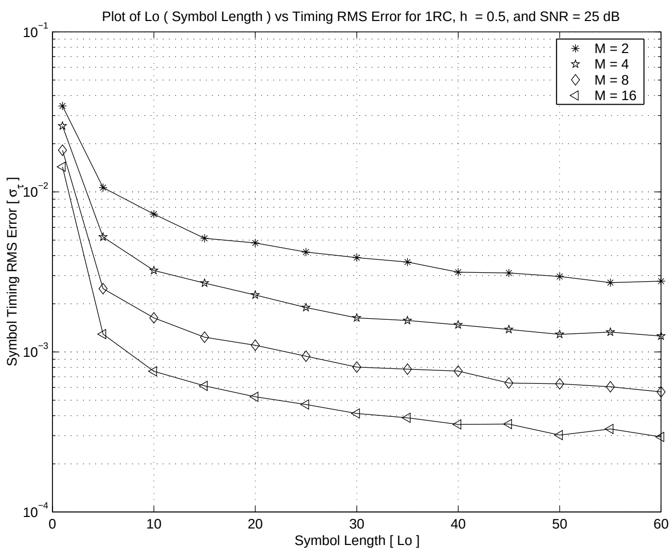
Param. by Length of Phase Response (L)

Plot of RMS Error as a Function of SNR

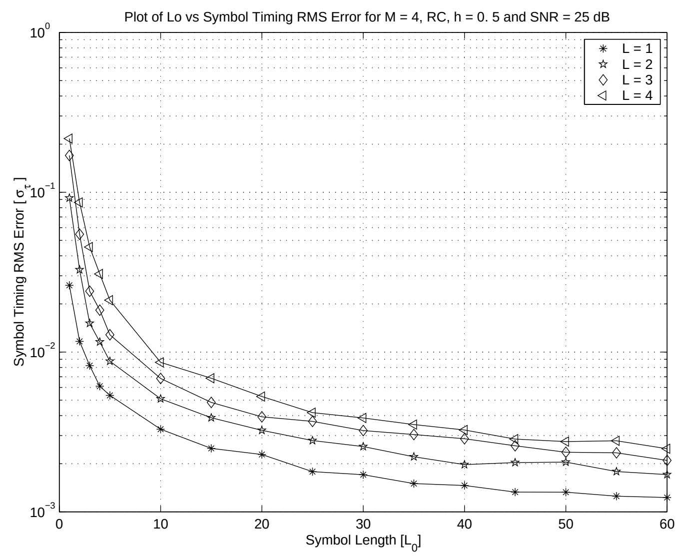


Param. by Preamble Length (K)

Plot of RMS Error as a Function of Preamble Length

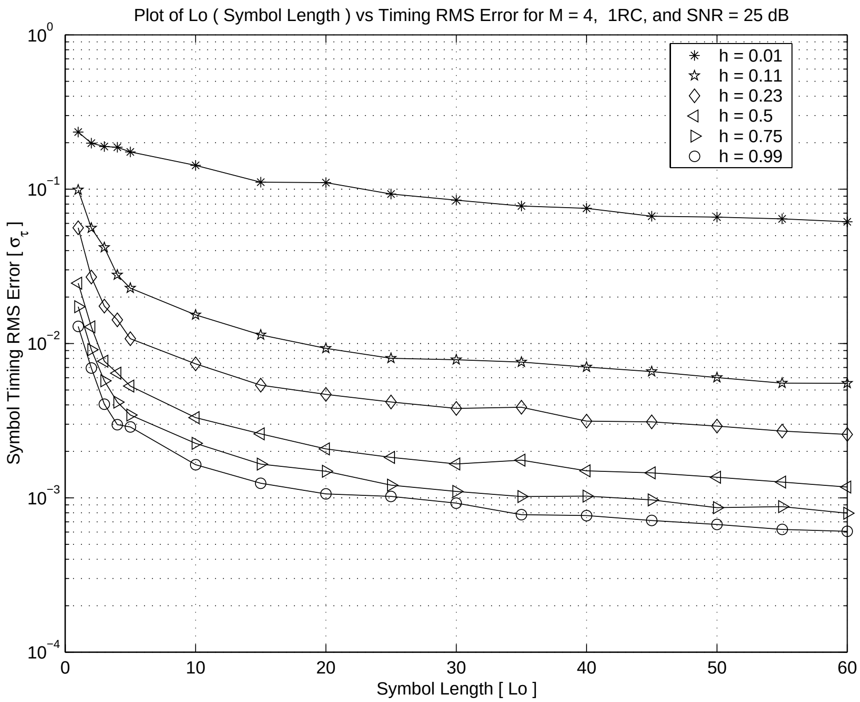


Param. by Size of Alphabet (M)



Param. by Length of Phase Response (L)

Plot of RMS Error as a Function of Preamble Length



Param. by Mod. Index (h)