Considerations for Measuring Loop Gain in Power Supplies

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Considerations for Measuring Loop Gain in Power Supplies

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Abstract

Loop gain measurements show how stable a power supply is and provide insight to improve output transient response. This presentation discusses the theory of open-loop transfer functions and empirical loop gain measurement methods. The presentation then demonstrates how to configure the frequency analyzer and prepare the power supply under test for accurate loop gain measurements. Examples are provided to illustrate proper loop gain measurement techniques.

I. INTRODUCTION

Loop gain is the product of all gains around a feedback loop. Figure 1 shows a simple system with negative feedback.

Figure 1 – Block diagram of a feedback system.

The loop gain of this system is defined as:

\[ T(s) = G(s) \cdot H(s) \]  (1)

A. Feedback Loop of a Power Supply

A power supply is usually a more complicated system than what is shown in Figure 1. Figure 2 shows a typical buck converter with voltage mode control.

Figure 2 includes a power stage, the output feedback resistor divider, pulse width modulation (PWM) comparator and error amplifier with compensation network. The power stage contains the power devices, magnetics and capacitors, which transfer energy from the input source to the output. The output is sensed by the feedback resistor divider. The error amplifier with the compensation network forms the compensator which amplifies the error between the output feedback (FB) and the reference voltage, \( V_{REF} \). The output of the compensator, COMP, is modulated by the PWM comparator which converts COMP, a continuous signal, into a discrete driving signal. When the driving signal is high, the buck converter low-side MOSFET is turned off and high-side MOSFET is turned on. When the driving signal is low, the buck converter high-side MOSFET is turned off and low-side MOSFET is turned on. For a voltage mode buck converter, the modulation gain is one divided by the amplitude of the ramp signal. The amplitude of the ramp signal is in reverse proportion to \( V_{IN} \) to maintain a constant DC gain over the input range. This technique is called input feed forward.

The loop gain, \( T(s) \), of the buck converter control system is the product of all the gains:

\[ T(s) = G_{PS}(s) \cdot G_{COMP}(s) \cdot F_m \]  (2)

\( G_{PS}(s) \) is the power stage gain from the duty-cycle to the output. \( G_{COMP}(s) \) represents the transfer function of the error amplifier, compensation network and resistor divider. \( F_m \) is the PWM modulation gain.

\[ \text{Figure 2} \text{ – A voltage mode buck converter control system.} \]
B. Loop Gain Relation to System Stability

Loop gain, \( T \), varies with frequency and can be expressed in the form of a Bode plot. Figure 3 shows the loop gain measured in Bode plot format for a buck converter.

![Figure 3 – Bode plot for loop gain of a buck converter.](image)

A Bode plot is a combination of a magnitude plot and a phase plot, both as a function of frequency. The blue plot in Figure 3 is the magnitude plot in dBs and the red plot is the phase shift in degrees. The phase margin (PM) is the difference between the phase and 0° at zero dB gain. When phase margin is zero or negative, the system is unstable.

Gain margin (GM) is another measure of loop gain that indicates the stability of the system. Gain margin is the amount of gain increase required to make the loop gain unity at the frequency where the phase shift is -180°. In other words, the gain margin is \( 1/G \) if \( G \) is the gain at -180° phase shift frequency.

In Figure 3, the phase shift measurement includes an additional phase-shift of 180° due to negative feedback. While a system is theoretically considered stable with even 1° phase margin, component variation or load/line variation can affect the frequency response and make the system unstable. The phase margin requirement is usually from 45° to 60°. The phase margin of this converter is only 15° at 22 kHz and the stability is considered marginal. Figure 4 shows the corresponding output load transient response. The purple trace is the load current waveform. The blue trace is the AC-coupled output voltage. The output voltage ringing after the load transient indicates that the system is on the brink of instability. In addition, this converter is also considered conditionally stable since the phase is less than 0° when the gain is greater than 1. The Bode plot gives the power supply designer information regarding the stability that can be used to adjust the compensation network accordingly.

![Figure 4 – Output load transient response of a buck converter with 15° phase margin.](image)

C. Loop Gain Bandwidth Relationship to Output Transient Response

When the system is stable, the output transient response is closely related to the bandwidth of the loop gain, as discussed in [1] and [2]. The loop gain bandwidth, \( f_C \), the frequency at which the magnitude of the loop gain equals one. Output transient overshoot and undershoot is inversely proportional to the loop gain bandwidth. Figure 5 shows a typical output load transient response.

![Figure 5 – Output load transient response of a buck converter [1].](image)
The overshoot or undershoot, $V_p$, can be estimated with:

$$V_p = \frac{\Delta I}{2\pi f_C C_{OUT}}$$  \hspace{1cm} (3)

where:

$\Delta I_{TRAN}$ is the transient load current step, $f_C$ is the loop gain control bandwidth and $C_{OUT}$ is the power supply output capacitance.

The delay time from the start of the current transient to the output transient peak or valley, $t_p$, can be approximated by:

$$t_p = \frac{1}{4f_C}$$  \hspace{1cm} (4)

Thus, the loop gain measurement assists the power supply designer in tuning the compensation network to improve the output transient response performance.

Given the importance of the loop gain, it is necessary to ensure loop gain measurement accuracy. This paper focuses on the methods and considerations for measuring loop gain experimentally. The goal is to provide guidelines for accurate loop gain measurement on which power supply designers can rely for compensation network design.

II. LOOP GAIN MEASUREMENT METHODS

By definition, a transfer function gain is the ratio between the output response and the input signal. Theoretically, loop gain can be measured by opening the loop, applying a DC bias and an input signal to point A, and then measuring the response at point B, as shown in Figure 6. However, it is impractical to measure loop gain in an open-loop setup.

The error amplifier usually has high DC gain for good output regulation. Even a small error applied to the input of the error amplifier would result in a tremendous error at the power supply output that could lead to damage. R.D. Middlebrook proposed measuring loop gain in a closed-loop setup in [3]. With the feedback loop closed, a small test voltage source or test current source is injected into the loop. By measuring the resulting response of the two points across the test voltage source or current source into the two points, the loop gain can be derived. Since the voltage injection method is the most practical and widely adopted method, this paper focuses on the voltage injection method.

![Figure 6 – Measuring loop gain of a power supply in open-loop setup.](Image)

When the feedback loop is broken at any point, such as that shown in Figure 6, looking backward, i.e. looking into point B, and applying Thevenin’s theory, the circuit is equivalent to a voltage-controlled voltage source (VCVS) in series with an output impedance, $Z_2$, as illustrated in Figure 7. The VCVS is controlled by the voltage of point A with a gain of $-M(s)$.

![Figure 7 – Equivalent circuit of a power supply looking backward from the breaking point.](Image)
The circuit looking forward from point A is equivalent to a network with an impedance, $Z_1$, as shown in Figure 8.

$$\tilde{V}_Y(s) = -M(s) \cdot \tilde{V}_A(s)$$

*Figure 8 – Equivalent circuit of a power supply with feedback loop.*

The system loop gain, $T$, is the product of the gain of the VCVS and the ratio of the divider, $Z_1$ and $Z_2$, when the loop is closed.

$$T(s) = M(s) \cdot \frac{Z_1(s)}{Z_1(s) + Z_2(s)}$$

If we find a point where $Z_2$ is zero, the loop gain, $T(s)$, equals the VCVS gain, $M(s)$.

$$T(s) = M(s)$$

With $Z_2 = 0 \ \Omega$, an AC voltage source, $\tilde{V}_{INJ}(s)$, is injected between point B and point A as illustrated in Figure 9.

$$\tilde{V}_Y(s) = -M(s) \cdot \tilde{V}_A(s)$$

*Figure 9 – Equivalent circuit of a feedback system with voltage injection.*

The response at point B, the output of the VCVS, is dependent on the voltage at point A:

$$\tilde{V}_B(s) = -T(s) \cdot \tilde{V}_A(s)$$

Equation (7) specifies the relation of $\tilde{V}_{INJ}(s)$, $\tilde{V}_A(s)$, and $\tilde{V}_B(s)$.

$$\tilde{V}_{INJ}(s) = \tilde{V}_B(s) - \tilde{V}_A(s)$$

(7)

The responses at point B and point A are derived as:

$$\tilde{V}_B(s) = \frac{T(s)}{1+T(s)} \cdot \tilde{V}_{INJ}(s)$$

$$\tilde{V}_A(s) = -\frac{1}{1+T(s)} \cdot \tilde{V}_{INJ}(s)$$

(8) (9)

The loop gain of the system is the ratio between the responses at point B and point A:

$$T(s) = \frac{\tilde{V}_B(s)}{\tilde{V}_A(s)}$$

(10)

Using the voltage injection method, the feedback loop is still closed and the DC operating point is maintained. The loop gain is derived from the closed-loop response of the control system.

**III. TEST SETUP AND EXAMPLES**

Figure 10 shows a typical loop gain measurement set up.

*Figure 10 – A typical loop gain measurement setup.*

Two test instruments are required: a frequency analyzer and an injection isolator. Then an appropriate injection point needs to be identified. Finally, we connect the equipment and the power supply together and start measurements over the frequency range of interest.
A. Setting Up the Frequency Analyzer

The frequency analyzer (FA) is the essential instrument for loop gain measurement. Figure 11 shows a typical frequency analyzer.

![Figure 11 – Frequency analyzer from AP Instruments. (Image reproduced with permission from AP Instruments, Inc.)](image)

The FA performs three functions for loop gain measurement. First, the frequency analyzer provides an AC voltage source for injection. Secondly, the frequency analyzer measures the response at point B and point A. Finally, the frequency analyzer calculates the loop gain using Equation (10) above. Since the AC voltage source has an output resistance of 50 Ω and the voltage is coupled through an injection isolator, the voltage source of the frequency analyzer is not equal to the voltage injected between point B and point A. Setting the amplitude of the AC voltage source will be discussed in detail in a later section.

When measuring voltage signals, all frequency analyzers have limitations. For example, the frequency analyzer shown in Figure 11 is capable of measuring voltage signal amplitudes as low as 5 μV_{RMS}[4]. For accurate loop gain measurement, the amplitude of signals at point B and point A must be greater than 5 μV_{RMS}. In the previous section, we know that the response at points A and B is a function of the loop gain, T, and the injected voltage, ̂V_{INJ}(s). To illustrate how the voltages at point A and point B respond to ̂V_{INJ}(s) over the frequency range, a buck converter with voltage mode control is used as an example. Figure 12 shows the magnitude plot of the transfer functions from ̂V_{INJ}(s) to ̂V_A(s) and from ̂V_{INJ}(s) to ̂V_B(s).

![Figure 12 – The magnitude plots of transfer functions from ̂V_{INJ}(s) to ̂V_A(s) and from ̂V_{INJ}(s) to ̂V_B(s).](image)

Most power supplies have high DC gain to maintain good output line/load regulation. Thus, at low frequencies, Equation (9) can be approximated by:

\[
\hat{V}_A(s) \approx -\frac{1}{T(s)} \cdot \hat{V}_{INJ}(s)
\]  

(11)

If the magnitude of the loop gain, T, at DC is 60 dB, to keep the amplitude of ̂V_A(s) greater than 5 μV_{RMS}, the injected voltage across points A and B, ̂V_{INJ}(s), should be greater than 5 mV_{RMS}.

At frequencies above the loop gain bandwidth, f_C, the magnitude of T is much less than 1. Equation (8) can be approximated by:

\[
\hat{V}_B(s) \approx T(s) \cdot \hat{V}_{INJ}(s)
\]  

(12)

When the magnitude of T is -20 dB, to keep the amplitude of ̂V_B(s) greater than 5 μV_{RMS}, the amplitude of ̂V_{INJ}(s) should be greater than 50 μV_{RMS}.

To demonstrate the impact of the injection voltage amplitude, a 2 A LDO, TPS7A8300, is used as an example. Figure 13 illustrates a TPS7A8300 loop gain measurement setup.

![Figure 13 – A TPS7A8300 loop gain measurement setup.](image)
An AC voltage source with constant amplitude is injected. The measurement results with different amplitude levels are shown in Figure 14. While the two measurement results match well at frequencies over 10 kHz, there are significant discrepancies at low frequencies. The DC gain is much lower than expected for the TPS7A8300, an LDO with good output regulation. The phase shift at low frequencies is also abnormal.

The voltage injection level is greatly increased. Results are shown in Figure 15.

The gain at lower frequencies is greatly improved by increasing the voltage injection level. However, it is also noted that the loop gain magnitude drops at high frequencies.

While insufficient voltage injection causes signal measurement error, too much voltage injection can cause problems as well. Injecting a voltage source into the loop causes a corresponding fluctuation in the power supply. For the TPS7A8300, too much voltage injection at high frequencies results in the LDO saturating.

For an LDO, it could be the transistor, the gate driver or the error amplifier that saturates during the measurement. When these circuits saturate, the gain drops. The measurement result does not represent the actual loop gain.

For a switch-mode power supply, too much voltage injection could saturate the duty-cycle or error amplifier, falsely triggering over-current protection.

The frequency analyzer used in the measurement provides a programmable voltage source level. Figure 16 shows the graphic user interface (GUI) of the frequency analyzer. The green curve shown in the window sets the voltage source level over the frequency range and can be adjusted by dragging.
In Figure 16, the voltage source amplitude is high at low frequency and decreases as the frequency increases. At frequencies much higher than the bandwidth, the amplitude is increased slightly. Figure 17 shows the comparison of the measurement results with a programmed amplitude versus low amplitude over the complete frequency range. With increased voltage injection at low frequencies, measurement accuracy is much improved.

In Figure 18, the measurement results with the programmed voltage source versus the high-amplitude voltage source are compared.

With a reduced voltage source at high frequencies, the measurement result is more accurate.

There are frequency analyzers that do not offer amplitude programmable voltage sources. They offer other means to adjust the voltage source amplitude over the frequency range.

Figure 19 shows the control menu of the model 3120 frequency analyzer by Venable. By enabling the servo control, the 3120 frequency analyzer can adjust the voltage source amplitude automatically. First, enter the AC volts out value. The AC volts out sets the initial voltage source amplitude for each measurement step. Use a small value, usually less than 10 mV to avoid circuit saturation. Secondly, enable Servo on and select “Smaller of CH1, 2” for Channel. AC volts in sets the servo threshold and AC out step size sets the adjustment.

Figure 19 – Control menu of the model 3120 frequency analyzer by Venable.
step. When the measured signal root mean square value is less than \( AC \) volts \( in \), the voltage source amplitude is increased by \( AC \) out size until the measured signal RMS value is greater than \( AC \) volts \( in \). To avoid damaging the circuit or equipment, set Max \( AC \) volts \( out \) to be less than 50% of the output voltage.

The higher the IF BW, the faster the sweep time. The narrower the IF BW, the better the noise immunity is. Figure 21 shows the loop gain measurement with two different IF BW settings.

Figure 19 – Control menu of frequency analyzer 3120 from Venable as generated by Venable Instruments’ Stability Analysis Software. (Image reproduced with permission from Venable Instruments)

The other important loop gain measurement setting is the intermediate frequency (IF) bandwidth (BW) as shown in Figure 20 or Integration time as shown in Figure 19. Both are related the sweep speed.

Figure 20 – Menu of Frequency analyzer AP300. (Image reproduced with permission from Ridley Engineering Inc.)

Integration time is an equivalent setting to IF bandwidth. The longer the integration time, the better the noise immunity is.

Some systems, such as a power factor correction (PFC) converter, have low loop gain bandwidth. When measuring loop gain for these systems, narrow IF bandwidth or long integration time is necessary for accurate results.

B. Selecting the Right Injection Isolator

Besides the frequency analyzer, another necessary instrument is the injection isolator. There are different types of injection isolators, passive injection isolators, such as shown in Figure 22 and Figure 23, and active injection isolators that support DC transfer function measurement, as shown in Figure 24.
Injection isolators are designed to function over different frequency ranges, so select an injection isolator with the proper frequency range.

C. Preparing the Power Supply for Testing

Figure 9 illustrates an ideal loop gain measurement setup using a voltage injection method. In a real testing situation, it is difficult to find an ideal point with $Z_2 = 0 \ \Omega$. Also, the voltage source injected into the loop has output impedance and is not an ideal voltage source.

i. **Identify the Correct Voltage Injection Point**

Figure 25 shows an equivalent feedback system with voltage injection where $Z_2$ is not zero. Let’s exam how $Z_2$ and $Z_1$, the impedance looking backward and forward respectively from the injection point, affect loop gain measurement.

The loop gain, $T$, of the system in Figure 25 is the product of the divider ratio of $Z_1$ and $Z_2$ and the gain of the voltage-controlled voltage source when $V_{INJ}(s) = 0$ V.

$$T(s) = M(s) \cdot \frac{Z_1(s)}{Z_1(s) + Z_2(s)}$$

(13)

When an AC voltage, $V_{INJ}(s)$, is applied across points B and A, the AC current through $Z_1$ and $Z_2$ is:

$$\dot{I}_{Z_1}(s) = \frac{\ddot{V}_A(s)}{Z_1(s)}$$

(14)

The voltage at point B is

$$\ddot{V}_B(s) = -M(s) \cdot \ddot{V}_A(s) - \frac{Z_2(s)}{Z_1(s)} \cdot \ddot{V}_A(s)$$

(15)

The measured loop gain, $T_M(s)$, is the ratio of $V_B(s)$ and $V_A(s)$:

$$T_M(s) = \frac{\ddot{V}_B(s)}{\ddot{V}_A(s)} = M(s) + \frac{Z_2(s)}{Z_1(s)}$$

(16)

Since $M(s) = T(s) \cdot \left(1 + \frac{Z_2(s)}{Z_1(s)}\right)$, as derived from

(13), Equation (17) shows the impact of $Z_2$:

$$T_M(s) = T(s) \cdot \left(1 + \frac{Z_2(s)}{Z_1(s)} + \frac{Z_2(s)}{Z_1(s)}\right)$$

(17)
To ensure the accuracy of loop gain measurement, the injection point should meet the following requirement:

\[ |Z_2(s)| \ll |Z_1(s)| \]  

(18)

Besides the requirement specified by (18), the voltage injection point should also be practical and accessible. In Figure 26, four possible injection points are shown for a buck converter with voltage mode control. Let’s examine each point.

**Figure 26 – A buck converter with voltage mode control and possible voltage injection points for loop gain measurement.**

Point 1 is between the phase-node of the buck converter and the output inductor. The impedance looking backward, \( Z_2 \), is the equivalent on-resistance of the two MOSFETs while the network looking forward is the output inductor, \( L \), in series with the output capacitor, \( C_{OUT} \). \( Z_2 \) ranges from several \( \text{m} \Omega \) to tens of \( \text{m} \Omega \). \( |Z_1| \) is usually much greater than \( |Z_2| \), meeting the impedance requirement. However, it is not a practical injection point. First, the voltage source of the frequency analyzer or the small resistor inserted in the loop could not survive the high current stress caused by the output inductor current. Secondly, the voltage at the phase-node is a pulse and not continuous.

Point 2 is between the output of the converter and \( R_{TOP} \), the top resistor of the feedback resistor divider. \( |Z_2| \) can be approximated by the impedance of \( R_{TOP} \). \( R_{TOP} \) is usually several \( \text{k} \Omega \). It meets the impedance requirement that \( |Z_2| \ll |Z_1| \).

Point 3 is between the middle point of the resistor divider and the inverting input of the error amplifier. Looking backward from this point, the impedance is that of the two resistors in parallel which is usually in \( \text{k} \Omega \). Because the op amp has negative feedback, the inverting input of the op amp is virtually the same as the positive input, which is a DC reference voltage, \( V_{REF} \). \( V_{REF} \) appears as an AC short, making the equivalent impedance looking forward almost zero. This violates the impedance requirement for loop gain measurement.

Point 4 is between the output of the error amplifier and the PWM comparator. The op amp output can be considered as an ideal voltage source with low output impedance while the input impedance of a comparator is almost infinite. This injection point meets the impedance requirement. However, this point is usually not accessible from outside of the controller. Thus, it is not available for loop gain measurement.

Thus, point 2 is the most widely used voltage injection point for loop gain measurement. For controllers with differential remote sensing, the preferred injection point is between the remote sensing point and the input of the remote sensing differential amplifier.

Figure 27 shows a realistic loop gain test setup example.

**Figure 27 – A real example of loop gain measurement setup.**
R\textsubscript{INJ} is a small resistor connected between point B and point A. V\textsubscript{SRC}(s) is the voltage source generated by the frequency analyzer and coupled across point A and point B through the injection isolator. R\textsubscript{OUT} is the output resistance of the voltage source. C is the DC blocking capacitor of the injection isolator.

**ii. Includes All Output Feedback Paths**

Loop gain is not only a stability indicator. Loop gain measurement results also assist the power supply designer to improve the compensation network design and output transient performance. This section will discuss how to measure loop gain so that it is useful for load transient response improvement.

The load transient response closely correlates to the peak output impedance of the power supply [2]. The lower the peak output impedance, the better the load transient response is.

A power supply system can be simplified as the block diagram in Figure 28.

**Figure 28 – Simplified block diagram of a power supply system with output load transient.**

\( Z\textsubscript{OUT(OPEN)}(s) \) is the open-loop output impedance of the power supply with the feedback loop disabled. The closed-loop output impedance of the power supply is the open-loop impedance divided by one plus the loop gain, T(s):

\[
Z\textsubscript{OUT(CLOSED)}(s) = \frac{Z\textsubscript{OUT(OPEN)}(s)}{1 + T(s)} \tag{19}
\]

At frequencies lower than the control bandwidth, the high loop gain reduces the output impedance. At frequencies higher than the control bandwidth, the feedback loop cannot provide a benefit to reducing output impedance. Thus the higher the control bandwidth, the lower the peak closed-loop output impedance and thus the better the output transient response [2].

A power supply designer can improve the output load transient response by increasing the bandwidth of the loop gain, T(s), as long as the system has sufficient stability margin. For a system with multiple feedback paths, this statement is true when T(s) is the sum of all the feedback path gains as indicated in Equation (20) and Figure 29.

\[
T(s) = H_1(s) + H_2(s) \tag{20}
\]

**Figure 29 – Simplified block diagram of multiple-feedback system with output load transient.**

When the loop gain is measured with one or multiple feedback paths remaining closed, the measured loop gain, T\textsubscript{M}(s), as shown in Figure 30 might no longer be useful for output transient improvement.

**Figure 30 – Multiple-feedback system loop gain with partial feedback path closed.**

The measured loop gain T\textsubscript{M}(s) in Figure 30 is:

\[
T\textsubscript{M}(s) = \frac{H_1(s)}{1 + H_2(s)} \tag{21}
\]

T\textsubscript{M}(s) measured with the setup shown in Figure 30 is not useful to assist the power supply designer for load transient improvement.
The following examples show different ways to measure the loop gain of a power supply with multiple output feedback paths.

The first example is an isolated converter using a shunt regulator, TL431, for secondary-side output regulation and compensation. Figure 31 shows the secondary-side error amplifier and compensation network with voltage injection for loop gain measurement.

Figure 31 – Secondary-side error amplifier and compensation network of an isolated converter with voltage injection for loop gain measurement.

There are two feedback paths from the output of the converter. One passes through the resistor divider and TL431, the error amplifier. The other path goes through the opto-coupler pull-up resistor, \( R_{\text{up}} \), and the TL431. In Figure 31, the injection point includes both feedback paths. Figure 32 shows the loop gain bandwidth to be 15kHz with the setup from Figure 31.

The total output capacitance is 424 \( \mu \text{F} \). Using Equation (3), we can estimate the overshoot and undershoot with a load step of 3.2 A.

\[
V_p = \frac{\Delta I}{2\pi f_C \cdot C_{\text{OUT}}} = \frac{3.2 \, A}{6.28 \cdot 15 \, \text{kHz} \cdot 424 \, \mu\text{F}} \approx 80 \, \text{mV}
\]

Figure 33 shows that the experimental overshoot and undershoot are about 80 mV, excluding the switching output ripple. Experiment results match the calculation well.

Figure 32 – Measured loop gain with test setup of Figure 31.

Figure 33 – Measured load transient response.

The second example is loop gain measurement of a buck converter with D-CAP™ control. Figure 34 shows a simplified block diagram of a buck converter with D-CAP control. Additional ripple
Injection is necessary when low equivalent series resistor (ESR) ceramic capacitors are used for output capacitors. Resistor, \( R_p \), and capacitor, \( C_p \), are connected between the phase-node and output and a triangular ripple is formed across the capacitor, \( C_p \). This triangular ripple is AC-coupled by the capacitor \( C_{ff} \) to the FB pin. The FB pin voltage is compared to the internal reference voltage generating the PWM gate drive. From the output voltage to the FB pin, there are two feedback paths: one is through the resistor, \( R_{TOP} \), of the divider and the other is through the ripple injection circuit capacitors, \( C_p \) and \( C_{ff} \).

Both setups in Figure 35 and Figure 36 meet the impedance requirement. The voltage injection point in Figure 35 includes only the path through \( R_{TOP} \). The voltage injection point in Figure 36 includes both feedback paths. The loop gain measurement results are different, as shown in Figure 37.

**Figure 34** – Block diagram of a buck converter with D-CAP™ control.

**Figure 35** and **Figure 36** show two different loop gain measurement setups for the buck converter with D-CAP control.

**Figure 37** – Measured loop gain with two different setups.
The bandwidth of the loop gain measured with the voltage injection point in Figure 35 is 14 kHz. Using Equation (4), the time delay from the start of load step to the peak of the overshoot, \( t_p \), is predicted to be 17.9 \( \mu \text{sec} \). The bandwidth of the loop gain measured with the voltage injection point in Figure 36 is 52 kHz. The predicted \( t_p \) is 4.8 \( \mu \text{sec} \).

An output load transient response was conducted and Figure 38 shows the transient response waveform of the same buck converter.

\[ t_p = 5.2 \mu \text{sec} \]

\[ V_{\text{OUT}}, 20 \text{mV/div} \]

\[ I_{\text{STEP}}, 2.5 \text{A/div} \]

*Figure 38 – Output load transient response of a buck converter with D-CAP™ control.*

The measured \( t_p \) is 5.2 \( \mu \text{sec} \). The loop gain measured with setup 2, which includes both output feedback paths, matches the load transient response result. The power supply designer can use the loop gain measured with setup 2 to tune the value of \( R_p, C_p \) and \( C_{\text{ff}} \) to improve the load transient response.

To make the measured loop gain useful for compensation network tuning and load transient performance improvement, it should use a voltage injection point that includes all output feedback paths.

### iii. Maintain Static Operating Point

Many power supplies have a frequency response that varies with the DC operating point. When conducting loop gain measurements, it is desirable to keep the DC operating point the same or similar to that during normal operation. Thus, a small resistor, \( R_{\text{INJ}} \), is connected between point B and point A as shown in Figure 27. By adding \( R_{\text{INJ}} \), regardless of the DC output resistance of the injection isolator, the resistance between point B and point A is smaller than \( R_{\text{INJ}} \). While a small \( R_{\text{INJ}} \) is desired to keep the output DC voltage close to that in normal operation, \( R_{\text{INJ}} \) being too small would make the amplitude of \( V_{\text{INJ}} \) lower given the same source voltage. It is recommended that \( R_{\text{INJ}} \) is selected so that the output change due to adding \( R_{\text{INJ}} \) is less than 2% of the output at normal operation but not less than 10 \( \Omega \). When a small value \( R_{\text{INJ}} \) is used, it is necessary to adjust the amplitude of the voltage source.

For advanced control topologies in which the FB is used for PWM modulation directly, the ripple at the FB pin affects the output DC set-point. The slope of the ripple also affects the frequency response. It is necessary to keep the ripple during testing the same or similar to that during normal operation.

D-CAP control is one of the advanced topologies that use the voltage at FB for PWM modulation. Figure 39 shows the simulated voltage waveforms of the buck converter in Figure 34 with \( R_p = 2 \text{k} \Omega \) and \( C_p = 100 \text{nF} \). The amplitude of the triangular ripple across the capacitor, \( C_p \), is 13.9 mV. The ripple voltage at FB is the sum of the output ripple and the ripple across \( C_p \). When low ESR ceramic capacitors are used for output capacitors, the output ripple can be ignored. The ripple voltage at FB is dominated by the ripple across \( C_p \). In this case, the amplitude of the ripple at FB is 13.9 mV.

\[ V_{\text{FB}} \]

\[ V_{\text{REF}} \]

\[ \text{PWM} \]

*Figure 39 – Simplified waveforms during normal operation with \( R_p = 2 \text{k} \Omega \) and \( C_p = 100 \text{nF} \).*

The amplitude of the ripple at FB affects the output set-point voltage. The falling slope of the ripple at FB determines the PWM modulation gain and thus affects loop gain.

Figure 37 in the previous section shows the proper voltage injection point for D-CAP control. A 20 \( \Omega \) resistor is inserted as shown in Figure 40.
Figure 40 – Loop gain measurement setup for a buck converter with D-CAP™ control with \( R_{\text{INJ}} = 20 \, \Omega \).

At the switching frequency, the impedance of \( C_p \) is much smaller than \( R_P \) and \( R_{\text{INJ}} \) and the pulsating voltage from “Phase” to \( V_{\text{OUT}} \) is distributed between \( R_P \) and \( R_{\text{INJ}} \). With \( R_P = 2 \, k\Omega \) and \( R_{\text{INJ}} = 20 \, \Omega \), the amplitude of the ripple across \( R_{\text{INJ}} \) is approximately 120 mV. The ripple across \( R_{\text{INJ}} \) and \( C_p \) is coupled to the FB pin by the capacitor \( C_{\text{ff}} \).

Figure 41 shows that the ripple at FB is greatly changed from Figure 39 because of the pulsating ripple across \( R_{\text{INJ}} \). The ripple voltage at FB is so high that it even triggers the over-voltage protection. It is necessary to minimize the ripple introduced by \( R_{\text{INJ}} \) during loop gain measurement.

One way to reduce the ripple across \( R_{\text{INJ}} \) is to use a larger value for \( R_P \). This also requires a smaller value for \( C_p \) to keep the same time constant of the ripple injection circuit. Figure 42 shows the simulated voltage waveforms with \( R_{\text{INJ}} = 20 \, \Omega \), \( R_P = 10 \, k\Omega \) and \( C_P = 20 \, nF \). To further minimize the ripple across \( R_{\text{INJ}} \), a bypassing capacitor can be added in parallel to \( R_{\text{INJ}} \) as shown in Figure 42. Figure 43 shows that the ripple across FB is greatly reduced by the bypass capacitor. The ripple at the FB pin is similar to that during normal operation as shown in Figure 39.

The red curve in Figure 37 is the magnitude plot of the loop gain measured with \( R_{\text{INJ}} = 20 \, \Omega \) with a 220 nF bypass capacitor in parallel. It proves that with the 220 nF bypass capacitor and the proper voltage injection point, the loop gain of the advanced control topology is measurable.

D. Considerations for Connection Wires

With the equipment ready and the power supply prepared for the loop gain measurement, we can connect them together to conduct the measurement.
i. How Connection Wires Affect Loop Gain Measurement

Connection wires are soldered to the two ends of the resistor, $R_{INJ}$, with the injection isolator and signal receivers clipped to the other end of the wires. Figure 44 is a photo of a real setup with one pair of connection wires.

![Figure 44 – Loop gain measurement setup with one pair of connection wires.](image)

With one pair of connection wires, the loop gain is measured with different connection wire lengths. Results are compared in Figure 45.

![Figure 45 – Loop gain measurement results with different connection wire lengths.](image)

The two measurement results have significant discrepancies at high frequencies. While the measurement results with long wires show only 8 dB gain margin, the result with short wires shows less phase drop at high frequencies. The gain margin is greater than 10 dB.

It is necessary to understand how the connection wires affect loop gain measurement. Figure 46 shows a detailed block diagram of a power supply under a loop gain measurement in which the connection wire impedance is taken into consideration.

![Figure 46 – Block diagram of a power supply under loop gain measurement.](image)

The two connection wires are in series with the $R_{INJ}$. When a voltage source is applied to the connection wires, there is a voltage drop across the wires. The voltage across $R_{INJ}$ is $\bar{V}_{INJ}(s)$. The voltage drop across the wires is:

$$\bar{V}_{DROP}(s) = \frac{Z_L(s)}{Z_{INJ}(s)} \bar{V}_{INJ}(s) \quad (22)$$

$Z_L(s)$ is the impedance of the connection wire and $Z_{INJ}(s)$ is the impedance between point B and point A. The signal receivers A and B include this voltage drop in the measurement.

The voltage measured by receiver B is:

$$\bar{V}_B(s) = \bar{V}_B(s) + \bar{V}_{DROP}(s) = \frac{T(s)}{1 + T(s)} \bar{V}_{INJ}(s) \quad (23)$$

The voltage measured by receiver A is:

$$\bar{V}_A(s) = \bar{V}_A(s) = \frac{-1}{1 + T(s)} \bar{V}_{INJ}(s) \quad (24)$$
The measured loop gain, $T'_M(s)$ is

$$T'_M(s) = \frac{-\bar{V}_B'(s)}{V_A'(s)} = \frac{Z_{INJ}(s) \cdot T(s) + Z_L(s) \cdot (1 + T(s))}{Z_{INJ}(s) + Z_L(s) \cdot (1 + T(s))}$$  \hspace{1cm} (25)$$

At low frequencies, when $|T(s)| >> 1$, the measured loop gain can be approximated by:

$$T'_M(s) = \frac{Z_{INJ}(s) \cdot T(s) + Z_L(s) \cdot (1 + T(s))}{Z_{INJ}(s) + Z_L(s) \cdot (1 + T(s))}$$

$$= \frac{(Z_L(s) + Z_{INJ}(s)) \cdot T(s)}{Z_{INJ}(s) + Z_L(s) + Z_L(s) \cdot T(s)}$$

$$= \frac{T(s)}{1 + \frac{Z_L(s)}{Z_{INJ}(s) + Z_L(s)}}$$  \hspace{1cm} (26)$$

At high frequencies, when $|T(s)| << 1$, the measured loop gain can be approximated by:

$$T'_M(s) = \frac{Z_{INJ}(s) \cdot T(s) + Z_L(s) \cdot (1 + T(s))}{Z_{INJ}(s) + Z_L(s) \cdot (1 + T(s))}$$

$$= \frac{(Z_{INJ}(s) \cdot T(s) + Z_L(s) \cdot (1 + T(s))}{Z_{INJ}(s) + Z_L(s) \cdot (1)}$$

$$= T(s) + \frac{Z_L(s)}{Z_{INJ}(s) + Z_L(s)}$$  \hspace{1cm} (27)$$

When $|T(s)| < \frac{Z_L(s)}{Z_{INJ}(s) + Z_L(s)}$, Equation (27) can be further simplified by:

$$T_M(s) = T(s) + \frac{Z_L(s)}{Z_{INJ}(s) + Z_L(s)}$$

$$= \frac{Z_L(s)}{Z_{INJ}(s) + Z_L(s)}$$  \hspace{1cm} (28)$$

Equations (26) and (28) help explain the discrepancy of the loop gain measurement results in Figure 45. It is obvious that lower wire impedance will render better measurement accuracy.

While shortening the connection wires will improve measurement accuracy, there are occasions where the connection wires cannot be shortened. For those occasions, a second pair of connection wires is recommended as shown in Figure 47.

By using separate wires for the voltage injection and signal measurement, the voltage drop across the connection wires is excluded from the signal measurement, as illustrated in Figure 48.

To verify the proposed solution, the loop gain is measured with two pairs of connection wires 6 inches long and compared to the earlier measurement in Figure 49. The measurement results with two pairs of connection wires and with one pair of short connection wires are similar.
Figure 49 – Loop gain measurement results of short and long connection wires and two pairs of connection wires.

**ii. Considerations for Bode™ Box**

Frequency analyzer vendors provide injection isolators, such as the Bode™ box, with the signal receiver connection in Figure 23. They are convenient tools that reduce the total connection points from six to three as shown in Figure 50. However, the signal receivers are connected to the output of the injection isolator. Thus, voltage drops across the cables from the output of the injection isolator to the power supply are included in measurements. Figure 51 compares the loop gain measurement results using a Bode™ box with 3 feet long cables to that with two pairs of connection wires. The voltage drop along the cable gives an erroneous gain margin of 1.3 dB.

Figure 50 – Block diagram of the loop gain measurement setup with Bode™ box.

Figure 51 – Loop gain measurement results with two pairs of connection wires and with Bode™ box.
For better accuracy, keep the connection cables as short as possible. For occasions when the connection cable length cannot be reduced, it is recommended to connect the signal receivers directly to the power supply.

E. Designate the Correct Reference Point

A reference point must be designated for all other nodes to refer to. For a loop gain measurement, the reference leads should be connected to the designated reference point. In most applications, this reference point is the ground. Figure 52 shows a BNC cable used for loop gain measurements.

![Figure 52 – A BNC-to-alligator coax cable used for receiver A and B.]

The black clip is connected to the reference point while the red clip is connected to the signal point. Figure 53 is a photo of an actual loop gain measurement setup.

![Figure 53 – A photo of a loop gain measurement setup. (Image used with permission from Venable Instruments and Ridley Engineering)]

For a single-ended system it is recommended to connect the reference leads to the signal ground of the controller. For a controller with an output remote differential sensing amplifier, it is recommended to use the negative input of the differential amplifier as the reference point and inject voltage at the positive input of the differential amplifier.

There are applications that the reference lead for the loop gain measurement is not connected to the system ground. Figure 54 shows an application circuit of shunt regulator LM4041-ADJ.

![Figure 54 – An application circuit of LM4041-ADJ.]

The loop gain was first measured using the typical measurement setup shown in Figure 10. Figure 55 shows the loop gain measurement result.

![Figure 55 – Loop gain measurement results using ground as the reference point and voltage injected between VOUT and top of the feedback resistor divider.]

The loop gain measurement result does not provide direct information on stability. It is necessary to examine the functional block diagram of LM4041-ADJ.
According to Figure 56, if the system ground is designated as the reference point, there are two feedback paths from the anode and the output. One is through the resistor divider; the other path is through the internal bandgap, $V_{REF}$, as shown in a simplified block diagram in Figure 57. The latter path is inside of the silicon and thus cannot be included in the voltage injection point. As discussed in Section III.C.ii on page 6-12, for a loop gain to be useful for compensation network tuning, all feedback paths should be included in the voltage injection point.

To solve the problem, the cathode, $V_{OUT}$, is designated as the reference point and the voltage injection point is between the ground and the bottom of the resistor $R_{BOTTOM}$ as shown in Figure 58.

In Figure 59, the positive error amplifier input is the virtual AC ground and the cathode is the output of the power supply. There is only one feedback path. The curves in Figure 59 are the loop gain measurement results with the test setup illustrated in Figure 58. Phase margin is only $20^\circ$. Based upon loop gain measurements, the output capacitor value is reduced and a phase-boosting forward capacitor is added in parallel to the bottom resistor to improve stability.

**F. Measuring Loop Gain for Power Factor Correction Converter**

Power factor correction (PFC) converters are required for many AC/DC adaptors and front-end converters. The PFC converter poses unique challenges for loop gain measurement because of the low loop gain bandwidth, high output voltage and the alternating input voltage.
First, the injection isolator should have a frequency range as low as 1 Hz. All PFC converter voltage loops have a bandwidth lower than 10 Hz to avoid distortion from the 100 Hz to 120 Hz output ripple. The frequency range can be found on the injection isolator as shown in Figure 22, Figure 23 and Figure 24.

Secondly, the frequency analyzer can accommodate a common mode voltage range greater than 400 V. Most PFC converter outputs are around 400 V. Figure 60 shows the control menu of a frequency analyzer with the input signal voltage range selection. “AC < 500V” is selected for Input coupling and sets the frequency analyzer to support up to 500 V DC voltage. It is a must to confirm that the frequency analyzer can support the high input voltage range and to set up the frequency analyzer correctly.

When a frequency analyzer with a high voltage rating is not available, an oscilloscope can be used for loop gain measurement. Please refer to [6] for instructions on how to measure loop gain with an oscilloscope. Figure 61 shows the oscilloscope screen shot of the resulting response at point A and point B with voltage injection.

The magnitude and phase shift of the loop gain can be derived from the oscilloscope waveforms. The input of the PFC converter is a semi-sinusoidal waveform. To make the measurement more accurate, a DC input is recommended for the loop gain measurement.

Figure 62 shows the measured loop gain of a PFC SEPIC converter at different DC input voltages.

**IV. MEASURING LOOP GAIN OVER THE WHOLE OPERATING RANGE**

For most power supplies, the frequency response varies with DC operating conditions. Figure 63 shows the loop gain of an active clamp forward converter under different load conditions as an example. Loop gain varies dramatically when the converter enters discontinuous current mode (DCM).

Compensation networks should be designed so that the power supply is stable over the whole operating range. Loop gain should be measured to verify stability over the following ranges, including, but not limited to:

1. Input voltage range
2. Output load current range
3. Temperature range
4. Output voltage range if output is adjustable.
Figure 63 – Loop gain of an active clamp forward converter under different load conditions.

V. SUMMARY

This paper reviewed the voltage injection method to measure the loop gain of a power supply. Equipment limitations and practical issues were discussed and analyzed. Solutions were proposed and explained. Following the guidelines provided in this paper will result in accurate loop gain measurements. The measurement results are then used to verify system stability and to guide the design of the compensation network to achieve the best transient performance.

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